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ERRATA.

P. 113, l 9 from bottom, *for* 'qualitative,' *read* 'quantitative.'

P. 270, l. 4 from bottom, *after* 'narrower limits,' *insert* 'if we apply the important general principle, that objective conditions are independent of variations in our cognitive procedure.' [An addition in Ed. 2, overlooked in adapting the translation to it.]

P. 421, l. 10 from bottom, *for* 'Ideas,' *read* 'ideas.'

P. 429, l. 2 from bottom, *for* 'in,' *read* 'of.'

BOOK I.

OF THOUGHT (PURE LOGIC).

INTRODUCTION.

I. At almost every moment of our waking life our senses are giving rise to various ideas, simultaneous or immediately successive. Among these ideas there are many which have a right thus to meet in our consciousness, because in the reality from which they spring their occasioning causes always accompany or follow one another; others are found together in us merely because, within the external world to whose influence we are accessible, their causes were as a fact simultaneous though not so inwardly connected as to ensure their similar combination in every recurring instance. This mixture of *coherent* with merely *coincident* ideas is repeated, according to a law which we derive from self-observation, by the current of memory. As soon as any idea is revivified in consciousness, it reawakens the others which have once accompanied or succeeded it, whether the previous connexion was due to a coherence in the matter of the ideas, or to the mere simultaneity of otherwise unconnected irritants. It is upon the first fact, the recovery of what is coherent, that our hope of arriving at knowledge is based: the second, the ease with which coincident elements hang together and push one another into consciousness, is the source of error, beginning with that distraction which hinders our thoughts from following the connexion of things.

II. The ever-changing whole of processes which results from this peculiarity of our psychical life is what we call the *current of ideas*. If it were in our power to observe this whole with omniscience, we should discover in every instance of it, in the sober course of waking thought, in the dreams of sleep, in the delirium of disease, a necessary connexion between its members. The application of universal laws, which hold good of all souls alike, to the particular conditions which are found to vary in each single instance, would exhibit the course of these inner processes in the light of an inevitable result. If we knew the permanent characteristics of a single particular soul, if we had a view of the form and content of its whole current of ideas up to the present time, then, the moment it had produced a first and a second idea on occasion of external irritants, we should be able to predict on the basis of those universal laws what its third and fourth idea in the next moment must be. But in any other soul, whose nature, past history, and present condition were different, the same first and second idea, developed at this moment by a similar external irritant, would lead with a similar necessity in the next moment to an entirely different continuation. An investigation of the subject would therefore have to recognise that any given current of ideas was necessary for that particular soul and under those particular conditions; but it would not discover any mode of connexion between ideas which was universally valid for all souls. And just because, under their respective conditions, every such series of ideas hangs together by the same necessity and law as every other, there would be no ground for making any such distinction of value as that between truth and untruth, which would place one group in opposition to all the rest.

III. Universal validity and truth are the two prerogatives which even ordinary language ascribes and confines to those connexions of ideas which *thought* alone is supposed to establish. Truth is familiarly defined as the agreement of ideas and their combinations with their object and its relations. There may be objections to this form of expression, which this is not the place to consider; but it will be innocuous if we modify it and say, that connexions of ideas are true when they follow such relations in the matter of the ideas as are identical for all consciousness, and not such merely empirical coincidence of impressions as takes one form in one consciousness, another in another. Now our ideas are excited in the first instance by external influences, and this leads us to regard thought as a reaction of the mind upon the material supplied by those influences and by the results of their interaction already referred to. The thinking mind is not content to receive and acquiesce in its ideas as they were originally combined by casual coincidence or as they are recombined in the memory: it sifts them, and where they have come together merely in this way it does away with their coexistence: but where there is a material coherence between them, it not only leaves them together but combines them anew, this time however in a form which adds to the fact of their reconnexion a consciousness of the ground of their coherence.

IV. I will connect the indispensable explanation of what I have just said with the elucidation of some obvious objections. It is not without a purpose, which I admit, that while I have represented the rest of the current of our ideas as a series of events, which happen in us and to us according to universal laws of our nature, I have represented thought as an activity which our mind exercises. There have been persons who doubted whether this opposition has any real significance, either in itself or in relation to thought; whether everything that we are in the habit of calling activity is not rather one amongst the events which simply take place in us. So wide a question does not of course admit of being decided here: if therefore I hold to the significance of the opposition, and expressly describe thought as an activity, this must be regarded as a presupposition which awaits proof elsewhere, but is at present open to dispute. It is necessary for the connexion of the whole to which I wish this view of thought to serve as an introduction; and it seems to me to be permissible, because, while it will determine decisively the general colour of my exposition, it will not alter unnaturally the internal relations of its subject-matter.

V. It is more profitable to meet another form of the same objection, which allows the general validity of the opposition in question, but holds that there is no occasion to apply it here. The connexion of the coherent (it is said), that is to say, Truth, is brought about in the same way, only not quite so soon, as the erroneous conjunctions of the casually coincident. The course of things itself ensures that those events which are inwardly connected exercise their combined effect upon us with incomparably greater frequency than those which have no inward bond, but are variously thrown together by chance. Owing to this more frequent repetition the connexion of what is coherent becomes fixed in us, while that of the merely coincident is loosened and disturbed by its want of uniformity. In this way the separation of the coherent from the incoherent, which we thought it necessary to ascribe to a special reaction of the mind, is effected by the current of ideas itself; and thus brutes, like men, acquire the mass of well-grounded information which regulates the daily life of both. It would be superfluous to point out that this account is perfectly correct if it purport to be no more than a history of the acquisition just mentioned; but I think it can be shown that this acquisition is just what neither characterises nor exhausts the specific work of thought.

VI. There is a common opinion which reserves the faculty of thought to man and denies it to brutes. Without seriously deciding for or against this view, I will use it for the convenience of my explanation. In the soul of a brute, which on this theory would be confined to a mere current of ideas, the first impression of a tree in leaf would only produce a collective image; there would be no power or even impulse to seek for any special coherence between its parts. Winter strips the tree of its leaves, and on a second observation the brute finds only a part of the former collective image, which tries to reproduce the idea of the rest, but is hindered by the present appearance. When the return of summer restores the old state of things, the renewed image of the whole tree in leaf may not, it is true, have the simple unquestioning unity of the first observation ; the recollection of the second intervenes, and separates it into the part which remained and the part which changed. I do not think we can say what precisely would take place in the soul of the brute under these circumstances; but even if we ascribe to it the additional faculty of comparing and surveying the current of its ideas and expressing the result, the expression could not say more than the fact that two observations were at one time together, at another not. Now it is true that the man, when he gives the name of leafy and leafless tree to the same observed objects, is only expressing the same facts; but the apprehension of the facts, which is indicated by these habitual forms of speech, involves a mental operation of quite a different kind. The name of the tree, to which he adds and from which he takes away the descriptive epithet, signifies to him, not merely a permanent as opposed to a changeable part in his observation, but the thing in its dependence on itself and in opposition to its property. The effect of bringing the tree and its leaves under this point of view is, that the relationship of thing and property appears as the justification both for separating and for combining these ideas, and thus the fact of their coexistence or non-coexistence in our consciousness is referred to the real condition upon which their coherence or non-coherence at the moment depends.

The same consideration may be extended to other instances. In the soul of the dog the renewed sight of the raised stick recalls the idea of his previous pain: the man, when he makes the judgment, 'the blow hurts,' does not merely express the fact of connexion between the two occurrences, but justifies it. For in representing the blow as the subject from which the pain proceeds, he clearly exhibits the general relationship of cause and effect as the ground, not of the mere follow one another. Lastly, the expectation of pain in the dog may be accompanied by the recollection that by running away, to which he was led before by an involuntary instinct, the pain is diminished; and this fresh conjunction of ideas will doubtless make him repeat the salutary operation as surely as if he reflected and concluded that threatening blows are prevented by distance, that a blow threatens him, and that therefore he must run away. But the man who in a similar or more serious case actually frames such a conclusion, performs an entirely different mental operation; in expressing a universal truth in the major premiss, and bringing a particular instance under it in the minor, he not merely repeats the fact of that salutary connexion between ideas and expectations by which the brute is affected, but he justifies it by an appeal to the dependence of the particular upon its universal.

VII. These examples, which embrace the familiar forms of thought, concept, judgment, and syllogism, will I think have made sufficiently clear what is the surplus of work performed by thought over and above the mere current of ideas; it always consists in adding to the reproduction or severance of a connexion in ideas the accessory notion of a ground for their coherence or non-coherence. The value of this work remains entirely the same, whatever opinion we may hold of its genesis: if we preferred to regard it, not as the outcome of a special activity, but only as a finer product of the mere current of ideas operating under favorable circumstances, we should then confine the name of thought to that particular stage of development in the current at which it gives birth to this new achievement. The peculiarity of thought, then, which will govern the whole of our subsequent exposition, lies, not in the mere correspondence of our apprehension with fact, but in the production of those accessory and justificatory notions which condition the form of our apprehension. We do not deny that, apart from thought, the mere current of ideas in the brute gives rise to many useful combinations of impressions, correct expectations, seasonable reactions; on the contrary, we admit that much even of what the man calls his thought is really nothing but the play of mutually productive ideas. And yet perhaps there is still some difference here. The sudden inspirations which enable us to make a decision in a moment, the rapid survey which arranges a complicated material in almost less time than would seem sufficient for the bare observation of its parts, the invention of the artist which remains unconscious of the grounds by which it is impelled, all these seem to us to be effects, not of a current of ideas which has not yet become thought, but of abbreviated thought. In the cases where these surprising operations are successful, they are so because mature thought has already in other cases developed into full-grown habits those accessory notions, which bring the impressions under universal principles of coherence; and this, like all other accomplishments which have acquired the ease of a second nature, has behind it a forgotten time of laborious practice.

VIII. In the examples which I have employed, the accessory notions, by which we justify the connexions of ideas, obviously coincided with certain presuppositions about the connexions of the real with which we cannot dispense. Without the opposition of things and their properties by which the whole matter of perception is articulated, without the assumption of a succession of effects from causes, and without the determining power of the universal over the particular, we could have no apprehension whatever of the reality which surrounds us. From this point of view, then, it seems a self-evident proposition that the forms of thought and the accessory notions which give them vitality are immediate copies of the universal forms of being and its connexions, and this *real* validity of thought and its operations has, in fact, been frequently maintained. The opposite view to this, which as its exact counterpart we might expect to find, has never been put forward so unreservedly. To the unprejudiced mind it is too natural to regard thought as a means of comprehending the real, and any interest in the scientific investigation of its processes is too dependent upon this presupposition for any one to assert the merely *formal* validity of all logical activity in the sense of denying *all* relation between it and the nature of being. Those, therefore, who have regarded the forms and laws of thought as being primarily peculiar results of our mental organisation, have not wholly excluded their correspondence with the essence of things; they have only denied the off-hand view which would make them immediate copies of the forms of being.

IX. In regard to this much debated question an introduction can only take up a provisional position. We shall certainly be right to confine our attention at starting to what is already clear, and to leave for a later stage the decision of uncertainties. Let us then go no further than the natural presupposition which regards thought as a *means* to knowledge. Now a tool must fulfil two conditions, it must fit the thing and it must fit the hand. It must fit the thing; that is, it must be so constructed as to approach, reach, and get hold of, the objects which it is to work upon, and find in them a point from which to operate; this requirement is satisfied in the case of thought if we admit that its forms and laws are no mere singularities of our mental organisation, but that, taken as they are, they show a constant and regular adaptation to reality. If, again, a tool is to fit the hand, it 2 must have such other structural properties as make it easy to grasp, hold, and move, having regard to the power, attitude, and position of the person who is to use it; and in the case of thought this second indispensable requirement limits the scope of the previous admission. Only a mind which stood at the centre of the real world, not outside individual things but penetrating them with its presence, could command such a view of reality as left nothing to look for, and was therefore the perfect image of it in its own being and activity. But the human mind, with which alone we are here concerned, does not thus stand at the centre of things, but has a modest position somewhere in the extreme ramifications of reality. Compelled, as it is, to collect its knowledge piece-meal by experiences which relate immediately to only a small fragment of the whole, and thence to advance cautiously to the apprehension of what lies beyond its horizon, it has probably to make a number of circuits, which are immaterial to the truth which it is seeking, but to itself in the search are indispensable. However much, then, we may presuppose an original reference of the forms of thought to that nature of things which is the goal of knowledge, we must be prepared to find in them many elements which do not directly reproduce the actual reality to the knowledge of which they are to lead us: indeed there is always the possibility that a very large part of our efforts of thought may only be like a scaffolding, which does not belong to the permanent form of the building which it helped to raise, but on the contrary must be taken down again to allow the full view of its result. It is enough to have thus raised a preliminary expectation, with which we wish our subject to be met; any more definite decision as to the limits which separate the formal validity of our thought from its real significance must await the further course of our enquiries.

X. I have purposely avoided postponing those enquiries by discussions which seem to me to encumber unjustifiably the approach to logic. What particular tone of mind is required for successful thinking, how the attention is to be kept up, distraction avoided, torpidity stimulated, precipitation checked, all these are questions which no more belong to the field of logic than do enquiries about the origin of our sense-impressions and the conditions under which consciousness in general and conscious activity is possible. We may presuppose the existence of all these things, of perceptions, ideas, and their connexion according to the laws of a psychical mechanism, but logic only begins with the conviction that the matter cannot end here; the conviction, that between the combinations of ideas, however they may have originated, there is a difference of truth and untruth, and that there are forms to which these combinations *ought* to answer and laws which they *ought* to obey. It is true that we may attempt by a psychological investigation to explain the origin of this authoritative consciousness itself; but the only standard by which the correctness of our results could be measured would be one set up by the very consciousness to be investigated. The first thing, then, that has to be ascertained is, *what* the contents of this authoritative conviction are; the history of its growth can only have the second place, and even then must conform to requirements of its own imposing.

XI. Having now said all that seemed necessary by way of introduction to my exposition, I will add a preliminary survey of its order. The examples which we have hitherto employed lead naturally to a first principal part, which, under the name of pure or formal logic, is devoted to thought in general and those universal forms and principles of thought which hold good everywhere, both in judging of reality and in weighing possibility, irrespective of any difference in the objects. We have only to mention concept, judgment, syllogism, to see how naturally these forms exhibit themselves as different stages of one and the same activity; and in treating of pure logic I shall endeavour to emphasise this thread of connexion somewhat more strongly than is usually done. The various forms of thought will be arranged in an ascending series, in which each higher member attempts to make good a defect in the preceding one, due to its failure to satisfy, in regard to its own particular problem, the general impulse of thought to reduce coincidence to coherence. This series will advance from the simplest formation of single impressions to the conception of the universal order in which this general impulse would lead us, if it were possible, to comprehend the world.

XII. Pure logic itself will show and explain that the forms of concept, judgment, and syllogism are to be considered primarily as *ideal* forms, which give to the matter of our ideas, if we succeed in arranging it under them, its true logical setting. But the different peculiarities of different objects offer resistance to this arrangement; it is not clear of itself what sum of matter has a claim to form a determinate concept and be opposed to another, or which predicate belongs universally to which subject, or how the universal law for the arrange-

ment of a manifold material is to be discovered. Applied logic is 2
concerned with those methods of *investigation* which obviate these defects. It considers hindrances and the devices by which they may be overcome; and it must therefore sacrifice the love of systematisation to considerations of utility, and select what the experience of science has so far shown to be important and fruitful. The boundlessness of the field of observation unfortunately makes it impossible to exhibit as completely as could be wished this most brilliant part of logic, which the inventive genius of modern times has made peculiarly its own.

XIII. The third part will be devoted to *knowledge*, that is, to the 3
question which our introduction touched without answering, the question how far the most complete structure of thought which all the means of pure and applied logic enable us to rear, can claim to be an adequate account of that which we seem compelled to assume as the object and occasion of our ideas. The currency in ordinary minds of this opposition between the object of our knowledge and our knowledge of that object makes me employ it without hesitation to describe in a preliminary way the subject of this third section; it may be left to the section itself to disclose the difficulties which this apparently simple antithesis involves, and to determine accordingly the more precise limits of the problems with which it has to deal.

Knowledge = an adequate account of that which we seem compelled to assume as the object & occasion of our ideas

CHAPTER I.

The Theory of the Concept.

A. *The formation of impressions into ideas.*

1. It is in relations within a manifold that the operations of thought usually show themselves to us, and we might therefore expect to have to look for the most original of its acts in some simplest form of connexion between two ideas. A slight reflexion, however, suggests to us to go a step further back. It is easy to make a heap out of nothing but round stones, if it is indifferent how they lie; but if a structure of regular shape is to be built, the stones must be already so formed that their surfaces will fit firmly together. We must expect the same in the case before us. As mere internal movements, the states which follow external irritants may exist side by side in us without further preparation, and act upon each other as the general laws of our psychical life allow or enjoin. But if they are to admit of combination in the definite form of a *thought*, they each require some previous shaping to make them into logical building-stones and to convert them from *impressions* into *ideas*. Nothing is really more familiar to us than this first operation of thought; the only reason why we usually overlook it is that in the language which we inherit it is already carried out, and it seems therefore to belong to the self-evident presuppositions of thought, not to its own specific work.

2. That which takes place in us immediately under the influence of an external stimulus, the sensation or the feeling, is in itself nothing but a state of our consciousness, a mood of ourselves. We do not always succeed in naming, and so making communicable to others, the manner in which we are thus affected; sometimes the formless interjection, the exclamation, is the only way we can find, though with no certainty of being understood, to give sound to what cannot be said. But in the more favorable cases, where we have succeeded in creating a name, what exactly is it which this creation effects and indicates? It is just what we are here looking for, the conversion of an *impression* into an *idea.* As soon as we give the name of green or red to the different movements which waves of light produce through our eyes, we have separated something before unseparated, our sensitive act from the sensible matter to which it refers. This matter we now present to ourselves, no longer as a condition which we undergo, but as a something which has its being and its meaning in itself, and which continues to be what it is and to mean what it means whether we are conscious of it or not. It is easy to see here the necessary beginning of that activity which we above appropriated to thought as such: it has not yet got so far as converting coexistence into coherence, it has first to perform the previous task of investing each single impression with an independent validity, without which the later opposition of their real coherence to mere coexistence could not be made in any intelligible sense.

3. We may describe this first operation of thought as the beginning of an *objectification* of the subjective; and I take advantage of this expression to guard against a misunderstanding and so illustrate the simple meaning of what I have said above. It is not objectivity in the sense of some sort of real existence which would subsist though nobody had the thought of it, that, by the logical act of creating a name, is accorded to the subject-matter to which that act gives rise. The true meaning of the first act of thought is best exemplified by those languages which have maintained the use of the article. The article, which had everywhere originally the value of a demonstrative pronoun, marks the word which it accompanies as the name of something to which we point; and what we point to is something which admits of being observed by another person as well as by ourselves. This can be done most easily with things which have an actual position in space between the speakers; but developed language makes an object of any other matter of thought in the same way. Such objectivity, therefore (which in these cases also is indicated by the article), does not entirely coincide with the reality which belongs to things as such; it is only the fact of their claiming such a reality, on the ground of the distinctive peculiarity of their real nature, which language has met and expressed in their names. When we speak of[1] 'the tooth-ache,' 'the day,' 'the franchise,' we do not imply that they could exist if there were no person to feel, to see, to enjoy them, respectively. Still less when we talk of 'the adverb' or 'the conjunction,' do we mean to indicate by the article that the subject-matter described by these words has any sort of existence outside thought. We only mean that certain special forms of resistance and tension, which we feel in the course of our ideas, are not only peculiarities of our own state and inseparable from it, but that they depend upon relations inherent in the matter of various ideas, which every one who thinks those ideas will find in them just as we do.

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[1] [The instances in the text are *der Schmerz, die Helligkeit, die Freiheit,* but none of the equivalents are used in the required sense with the article in English. The same applies to the instances in the following sentence, *das Zwar, das Aber, das Dennoch.*]

The logical objectification, then, which the creation of a name implies, does not give an external reality to the matter named; the common world, in which others are expected to recognise what we point to, is, speaking generally, only the world of thought; what we do here is to ascribe to it the first trace of an existence of its own and an inward order which is the same for all thinking beings and independent of them: it is quite indifferent whether certain parts of this world of thought indicate something which has besides an independent reality outside the thinking minds, or whether all that it contains exists only in the thoughts of those who think it, but with equal validity for them all.

4. But the objectification of the matter so first constituted is not the whole of this first act of thought; consciousness cannot simply present the matter to itself, it can only do so by giving it a definite position; it cannot simply distinguish it from an emotional mood of its own, without accrediting it with some other sort of existence instead of that which belonged to it as such a mood. The meaning of this requirement (for I admit that my expression of it is not immediately clear) is most simply shown by the way in which language actually satisfies it. It is only in the interjection, which is not a name of definite content, that language retains the formlessness which belongs to it as the mere expression of excitement; the rest of its stock of words is articulated in the definite forms of substantives, adjectives, verbs, and the familiar *parts of speech* in general. And it is hardly necessary to insist that the various characters thus impressed by language upon its material are the indispensable condition of the later operations of thought; it is obvious that neither the combination of marks into the concept, of concepts into the judgment, or of judgments into the syllogism would be possible, if the matter of every idea were equally formless or apprehended in the same form, if some of them were not substantival and did not express fixed and independent points of attachment for others which are adjectival, or if

others again were not verbal, exhibiting the fluid relations which serve to bring one thing into connexion with another. I do not think it advisable to separate this particular conformation of the matter of ideas, as a second act of thought, from the first act, to which we ascribed its objectification; I prefer to comprise the primary activity of thought in a single operation, which may be indifferently represented as that of giving to the matter of ideas one of these logical forms by making it objective for consciousness, or as that of making it objective by giving it one of these forms.

5. The three parts of speech which I have noticed remind us inevitably of three concepts which are indispensable for our judgment of reality. It is impossible to have even an expressible idea of the world of perception, without thinking of things in it as fixed points which serve to support a number of dependent properties, and are connected together by the changing play of events. If metaphysic is the investigation, not of the thinkable in general, but of the real or that which is to be recognised as such, these concepts of thing, property, and event are metaphysical concepts; not perhaps such as metaphysic would finally allow to stand without modification, but certainly such as at its outset purport to represent immediately the proper essence and articulation of what is.

It would seem at first sight that the logical forms of substantivity, adjectivity, and verbality coincide with these concepts: but a second view shows the same difference between the two series as that which separated the logical objectification of an idea from external reality. Nothing passes with us for a thing or a substance which has not reality outside us and permanence in time, producing changes in something else and capable of undergoing changes itself; but we apprehend as substantives not only things but their properties; as substantives we speak of 'change,' 'occurrence,' even of 'nothing,' and so in innumerable cases of that which has no existence at all or none except in dependence on something else. Thus the substantival form invests its content, relatively to the future predicates to which it is to serve as subject, with only the same priority and independence as belong to a thing in contrast with its properties, conditions, and effects, but by no means with that concrete and independent reality and activity which place a thing above a mere object of thought.

Verbs, again, express most frequently an event which as a fact takes place in time; but when we say that things 'are,' or 'are at rest,' or that one 'conditions' or 'equals' another, it is clear that the verbal form too does not universally give to its content the meaning of an event, but only finds it there usually. In order to conceive fully the sense of such verbs as we have just instanced, we have to connect several distinct contents together by a movement of thought, and this movement, though it implies time for its execution, is, as regards its meaning and intention, quite independent of time. In a word, the general sense of the verbal form is not an event, but a relation between several related points; and this relation may just as well occur between contents which are out of time and coexist only in thought, as between those which belong to reality and are accessible to temporal change.

Lastly, while it is true that radical adjectives, such as 'blue' and 'sweet,' express primarily what appears to our first apprehension as a real property of things, every developed language knows words like 'doubtful,' 'parallel,' 'allowable,' which, as the least reflexion shows, can no longer mean in the same simple sense as the former a property attaching to actual things; they are abbreviated and condensed expressions of the result of all sorts of relations, and it is only for purposes of thought that we represent the contents of such adjectives as related to those of substantives in the way in which we imagine an attribute to be related to its subject. Speaking generally, then, the logical import of the parts of speech is only a shadow of the import of these metaphysical concepts: it only repeats the formal characteristics which the latter assert of the real; but by not confining their application to the concrete external reality, it loses that part of their significance which they only possess in that application.

6. Lastly, if we found in the forms of the parts of speech the most original activity of thought, we must also understand how to distinguish this from its linguistic expression. Now that man has come to use the language of sounds for the communication of his thoughts, that activity is, it is true, most clearly manifested in the forms of the parts of speech; but in itself it is not inseparably bound up with the existence of language. The development of which the ideas of the deaf and dumb are capable, though guided in the first instance by those who can speak, is enough to show that the internal work of logic is independent of the possibility of linguistic expression. That work consists merely in the fact that we accompany the content of one idea with the thought of its comparative independence, while we think of another as requiring support, and of a third as a connecting link which neither subsists on its own account nor rests upon something else but mediates between two others. No one doubts the extremely effective support which language gives to the development of thought by making the formations and transformations of ideas vividly objective to consciousness by means of sharply defined sounds and their regular changes; still, if some other mode of communication were natural to man instead of the language of sounds, the same logical associations would find in it a corresponding expression though of a different kind. And if in some languages the poverty of forms does not always allow these associations to take shape, cannot, for instance, distinguish between substantival and verbal construction, yet there is no doubt that the mind of those who speak them maintains the logical distinctions while forming ideas which are vocally undistinguished. Wherever there is this inward articulation, there is thought; where it is wanting, there is no thought. For this reason music is not thinking; for however manifold and delicately gradated are the relations of its tones, it never brings them into the position of substantive to verb, or into dependence such as that of an adjective on its noun or a genitive on the nominative by which it is governed.

7. In mentioning hitherto only three out of a greater number of parts of speech, the three without which the simplest logical enunciation would be impossible, I do not wish to deny the value of the others. But the road which we have to traverse is too long to allow us to make further circuits into the attractive field of philological enquiry, circuits which, considering how thought has just been said to be independent of its mode of expression, must for our purpose remain circuits. The articulation and usage of language do not fully cover the work of thought. We shall find later that they frequently do not express the complete structure of the thought; and then we have for the purposes of logic to supplement what is said by what was meant. On the other hand language possesses technical elements which do not depend, or only depend with various degrees of remoteness, upon characteristics essential to logic: in such cases we should not be justified in distinguishing a different logical operation of thought for every grammatical or syntactical difference of form presented by language. There are not only interjections, but particles too, which, like the tone of the voice, hardly indicate more in ordinary usage than the interest which the speaker feels in what he is saying, and contribute nothing to its substantial logical meaning. When language introduces the distinction of gender into all substantives and adjectives, it follows an æsthetic fancy which has no interest for logic; when on the other hand it determines the gender of the adjective by that of its substantive, this consistency in an arbitrarily adopted custom points back to a logical relationship which we shall become

acquainted with. When in the inflexions of the verb it distinguishes the person speaking from the person spoken to and the third person not present, it emphasises an extremely important fact in a way which is indispensable for the living use of speech, and yet there is no corresponding distinction in logic proper. It is nothing but the same reason which justifies grammar in considering pronouns as a specific class of parts of speech: logically, the personal pronouns must be reckoned entirely among substantives, with which in formal position they are identical; the possessive and demonstrative we have no ground for separating from adjectives; the relative we should regard as the most specifically technical element in language, serving only the need of methodical communication, and based on no other logical relation than its counterpart the demonstrative. Numerals are treated by grammar as distinct parts of speech; in the actual usage of language they are equivalent to adjectives, and that logically they belong to the latter we cannot doubt, when we remember that logically the form of adjectivity belongs to all characteristics of a subject-matter which are not self-dependent, and not only to those which attach to it in the sense of properties. Adverbs, lastly, stand in precisely the same relation to the meaning of verbs as adjectives to that of substantives, so that logic would have no occasion to consider them as a distinct part of speech or a peculiar form of the content of thought.

Thus there would only remain prepositions and conjunctions to put forward such a claim, and of them I think we must admit that, however they may be derived linguistically, they form an indispensable element in the world of our ideas. They cannot be derived from the concept of relation, with which at first they seem to be connected: whenever two members are connected by a relation, there is involved the thought of a certain position which those members occupy within the relation itself, and this position need not be the same for both; on the contrary, it is generally different, the one embracing, containing, and conditioning the other. Now it will be found upon trial to be impossible to express this difference of value between the related points, without which the relation has no meaning, in a merely verbal form: somewhere or other we shall need a preposition, a conjunction, or at least one of the various case-forms in which many languages express some of these accessory notions still more shortly. In what linguistic form they appear, is of course quite indifferent to logic; just as we oppose the nominative, as that which conditions, sometimes to the genitive, as that which is conditioned, sometimes in a different sense to the accusative, so, if language had produced or preserved a still greater wealth of cases, all prepositions would be superfluous, as all conjunctions would be if there were a similar variety of moods. This would make no change in the logical needs of thought; in one way or another, the meanings of substantives, adjectives, and verbs would have to be supplemented by a number of ideas, indicating, either, like prepositions, the position of two supposedly simple objects in a simple relation, or, like conjunctions, the comparative position and value of two relations or judgments.

8. If we glance at the developed structure of the world of our thoughts and ask what the conditions are upon which its construction depends, the objectification of impressions and their concomitant formation in the sense of the parts of speech must always appear as the most indispensable, and in that sense the first, of all operations of thought. It is certain that without it the framing of sentences, simple or complex, through which we express the work and results of our thinking, would have been quite impossible. But we must not be taken to mean that the logical spirit, at the beginning of its intellectual work, before it ventured a step further, performed this, the first of its necessary operations, on the entire matter of its ideas once for all. The infinitude of possible impressions, of which every moment may bring a new one, would be enough to make such a task impracticable: it is made still more impracticable by the fact that in working up the matter that is given to it thought is constantly producing new matter, and has to bring this again into the same logical forms of which, as applied to a simpler matter, it is the result. Thus it is that every developed language possesses, in the form of simple substantives, adjectives, or verbs, numerous ideas which could not have been framed, and cannot be fully understood, without manifold intellectual operations of a higher kind, without employing judgments and syllogisms, and even without presupposing systematic scientific investigation.

This obvious reflexion has given rise to the assertion, that in logic the theory of judgment at least must precede the treatment of concepts, with which it is only an old tradition to begin the subject. I consider this to be an over-hasty assertion, due partly to a confusion of the end of pure with that of applied logic, partly to a general misconception of the difference between thought and the mere current of ideas. For if those judgments, out of which the concept is said to result, are to be really judgments, they themselves can consist of nothing but combinations of ideas which are no longer mere impressions; every such idea must have undergone at least the simple formation mentioned above; the greater part of them, as experiment would show, will already practically possess that higher logical form to which the very theory in question gives the name of concept. The element of truth in this proposed innovation reduces itself to the very simple thought, that in order to frame complex and manifold concepts, more especially in order to fix the limits within which it is worth while and justifiable to treat them as wholes and distinguish them from others, a great deal of preparatory intellectual work is necessary; but that this preparatory work itself may be possible, it must have been preceded by the conformation of simpler concepts out of which its own subsidiary judgments are framed. Without doubt, then, pure logic must place the form of the concept before that of the judgment: it remains for applied logic to tell us how, in framing determinate concepts, judgments consisting of simpler concepts may be turned to account. A proposal to reverse this order can only commend itself to those who regard thinking in general as merely the interaction of impressions excited in us from without, and overlook the reacting energy which makes itself felt at every point in the current of ideas, separating the merely coincident, combining the coherent, and thus already giving form to the individual elements of future thoughts.

B. *Position*[1], *Distinction, and Comparison of the Matter of Simple Ideas.*

9. If we recognise in these first formative acts the specific contribution which the operative energy of thought makes to the whole of our intellectual world, we are easily led to the view that the logical spirit has certain ready-made modes of apprehension with which it meets the impressions as they come; and this again raises the question, how it contrives to bring the matter of each impression under that particular form which is appropriate to it. But such a view is inadmissible, and such a question therefore has no point, or at any rate leads to an answer different from that which it expects. Thought does not stand fronting the impressions as they arrive with a bundle of logical forms in its hand, uncertain which form can be fitted to which impression, and therefore needing some special expedient to discover how to pair them properly. It is the relations themselves, already subsisting between impressions when we become

self, as different from others, and as comparable with others.

10. This second operation of thought, like the first, is one which nherited language has already carried out for all those who speak it; ike the first therefore it is easily overlooked, and not reckoned as art of the work of the mind. But logical science, expressly devoted o the self-evident, must not treat a part of its subject as a still more elf-evident presupposition which may be excluded from the proper bjects of its consideration. Still, the first at any rate of the three eads under which we expressed this new operation of thought does ot need a detailed explanation. It is at once obvious how every name, sweet' or 'warm,' 'air' or 'light,' 'tremble' or 'shine,' gathers up the atter which it indicates in some sort of coherent unity with a meaning of its own; it is not only (though it is most emphatically) matter n the substantival form that is thus lifted into unity with itself by the prefixed article; the same indicative force resides, under a different form, in the infinitive of the verb, and even when language has no distinctive expression for it, this accessory notion of singling out and giving position to the matter indicated accompanies every form of word. It may be doubted whether the process which we would understand by *giving position* is not already contained in the objectification which we represented as converting the passive impression into an idea; and it is true that we can neither have an idea without thus giving position to its content, nor give it position in any intelligible sense without objectifying it. Practically therefore it is a really inseparable operation which we are considering from different sides; before, we contrasted the presented idea to which we are related as presenting, with the impression by which we are simply affected; now, when the multiplicity of the matter presented begins to excite our attention, we lay stress upon the unity and independence in virtue of which the matter thus singled out by attention is what it is and differs from everything else.

11. By the last words I wished to convey clearly the close connexion in which the affirmative position given to a content stands with the terms which we are obliged to employ to express the simple sense of the first are only made perfectly clear by adding the accessory notion of the second. We can only explain what we mean by the unity of position given to a content by emphasising its difference from others, and saying, not only, it is what it is, but also, it is not what others are. The affirmation and the negation are one inseparable thought, and accompany in inseparable union every one of our ideas, even when we do not expressly attend to the others which are tacitly negated. But the accessory notion thus amalgamated with our ideas only determines the logical setting which we give to their content; it does not produce that content in the first instance. It cannot be said that we have the idea of red as red only when we distinguish it from blue or sweet, and only by so distinguishing it, and again the idea of blue as blue only by a similar opposition to red. There could be no conceivable occasion for attempting such a distinction, nor any possibility of succeeding in the attempt, unless there were first a clear consciousness of what each of the two opposites is in itself. Without doubt the peculiar impression which we experience under the influence of red light will be entirely the same before we have had our first experience of blue light as it will be afterwards; the possibility of comparison and distinction which the latter experience gives may indeed, at any rate in a matter more complex than these simple colours, draw the attention to parts of the impressions which had been previously overlooked, and so make both of them more complete; but even in this case, which is quite outside our present consideration, the new element is not discovered by the distinction, but by the immediate sensation of which the comparison was merely the occasion. It is always affirmative position therefore which makes negative distinction possible, while it is never the case that the act of distinction gives rise to the matter distinguished. Only our accessory notions about the matter of our ideas, only its logical setting, gains in definiteness by adding to the affirmation of itself the negation of others; and even this gain would seem to me small if it went no further, and were not supplemented by that third operation of positive comparison, which, in the above account of this second act of thought, was mentioned last.

12. I will introduce the consideration of this third operation, which I regard as the most essential part of the logical work to be here explained, by recalling a familiar fact which is commonly used to support other conclusions. Words never denote impressions as they can be experienced; we can only experience or actually perceive a

sensation has undergone, ny regarded as an [illegible]
xactness of language, perhaps even of the thought which language ves to express. Unable or not accustomed to make a definite me for every single impression, language (it is supposed) blurs the ght differences between them, and retains only what is immediately perienced in sensation as common to them all: by this reduction its means of expression to a moderate number it certainly makes e communication of ideas possible, but diminishes proportionately e exactness of that which has to be communicated. I do not think at this view does full justice to the significance of the fact.

13. First of all, to regard the universalisation in question as a sort falsification of the impressions is to pass too lightly over the very markable circumstance, that in a number of different impressions ere *is* something common which can be thought apart from their fferences. This is by no means such a matter of course that the posite is out of the question; on the contrary, it is quite conceiv-le that every one of our impressions should be as incomparably fferent from every other as sweet actually is from warm, yellow om soft. The fact that the thinkable world itself is so constituted at this is not the case, is one which it is worth while to take into nsideration. Nor again can I regard the want of exactness, which e application of the universal terms of language undoubtedly gives the communication of ideas, as sheer loss. Moreover, when perfect actitude is felt to be important, the shortcomings of these simplest oducts of rudimentary thought can always be supplemented by its ore advanced activity: science has long taught us to measure every gree of heat, and in case of necessity would find out how to measure mate expressions because they are themselves only approximations to these fixed points, or connecting links between them. If we really had particular and mutually independent names for every single shade of blue, and our ideas answered to this form of expression, we should have achieved in a onesided way the separation of each from every other, but we should have overlooked completely the positive relations which subsist between them all. If on the contrary we speak of bright blue, dark blue, black blue, we arrange this manifold in a series or a network of series, and in each series a third member results from a second by intensification of the same sensible change in a common element as that which gave rise to the second out of the first. It must be already perfectly clear that a presentative activity which did not involve this comparison of the diverse, but was confined to the bare separation of each from each, would not offer to the later operations of thought adequate grounds for contrasting two ideas, as in some way or other cohering, with two others as not cohering. We therefore apprehend this second act of thought, of which we are here speaking, not merely as that of giving simple position to *a* or *b*, not merely as that of simply distinguishing every *a* from every *b*, but also as that of determining the extent and peculiarity of the distinction, which is not everywhere the same in degree and kind, but is different between *b* and *c* and between *a* and *b*. I do not mean to say that every single idea, *a*, must be accompanied by the developed idea of all its relations to the infinite number of all other ideas; the general accessory notion, that every idea is enclosed on all sides in such a network of relations, does indeed in our logical consciousness envelop every idea; but these relations are only followed out in each particular case so far as a special requirement suggests.

14. This comparison of the diverse clearly presupposes a common element to which in the several members of the series specific differences attach. Such a common element is usually considered by logic only in the form of a universal concept, and in this shape it is a product of more or less numerous acts of thought. It is therefore important to point out that this *first universal*, which we find here involved in the comparison of simple ideas, is of an essentially different kind; that it is the expression of an inward experience which thought has merely to recognise, and that just for this reason it is, as will be seen later, an indispensable presupposition of that other finable series of intellectual operations, connecting, separating, or relating a number of simple ideas assumed to be known; when this logical work is completed, we suppose him to have before his mind the same object-matter which we wished to impart to him. But we cannot explain by the same means wherein the universal blue or the universal colour consists, which accompany our ideas of bright and dark blue or of red and yellow. We can indeed direct another person to think of all single colours or all shades of blue, and by eliminating their differences bring out what is common to his ideas in the two cases; but it is only in appearance a logical work which we are here prescribing; all that we really call upon him to do is to see for himself how he executes the task. How he is to set to work to discover whether there really is any common element in red and yellow, and how he is to contrive to separate it from the differences, this we cannot tell him; we must simply trust to his having an immediate sensation, feeling, or experience of the connexion which exists between red and yellow, of the fact that they contain a common element; his logical work can consist only in the recognition and expression of this inward experience. This first universal, therefore, is no product of thought, but something which thought finds already in existence.

15. I will insert an observation here which with slight modification may be extended to all universals, but is most easily illustrated in this simplest instance, the first universal. That in which red and yellow agree and which makes them both colours cannot be separated from that which makes red red and yellow yellow, not separated, that is to say, so as to form the content of a third idea similar in kind and order to the two compared. It is always, as we know, only a single definite shade of colour, only a tone of definite height, strength, and quality, which is the object of sensation; and it is only these definite impressions which are so repeated in memory as to present substantial and perceptible images to consciousness. Universal ideas never have this perceptibility. If we try to apprehend the universal element of colour or tone, we shall always find that either we have before our perception a definite colour and a definite tone, only with the accessory notion that every other tone and colour has an equal right to serve as a perceptible instance of the ever imperceptible universal; or else our memory will produce a number of colours and tones in succession, with the same accessory notion that it is not these individuals that are really meant, but the common element in them which cannot as such be apprehended in perception. If therefore we understand by idea[1] (as ordinary usage certainly inclines us) the consciousness of something standing at rest before the mind, or a perception of something capable of being presented to it, the universal cannot claim to be called an idea. Words like 'colour' and 'tone' are in truth only short expressions of logical problems, whose solution cannot be compressed into the form of an idea. They are injunctions to our consciousness to present to itself and compare the ideas of individual tones and colours, but in the act of so comparing them to grasp the common element which our sensation testifies them to contain, but which cannot by any effort of thought be really detached from their differences and made the material of a new and equally perceptible idea.

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[1] [Vorstellung.]

16. Let us now direct our attention to the differences, which, within the first universal, separate the various instances of it. It is clear that what distinguishes one sensation of warmth from another, a gentler from a louder sound, bright from dark blue, is a more or a less of a common sensible element, which in itself, undetermined by any degree, is no object of perception. We shall find ourselves brought back to the same ground of distinction in all other ideas; it is only in giving an account of the universal, to which this quantitative comparison applies, that we meet with a difficulty, which after the above remarks is intelligible. The louder tone is no doubt distinguished from the gentler by a certain intensification, but so also is the higher from the lower; yet it is only in the former case that we feel able to express directly, by the term 'strength,' the common element which undergoes this change; in the latter we express it by the metaphor of height. Red and yellow seem to be still more essentially different and underivable one from the other by increase or decrease of a common element; only the intermediate colours, reddish yellow or yellowish red, are intelligible to us as mixtures containing more or less of one or the other. Nevertheless no one denies that one of the fundamental colours is more nearly related to a second than to a third, red to yellow than to green; and these grades of resemblance cannot be conceived without a more or a less of some common element, which we are conscious of in passing from one member of the series to the next and from this to the third. To determine in each particular case what this common element consists in, to decide whether a number of ideas are separated merely by differences in degree of one simple universal, or by differences in value of several mutually determined ones, and whether accordingly the ideas are to be grouped in a linear series or plane-wise or in still higher forms, these are all attractive objects of enquiry, but they are not objects of

:es of degree do exist in the same matter, as well as the ɩr fact that the degree of *a* exceeds that of *b*, can only be ɩced, felt, or recognised as part of our inward consciousness. ever artificial contrivances we may seek to increase scientifi- : exactness of a measurement, everything must depend ulti- ›n the capacity to recognise two sensuous perceptions as like or e, and not to be deceived as to which has the more and which

f inward experience were confined to bringing out resem- and differences in the various object-matters, thought would be called upon to arrange ideas in an unalterable system, like ;ical scale, in which all tones have once for all their fixed and :able places. But logic has to do with thought, not as it ›e under hypothetical conditions, but as it is. Now owing to chanism which controls the interaction of its inward states, all hought has necessarily more opportunities of stimulation than ›ve hypothesis would imply; the manifold matter of ideas is t before us, not only in the systematic order of its qualitative ships, but in the rich variety of local and temporal combina- and this fact, like the other, belongs to the material which thought in its further operations and must be given it to start The combinations of heterogeneous ideas produced in this rm the problems, in connexion with which the efforts of t to reduce coexistence to coherence will subsequently have to le. The homogeneous or similar ideas on the other hand give n to separate, to connect, and to count their repetitions; and : ideas of unity and multiplicity those of greatness and smallness consciousness, into the order in which one of them may condition the origin of another, and into the different importance of perceptions of time and space in their formation. However attractive these questions may be, it would lengthen our way unnecessarily to answer them; logic is not concerned with the manner in which the elements utilised by thought come into existence, but with their value, when they have somehow or other come into existence, for the carrying out of intellectual operations. Now this point, which I conceive to have been unduly neglected, I wish to emphasize here, and shall subsequently keep in view, viz., that all ideas which are to be connected by thought must necessarily be accessible to one of the three quantitative determinations which have just been mentioned. The other thing which I exclude is the investigation of the consequences which may be drawn from these quantitative determinations as such: they have long ago developed into the vast structure of mathematics, the complexity of which forbids any attempt to re-insert it in universal logic. It is necessary, however, to point out expressly that all calculation is a kind of thought, that the fundamental concepts and principles of mathematics have their systematic place in logic, and that we must retain the right at a later period, when occasion requires, to return without scruple upon the results which mathematics have been achieving, as an independently progressive branch of universal logic.

19. If we take a general survey of this second act of thought, in which I now include that of giving affirmative position to the object-matter, that of distinguishing it negatively from all others, and that of estimating by quantitative comparison its differences and resemblances, we may observe that the significance of this new logical operation is somewhat different from that of the first, by which impressions were shaped into ideas. In the former case there was a temptation (which, it is true, we resisted) to regard the forms of substantivity, adjectivity, and verbality as modes of apprehension which thought is ready to put in practice upon its object-matter before receiving any solicitation from it; but though we set aside this claim at once, it remains true that in those forms thought does not merely respond to and reproduce the actual current of ideas, but gives them the shape without which the logical spirit could not accept them. The independence which the substantival form gives to its matter, most adjectival form present, as such, in the fact which stimulated the mind to characterise it by that form; so that we may continue to assert, in a certain sense, that in this first act thought dictates its own laws to its object-matter.

If, using an expression which we shall otherwise avoid, we represent this procedure as a proof of spontaneity, the second act of thought has the character of receptivity; it is a recognition of facts, and adds no other form to them except this recognition of their existence. Thought can make no difference where it finds none already in the matter of the impressions; the first universal, as we saw, can only be experienced in immediate sensation; as so experienced it can be named, but this is the only contribution which logic can make to the further fixing of its character; all quantitative determinations, to whatever extent thought may develop them by subsequent comparison, always come back to an immediate consciousness of certain characteristics given in the object-matter. I should wish this fact to be considered from two points of view. In the first place, logic is guilty of a certain carelessness in assuming at almost every moment in its later stages the comparability of ideas and the possibility of their subordination to a universal, without observing that that possibility, and the success of its own procedure in general, depends upon this original constitution and organisation of the whole world of ideas, a constitution which, though not necessary in thought, is all the more necessary to make thinking possible. For I must repeat that there is no inherent contradiction in supposing that every idea was incomparably different from every other; that in the absence of all qualitative comparability there was no standard of more or less; that the same idea never presented itself twice to perception; and that, as there was no repetition of the homogeneous, the ideas of larger and smaller also vanished. The fact that this is not the case, but that the world of ideas is organised as we have found it to be, must be emphasized as of the highest importance; but logic ought not in case of need to appeal to it incidentally as a self-evident truth derived no one knows whence. And this brings me to the other observation which I had to make. If thought is a reaction upon a stimulus found in the current of ideas, a systematic survey of its functions will show clearly at certain points the influence exercised upon them by the thinkable world; as it is here the second member in the first triple series of operations, so at a later stage also it will be the second member of the following more highly developed group in which we shall see the peculiar dependence of thought upon the material to which it is directed. I do not however claim to do more by this preliminary indication than to throw a preliminary light over the system which I have followed in my exposition; the system itself can only find its justification in the advantages which in its successive stages it will be found to secure.

C. *The Formation of the Concept.*

20. To separate the merely coincident amongst the various ideas which are given to us, and to combine the coherent afresh by the accessory notion of a ground for their coherence, is the further task of thought. It will be useful, with a view to making its meaning clear, to review the different senses in which any combination of manifold elements occurs in our mental world. In the first place, no later intellectual activity is possible, unless the various ideas upon which it is to be exercised meet together in one and the same consciousness. The fulfilment of this condition is secured by the unity of the soul and the mechanism of memory, which, by bringing together impressions separated in time, makes their interaction possible. This union of the manifold may be called the synthesis of apprehension; it is not a logical act; it merely lumps the manifold together into a simultaneous possession of consciousness, without combining any two of its elements in a different order from any other two. Such an order comes in with the second form of connexion, the synthesis of perception, that is, with figures in space and succession in time, in which the individual impressions take up definite and non-equivalent positions. This connexion also is supplied by the inward mechanism of consciousness without any action of thought, and however firmly defined and finely articulated it may be, it exhibits nothing but the fact of an external order, and reveals no ground of coherence justifying coexistence in that order. From the second stage I pass at once to the fourth, to a synthesis in which the last-mentioned requirement would be completely satisfied in regard to any given object-matter. In such a synthesis we should have before our mind, not the mere fact of manifold elements in order, but also the value which each element possessed in determining the coalescence of the whole. If what we thus apprehended were an object in real existence, we should see which were the prior, determining, and effective successive growth of them all: if, like the figures of geometry, it was something which had no reality out of our consciousness and no growth or development in time, we should here too attempt at any rate (though, as we shall see later, with limited success) to arrange the elements of the whole in a hierarchy in which those that conditioned others should take precedence of those that were conditioned, according to their stages of dependence. It is easy to see that a synthesis of this sort would be neither more nor less than the knowledge of the thing; as the goal of all intellectual effort, it lies as far above the province of logic as the first and second modes of connexion lay beneath it; it is in the space between that we must place the third and logical form of synthesis, the character of which has now to be examined.

21. When a person who has no special knowledge speaks of 'credit' or of 'banking,' we trace in these expressions his conviction that a number of businesses and institutions form a connected whole; but he would not be able to say where the nerve of the connexion lies, or what limits separate the whole from that which does not belong to it. In this accessory notion, that the various elements are not merely there in a sort of heap, but form a whole of parts with self-imposed limits and a unity included by those limits, the general impulse of thought leaves its mark upon the given object in a formal way, without as yet attaining material fulfilment. If we pass our mental world in review, we are in this position as regards a very large part of its contents; indeed we shall be surprised to find that words of great significance betray this imperfect apprehension of their objects; for the more complex, important, and various any matter is, the more easily will persuasive impressions derived from repeated observations awaken the feeling of its individuality, completeness, and self-inclusiveness, without necessarily giving any real insight into its structure. Such words as 'nature,' 'life,' 'art,' 'knowledge,' 'animal,' and many others have no more significance than this in ordinary usage; they merely express the opinion that a certain quantity, usually not exactly definable, of individual objects, attributes, or events, which attach to one another, form somehow an inwardly connected whole, which can neither have any part taken away without being destroyed, nor admit any casual additions within the bounds of its unity. But how little the nature of this connexion is really known, appears from the failure of the attempt to describe the limits which include what belongs to the unity and exclude what does not. So long as the logical work of holding the manifold together has not gone further than this, I should hesitate to speak of 'concepts,' though I do not attach any value to the invention of a special technical term for such imperfect apprehension. Suppose we call it an imperfect or growing concept; then we shall not feel that we have got a perfect or fully developed concept, until the vague suggestion of some sort of whole has grown into the pervading thought that there is a definite ground for the co-existence of these particular attributes, in this particular combination and to the exclusion of certain others, and that this ground is an adequate one.

22. The question now arises, how we get at this ground and condition. If we merely continued to observe a composite form *a b c d* in its isolation, we should never discover, however long we looked, which of its parts only coexist, which really cohere, and in what degree the existence of one depends upon that of another. But if we compare *a b c d* with other forms like it, that is, with such as we are led from it to observe, not by any special logical effort, but by the natural current of our ideas, and if we find that in *a b c d*, *a b c f*, *a b c g*, etc., a similar group *a b c* occurs with various dissimilar additions, we regard the latter as loose and separable appendages of the permanent stem *a b c*. Nor does the common group *a b c* contrast with the rest merely as the centre to which as a matter of fact they attach; on the general assumption that we have before us a whole of interdependent parts, this solid kernel becomes the expression of the constant rule which allows the accretion of the several accessory elements, and determines the manner in which it takes place. If we wish for practical purposes to ascertain in any creature, object, or arrangement, what is the line which divides what is inwardly coherent from casual accessions, we put the whole in motion, in the belief that the influence of change will show which parts hold firmly together while foreign admixtures fall away, and in what general and constant modes those parts combine while changing their relative positions in particular cases: in this sum of constant elements we find the inner and essential cohesion of the whole, and we expect it to determine the possibility and the manner of variable accretions. The first of these methods, that of bringing out the common element in different instances when at rest, is the one which has been usually followed by

nts as coherent, and as the ground for the admissibi
ıissibility of fresh elements.

Abstraction is the name given to the method by whi
:sal is found, that method being, we are told, to lea
is different in the particular instances compared and
ıer that which they possess in common. If we look
procedure of thought, we do not find this account con
silver, copper, and lead differ in colour, brilliancy, weig
ty; but their universal, which we call metal, is not founc
arison by simply leaving out these differences without cc
ı. Clearly it is no sufficient definition of metal to say neg
neither red nor yellow nor white nor grey; the affirmatic
; at any rate some colour, is equally indispensable; it h
d this or that specific weight, this or that degree of bri
he idea of it would either cease to have any meaning
ould certainly not be the idea of metal, if it contained no t
ever of weight, brilliancy, and hardness. Assuredly we
he universal image of animal by comparison, if we leave
ninds entirely the facts of reproduction, self-movemei
ation, on the ground that some animals produce their
others lay eggs, others multiply by division, that some
he through lungs, others through gills, others through th
that lastly many move on legs, others fly, while some
ole of any locomotion. On the contrary, the most e
of all, that which makes every animal an animal, is tha
mode or other of reproduction, of motion, and of resp
ll these cases, then, the universal is produced, not by
ıg out the different marks p^1 and p^2, q^1 and q^2, which occu
iduals compared, but by substituting for those left (
rsal marks P and Q, of which p^1 p^2 and q^1 q^2 are particula
simple process of leaving out only takes place when

ascribe two marks jointly to both the objects compared, but to assume them to be at zero in the plant, while in the animal they always occur in an appreciable quantity. To express the matter somewhat differently, it may be asserted from the point of view of logic that compensation by the corresponding universal for omission of individual marks is the regular rule of abstraction, while the uncompensated omission applies to exceptional cases, where we can find no logically common mark, of which the presence and absence of some individual mark might be held to constitute different species. So formulated, our rule of abstraction covers these cases of mere omission; on the other hand, a rule which made omission its sole starting-point could find no way to bring in compensation afterwards; and the importance of compensation in forming the universal will be confirmed at every step in the later stages of logic.

24. After the considerations urged in the preceding section, the necessity of which to what was to follow will now be clear, the apparent circle involved in the injunction to form universals by putting together universals, will not give serious offence. We have seen that the universal marks *P* and *Q* which we require here, the 'first universal' of the section referred to, come to us without logical effort as simple facts of observation in our mental life; and just for this reason they can be applied in building up this second universal, which we do produce by logical effort. That the yellow of gold, the red of copper, and the white of silver are only variations of a common element which we proceed to call colour, this is a matter of immediate sensation; but to a person who could not be made sensible of it, it could never be explained by logic either that these particular impressions are species of this universal, or what is meant by a universal as such and the relation of its particular to it. It is just this point to which I would again draw attention here, that the immediate perception of a first universal and the application of some kind of quantitative ideas is the condition of the formation of the second universal in all cases, not only in those like metal where there is no difficulty in regarding the marks of colour, brilliancy, and hardness as stable properties of that which they describe, but also where, as in the case of the animal powers of reproduction and motion, they are merely short adjectival descriptions of conditions which we cannot think completely

dentical or similar process takes place in them, or of the
elated points between which it takes place, or of the
form to which it is liable owing to variations in the
nese related points, the intimacy of their relations, and
positions in space and time, these last, like the rest, being
variations. If we take away this quantitative gradation
bility, which extends, though of course in different ways,
g, whether simple properties, or their relations, or com-
events simultaneous or successive, the formation of a
comparison of different groups of ideas, would, at least
in which it has any value for thought, be impossible.

now mention some traditional technical expressions. If
ally give the general name of concept (*notio*, *conceptus*) to
e idea which we think as a connected whole, the sum of
eas or marks (*notae*) *a*, *b*, *c*, *d*, etc., through which a concept
ught and distinguished from all other concepts Σ, is called
' (*materia*); while its 'extent' (*ambitus*, *sphaera*) is the
dividual concepts s^1, s^2, s^3, etc., in each of which the con-
t is, the group of marks *a*, *b*, *c*, *d*, in some one or other
ible modifications, is contained. Thus colour, *a*, weight,
d, and the like, would together form the content of metal,
pper, s^1, silver, s^2, gold, s^3, and the like, taken together,
nt. It is usual also to speak of the individual marks *a*, *b*, *c*,
ted' in the content of *S*, and of the individual species,
coordinated' in the extent of *S*: the relation of the species
e universal itself which forms their genus, is called 'sub-
while both the species and the genus are said to be 'sub-
r each of the universally expressed marks, which make
nt of *S* and consequently also of s^1 s^2 s^3. Lastly it is

concepts *S* and *T* originally compared, the poorer will it be in content and the larger in extent; and conversely, if we descend from the highest universals *W* through *V* and *U*, *S* and *T*, to the species of *S* and lower, the content will increase with the decreasing extent and become greatest in those completely individual ideas to which logic hesitates to give the name of concept at all.

26. The value of these distinctions is unequal, but on the whole slight. I will begin what I have to say about them by fixing the terminology which I shall myself use in future. I speak of any composite matter *s* as conceived or as a concept, when it is accompanied by the thought of a universal *S*, which contains the condition and ground of the coexistence of all its marks and of the form of their connexion. After this explanation we shall not hesitate to speak of concepts of perfectly individual things (singular concepts, in the old logical terminology), and we believe this to be quite consistent with the usage of language. For when we observe a new object *s* for the first time, and, not content with the perfectly clear sensible perception of it, go on to ask what it really is, we clearly want to know the rule which connects the perceived marks in the observed fact and converts them into a coherent whole of a definite and predictable character. If we then find that this *s* is *S*, an animal or a plant, we suppose ourselves to have a conception of *s*; it is the idea of it which is raised into a concept by the accompanying thought of the universal *S*. Every proper name is an illustration of this. 'Alcibiades,' for human thought, never means merely a multiplicity of differently coloured points, which are combined in space in a definite though not quite invariable outline, and resist the attempt to separate them; nor does the name express merely the accessory notion that this multiplicity in some unexplained way forms a whole; it suggests to the mind a definite general image of a man or a human being, which lays down the lines for our view of the connexion of the observed marks with one another and with the future behaviour to be expected from them. A view so determined cannot be appropriately called either a perception, or an idea merely, but only a *singular concept*.

27. On the other hand, it seems to me quite out of place to call the universal *S* itself, the accompanying thought of which makes the ordinarily used expresses no more than an image of this kind; reflexion, by subordinating it to the universal 'animal,' easily makes it into a concept; but then 'animal' remains a general image, which only the naturalist, for the uses of his science, converts into a concept by thinking 'organic being' along with it. It is upon such incomplete logical activity, which brings into relief only a single link in the chain, the connexion of the individual with its nearest universal, but leaves all beyond it in darkness, that the concepts which occur in ordinary thinking are based; as however scientific investigations, to which logic is primarily intended as an introduction, do really aim at extending the conceptual form from the concept itself to the higher universals under which it successively falls, it is enough to have made the above remark without rigidly enforcing it, and I shall follow ordinary usage in conceding the name of concept to those general images as well. I can do this the more easily because the name 'concept' does not seem to deserve in logic that exalted significance which the school of Hegel has given it, and in which it claims to express the knowledge of the essential nature of the object. The difference between logical forms and metaphysical ideas must be taken into account here as elsewhere. There may be a privileged concept, which follows the thing itself in its being and development, or takes up a point of view at the very centre of the thing, the fountain-head of its self-determination and self-organisation; but it is not the function of logic to reserve its concept-*form* for so very select a filling. By the logical concept we understand such a form of apprehending any matter of thought, from whatever point of view, that consequences admit of being drawn from it which coincide again at certain points with results flowing from that matter, that is, from the thing itself; and as the thing projects itself differently at every different point of view, there may be various equally right and equally fruitful logical concepts of the same object. We may therefore continue to call 'concept' any apprehension which, though only with the help of a general image which is not further analysed, has the effect of bringing the given object under a rule of behaviour which agrees, when applied, with its actual behaviour.

28. The asserted coordination of marks in the content of the concept raises serious difficulties. To begin with, it is a misfortune that we have no appropriate name for the elements of which we compose the concept; for 'mark' and 'part' only apply in certain cases. They give rise to the current delusion that the elements of a concept are universally of equal value, connected in the same way each with the whole and each with each. The ordinary instances of logic, taken from simple natural objects, are specially calculated to lead us into this error. It is true that gold is yellow only in the light, ductile only under a certain power of traction, heavy only for the body upon which it presses; but these various conditions easily present themselves to our imagination as stable properties, collected in a definite point of space, and inhering, in a manner identical but otherwise unexplainable, in the reality which on their account we call gold. Here the name 'marks' is appropriate, and here the marks are certainly coordinated in the content as has been asserted; but this coordination merely means that they are all equally indispensable to the whole, but have *not* any other sort of order. If we leave such simple instances, and consider concepts like 'triangle,' 'animal,' or 'motion,' we require, in order to think them properly, a quantity of part-ideas which are no longer mutually equivalent, but have to be placed in the most various relations to one another. The three sides of a triangle are not merely there *as well as* the three angles; they must form the angles by their intersections: the concept of motion does not merely contain the part-ideas of place, change, direction, and speed; direction and speed are, each in a different sense, determinations of change; place, being that which is left behind, can least of all be called a mark of the concept; it is a point of reference for the idea of change, to which its relation is expressed by that of the genitive to the nominative which governs it. To follow out these points in detail would take too long, but it would evidently lead us to the conviction that, as a rule, the marks of a concept are not coordinated as all of equal value, but that they stand to each other in the most various relative positions, offer to each other different points of attachment, and so mutually *determine* each other; and that an appropriate symbol for the structure of a concept is not the equation $S=a+b+c+d$, etc., but such an expression as $S=F(a, b, c, \text{etc.})$, indicating merely that, in order to give the value of S, a, b, c, etc., must be combined in a manner precisely definable in each particular case, but extremely variable when taken generally; If in any particular instance $S = a[b^c \sin d] + (e - \frac{f}{g})\sqrt{h}$, this formula, however foolish it would be if it professed to mean anything more, would give a better picture than the above inadequate formula of addition of the different ways in which the several marks a, b, c, etc. contribute to the construction of S as a whole.

the subordination of the species to the genus, and that of the universal S along with its species to the universal marks a, b, (ductile, coloured, etc.). The nature of the universal, metal, completely dominates the nature of its species, gold and copper, and no property of the latter escapes its influence; many things are yellow or red, but the glistening red and yellow of copper and gold belong to metal alone; many things are ductile, but the amount and other peculiarities of the ductility exhibited by gold and copper are heard of only in metals; and only metallity explains their degree of specific gravity. Similarly the universal animal determines every property and every movement of its species; animals move, grow, and rest differently from plants and lifeless things. If we symbolise the universal metal by a circle S, the smaller circle of gold, s^1, lies entirely within it, and by the side of this, separate from it but also completely inside S, the circles s^2, s^3, copper and silver. Applying differently two names which are generally used as equivalents, I describe the true subordination to a dominant universal as *subordination* to the genus, while I call the subordination of gold to yellow or ductile *subsumption* under the mark. These universal marks obviously do not rule and penetrate the whole nature of gold; each of them expresses only one side of it, which it shares with other objects of an entirely different kind, from which, so far as logic can see, no sort of inferences can be drawn as to the other properties of gold. Thus the lesser circle s, gold, occurs only in a particular place in the larger G, yellow, and intersects it without lying wholly within it; G is similarly intersected in other places by the circles of other yellow objects, and they all remain partially outside it.

30. Starting from the universal S, which was the rule for s^1, s^2, s^3, the original objects of comparison, we were able to mount to higher and higher universals T, U, V, W. In natural history, where such a series is of value, its several members in an ascending scale have been named species, genus, family, order, class: there is however a difference of opinion as to what functions a universal concept must perform in order to represent even a species or a genus, and the other names are applied still more divergently, and always from points of view depending for their justification on the special nature of the subject-matter. If we dispense with this plea, the plea from the side of the specialist for the significance and importance of these distinctions, the only way to give some sort of fixed logical value to species and genus is as follows. The only thing which suggests to the natural mind to look for a universal, is the comparison of individual instances which are

not identical but similar. To seek for a concept which included under it cucumbers and mathematical principles, could only be an ingenious joke; but all varieties of human beings, big and little, old and young, fat and thin, black and white, provoke the natural mind to the search. Their sensible appearances produce similar images, at the corresponding points of which only such marks occur as are immediately felt to be species of the same universal mark, such as hardness or colour; and the relations between any two of these points are in all cases merely modifications, differing in degree and amount, of one and the same universal relation. The comparison of individual men, therefore, produces a universal image; not indeed in the sense that the universal man can really be painted, but in the sense of the illustrations in a natural history, which purport by one camel or horse to exhibit all camels or horses clearly to perception, in a form which is more than a mere scheme or symbol; or again in the sense of geometry, in which a drawn triangle, though necessarily individual with others existing beside it, yet represents all these others, and in a similarly perceptible form. But this possibility vanishes when we ascend to higher universals, in which these universal images are themselves included in their turn as species: the universal mammal, which is neither horse nor camel nor is otherwise named, cannot even be drawn in a schematic form, any more than the polygon can which has neither three, four, or any other definite number of sides. Thus these higher universals are no longer apprehended in perception, but only in thought, by means of a formula or equation, which prescribes essentially the same relation between various related points, but leads to quite different perceptible configurations, accordingly as the previously undetermined values of these points and their various connexions are differently determined in thought. I would then call a universal which still admits of an image, a species, and the first of those which can only be expressed by a formula, a genus, in agreement, as I believe, with the instinct of language, and incidentally also with the old terminology of Aristotle; for in his choice of the words εἶδος and γένος he was no doubt determined by their original meanings; εἶδος, the species, which includes only individuals under it, is the common element in the look or appearance of things, while γένος comprehends things which differ in form, but in their procepts; this seems to me to be untrue where its truth would be important, and to be comparatively unimportant where it is true. The number of marks, of which we compose our concepts, is not infinite; the words of language, numerous but not innumerable, suffice to denote them. It may therefore easily happen that a group of them, say *i k l*, occurs in several universal concepts, *S T* and *V*, at once, without its therefore representing a higher universal containing all species of *S T* and *V*. We may class cherries and flesh under the group *i k l* of red, juicy, edible bodies, but we shall not suppose ourselves thereby to have arrived at a generic concept of which they deserve to be called species. I do not say that in giving exclusive prominence to such groups there is always as little sense as in this absurd instance; we shall see later how valuable the process may be; it helps to show, what is often useful and necessary, that different subjects, though otherwise quite foreign to one another and not subsumable under any common generic concept, are nevertheless, in consequence of a single or a few common marks, jointly liable to certain inevitable consequences. If then anyone chooses to go on to call these groups of marks universal concepts, he is certainly right about the inverse ratio of their content and extent: the fewer members there are in the group, the more sûre will it be to occur in all sorts of concepts, and again, the greater the number of different ideas compared, the smaller will be the group of marks in which they all agree. Of the true universal, on the other hand, which contains the rule for the entire formation of its species, it may rather be said that its content is always precisely as rich, the sum of its marks precisely as great, as that of its species themselves; only that the universal concept, the genus, contains a number of marks in a merely indefinite and even universal form; these are represented in the species by definite values or particular characterisations, and finally in the singular concept all indefiniteness vanishes, and each universal mark of the genus is replaced by one fully determined in quantity, individuality, and relation to others. It is true that instances may be alleged against the universal validity of this assertion, like that mentioned above of organic being, to the concept of which we subordinate plants and animals; it may be called a logical caprice to retain the marks of sensibility and motivity in this concept, with the tacit reservation that they are both at zero in plants. But what this instance properly shows is rather, that the higher universals, from the genus upwards, really cease to be true universal *concepts*, and pass over into groups of conditions, imposing uniform consequences upon various genera, more properly so called. The concept of organic being is such a group of marks, $i\,k\,l$, which does not occur in any independent form of its own, but in the genera in which it does occur, plants and animals, gives rise necessarily to the same results.

32. By the preceding remarks I neither hope nor aspire to bring about a permanent change in the traditional terminology: they were intended merely as helps to a clearer insight into the structure of concepts in general. With the same object I add the following. I express the genus G, so far as its concept gives the rule of combination for a number of individual marks $A\,B\,C$, etc., by $F\,[A\,B\,C]$, and I assume that each of the marks admits of particular forms, which we may call $a^1\,a^2\,a^3\,..\,b^1\,b^2\,b^3\,..\,c^1\,c^2\,c^3$; also that the principle of combination F has freedom to assume various forms, of which we may indicate three by f, ϕ, and $\mathfrak{f}$. Now as the marks $A\,B\,C$ may be of very different value for the whole G, it is possible that the different values assumed e. g. by A may be of decisive importance for the configuration of the whole, and may also exercise a transforming influence upon the combination of the other marks. The consequence of this may be that, as A assumes one or other of its values, the organisation of the whole, F, changes from one of its particular modes to another; the sum total of the species of G would then be,

$$G = f\,(a^1\,B\,C\ldots) + \phi\,(a^2\,B\,C\ldots) + \mathfrak{f}\,(a^3\,B\,C),$$

omitting for shortness' sake to express the corresponding changes in B and C. These decisive marks, $a^1\,a^2\,a^3$, are in this case the *specific* differences, *differentiae specificae*. Thus Aristotle, who gives them the name of διαφορά, when he subordinates man to the genus animal, usually describes the faculty of rational thought as that peculiar characteristic, a^1, of the universal psychical life, A, by which man is distinguished from all other animals; to this we may now add, following out what I have indicated above, that this a^1 not only separates man from brutes, but also determines the values of B and C peculiar to him, as also the mode of their combination, i. e. the general character by which man is distinguished from the brutes with their peculiar organisation ϕ or $\mathfrak{f}$. It may further happen that the particular values which one or more of the generic marks have assumed in a single species, are possible in this and no other species, and that yet they have no important influence upon the shaping of its other marks, and but without exhausting his nature. There are also, according to Aristotle, marks which do not belong to the rigid constitution of a concept, but indicate something which comes in contact with or happens to it; every verb which says, e.g., Socrates 'is sitting' or 'standing,' is an example. Translators torment themselves in vain to find an equivalent for both the real and the etymological sense of Aristotle's expression *συμβεβηκός*; what is important and true in it answers completely to what we call *state*; that this word does not nevertheless cover the usage of Aristotle seems to me to be the fault of an inexactitude of his own, which it is scarcely worth while to enter into. As to the relation in fact between the concept as a whole and this species of mark, its consideration belongs to the theory of the judgment. In the introduction of Porphyrius to the Aristotelian logic there is material enough for further reflexion, though indeed of a mostly unprofitable sort, about the likenesses and differences of the logical determinations here touched upon; we have used them primarily to illustrate the complex organisation of concepts, and with this view have not always agreed with Aristotle in the form of our exposition.

33. And now, where do we get to at last if we go on looking for higher and higher concepts above those which we have already found? What form does the entire system of our concepts assume if we suppose this task completed? It must be a structure resting on a broad base, formed by all singular concepts or ideas, and growing gradually narrower as it rises. The ordinary view, in fact, gives it the form of a pyramid, ending in a single apex, the all-embracing concept of the thinkable. I cannot see much point in this notion; it rests entirely upon that unmeaning subsumption under a mark, the logical value of which we have already depreciated. A single step suffices to bring everything at once under the head of the thinkable; we may spare ourselves the trouble of climbing up to this result by a pyramidal ladder; and moreover the result itself ignores in the most absolute and unmeaning way everything which gives substance and character to thought. If on the other hand we follow the method of subordination to the genus, and arrange the manifold only under such universals as still imply the notion of universally regulating its specific conformations, we arrive not at one but at several ultimate concepts not reducible to one another, in which we are not surprised to recognise those very meanings of the parts of speech which at the outset we found to be the primary logical elements. All substantives go back to the radical concept of something, all adjectives to that of quality, verbs to that of becoming, and the rest to that of relation. It is true that all these radical concepts have the common mark of being thinkable; but there is no common genus over them of which their several essences form species, nor does any one of them occupy this position in regard to the rest; it is not possible to apprehend something as a species of becoming, or becoming as a species of something. From this point of view the entire structure of our concepts rises like a mountain-chain, beginning in a broad base and ending in several sharply defined peaks.

Transition to the form of the Judgment.

34. It was this image of a conceptual world building itself up without a break, upon which the vision of Plato dwelt. The first to recognise the eternal self-identity of every concept and its significance as against the variableness of the real world, he might well feel the charm of tracing out all the simple elements of thought, of combining all that could be combined, and of setting up in the organic whole of a world of ideas the eternal pattern of which the created world is an imperfect imitation. But neither he nor his successors have attempted actually to execute this essentially impossible task: still less should we now be inclined to regard its execution as desirable. And this not only because reality, things as they are, suggests riddles too many and too hard to leave us any time for drawing up an inventory of what might be but is not; for even a perfect knowledge of the ideal world would give us little support in understanding the real. The utmost that we could attain by such means would be merely the image of a fixed order, in which simple and composite concepts stood side by side, each unchangeably self-identical and each bound to its place in the system by invariable relations to all the rest; whereas what reality shows us is a changing medley of the most manifold relations and connexions between the matter of ideas, taking first one form and then another without regard to their place in the system. This great fact of change does not cease to be a fact because, in the spirit of antiquity, we find fault with it as an imperfection compared with the solemn rest of the world of ideas: the current of our thoughts is perpetually bringing it before us again, and the mind, receiving as it same step next. When new marks, of which we were not before conscious in a concept, attach themselves to it without its apparently being changed, we are directly stimulated to ask what ground can be conceived for such a variable connexion of the two. But also when we compare different instances of a universal, in the universal marks of which we have already included the possibility of many particular ones, it may still be asked on what ground a particular mark in each instance coheres with the rest of the content, and why this particular mark is privileged above all the others which remain absent, though, as species of the same universal, they might equally well be present. Lastly, as we think of every concept as uniting a number of marks, and these marks, though not essentially related as members of one and the same systematic series, but rather heterogeneous and foreign to one another, nevertheless determine each other and in their combination influence the accession of others, the question again recurs, what is the ground of the apparent coherence in this coexistence of heterogeneous elements. We are conscious that when, in considering the concept, we attributed to a certain combination of marks this position of a dominant logical substance, operating in a number of different or changing forms, we required and presupposed a view which we have yet to show to be logically practicable. This then is our present problem, either to break up these presupposed combinations again, or, if they can be justified, to reconstitute them, but in a form which at the same time expresses the ground of coherence in the matter combined. In seeking to solve this problem, the form in which thought will move will obviously be that of the *judgment*. In this a permanent conditioning member, the whole content of a concept, appears as *subject* over against the variable or conditioned members or the sum of them, as *predicates*; the relation of the two, explaining and justifying their connexion, lies in the *copula*, that is, in the accessory notion which, more or less fully expressed in language, holds together the two members of the sentence.

CHAPTER II.

The Theory of the Judgment.

Preliminary observations on the meaning and customary division of Judgments.

In accordance with the general plan of my exposition, I should now have to develop the various forms of judgment systematically as members of a series of intellectual operations, each one of which leaves a part of its problem unmastered and thereby gives rise to the next. Before beginning this attempt, I must say a few words about other usual modes of treatment, and my reasons for deviating from them.

36. Every judgment formed in the natural exercise of thought is intended to express a relation between the matters of two ideas, not a relation of the two ideas themselves. Of course some sort of relation between the ideas follows inevitably from the objective relation in the matter which they represent; but it is not this indispensable relation in the mental media through which we endeavour to grasp the matter of fact, but this matter of fact itself, which is the essential meaning of the act of judgment. When we say, 'gold is yellow,' it is indisputable that in this judgment our idea of gold lies within the sphere of the idea of yellow, and that accordingly the predicate is of wider extent than the subject; but it was certainly not this that we intended to express by the judgment. We wanted to say that yellow itself belongs as a property to gold itself, and only because this relation of fact is already presupposed to exist (whatever difficulties this may involve), can it be reproduced in a sentence in which the idea of gold is contained by that of yellow. Logic indeed has already drawn attention to the fact that we are not quite right even in making this sentence;

imperfection of this whole view of the judgment is not mended; for it does not tell us what is after all the relation between the two members so corrected, and it loses sight entirely of the great possible variety in the modes of their connexion. Thus gold is not yellow in the dark; its colour therefore only attaches to it under a condition, that of the presence of light; and if we wished to connect this new experience with the previous one in the phraseology of the view which we are now considering, we should have to say, the idea of gold lies simultaneously within the spheres of that which is yellow in the light and of that which is not yellow in the dark; but this form of expression seems to me only to betray a disposition to leave the really important point, the mention of the conditional relation, and to go off upon results which are true but quite without significance. Doubtless these relations of extension between the ideas combined in the judgment have their logical value; but where the want of them is felt, they are not so difficult but that they can be mastered at the moment without special effort: to give them a chief place in the consideration of the judgment seems to me to be as erroneous as it is wearisome.

37. The technical expressions of logic point to the view which I have taken here. In the judgment above the *subject* in the sentence, that is, the grammatical subject, is the *word* gold, the subject in the judgment, the logical subject, is, not the idea of gold, but gold; for it is to this only that yellow belongs as that which is predicated of it, and predicated in a definite sense indicated by the copula. On the other hand, the idea of yellow is not a property of the idea of gold in the same sense in which yellow is of gold; the one idea is not affirmed or predicated of the other; the relation which exists between them is primarily no more than this, that whenever, or whenever under certain conditions, the one idea, gold, is found, there the other idea, yellow, is also found, but that the former is not always present when the latter is. But to explain and express what it is which makes this relation possible, justifiable, or necessary, is the problem of the logical judgment alone, and it solves the problem by exhibiting through its copula the relation between the object-matters of the two ideas, a relation due to that which the ideas represent and differing in different cases. On the other hand, it is only between these object-matters that a logical copula is conceivable; between the ideas there is no relation but that of the psychological connexion mentioned above, and that of the monotonous, unmeaning inclusion of the one within the other.

38. It is now clear that for us there can be only so many essentially different forms of judgment as there are essentially different meanings of the *copula*, that is, different accessory notions which we form of the connexion of the subject with its predicate, and to which we give more or less complete expression in the syntactical form of the sentence. Thus many other distinctions which meet us in logic have no use or place in our systematic survey, though they may still have a logical value of some other kind. To secure clearness in what is to follow, therefore, it is desirable to give a preliminary explanation of traditional views; but I think I may confine it to that division of judgments to which Kant has given currency in Germany, though it is itself of much older date. According to Kant, as we know, the character of every judgment is determined in four respects, quantity, quality, relation, and modality, and in each respect every judgment has necessarily one of three mutually exclusive forms. I may exclude the third member of this division from these preliminary considerations, for *relation* (between subject and predicate), in respect of which Kant distinguishes categorical, hypothetical, and disjunctive judgments, clearly concerns just those essential characteristics of the judgment which we are looking for, and which I shall have subsequently to expound myself. If the categorical judgment connects its subject *S* and its predicate *P* *absolutely*, as the phrase is, or on the simple model of the relation of a thing to its property, while the hypothetical assigns *P* to *S*, not immediately, but only on the assumption that a certain condition is fulfilled, and the disjunctive gives *S* no definite predicate, but imposes on it the necessity of choosing between several mutually exclusive ones, there is no doubt that in each of these three forms the sense of the copula, the mode of connexion between *S* and *P*, is different and peculiar; these three will form the series of judgments which we shall have subsequently to construct; only the nine remaining ones call for the following preliminary remarks.

39. In respect of their *quantity* judgments must be either *universal* or *particular* or *singular*. If we express these distinctions by the usual formulæ, 'all *S* are *P*,' 'some *S* are *P*,' 'this *S* is *P*,' it is clear that they indicate merely the different extents to which a connexion between *S* and *P* is supposed to hold good; the nature of the connexion in all the cases is the same, and must be the same, because the universal judgment, according to this view of its meaning, admits therefore of importance where the connexion of ideas requires the application of a judgment, the import of which depends upon the circuit over which it holds good; but no special advance in logical activity is indicated by these distinctions as they are here formulated. I say 'as they are here formulated,' because certainly the quantitative differences of judgments are really connected with important logical differences in the mode of connexion between *S* and *P*; for doubtless that which belongs to all *S* has also a different hold upon the nature of its subject from that which belongs only to some; but the quantitative formulation of the judgment, which merely *counts* the subjects, just fails to seize this important accessory notion, and makes the relation of the predicate to its subject, often in violation of the fact, appear the same in all cases.

40. In respect of *quality* Kant distinguished *affirmative*, *negative*, and *limitative* judgments. Nothing is clearer than that the two sentences '*S* is *P*,' '*S* is not *P*,' so long as they are supposed to be logically opposed to one another, must express precisely the same connexion between *S* and *P*, only that the truth of that connexion is affirmed by the one and denied by the other. It is useful, though certainly not necessary, to make this clear to ourselves by splitting each of these judgments into two. We think of a certain relation, whatever it may be, between *S* and *P* expressed in the judgment '*S* is *P*' as an idea still open to question; this relation forms the object-matter upon which two opposite judgments are passed; the affirmative gives it the predicate of validity or reality, the negative refuses it. In the connexion of our thoughts it is of course of the greatest importance which of these judgments is subsequently passed upon a given connexion between *S* and *P*; but this difference does not give rise to two essentially different kinds of judgment as such; validity or invalidity are rather to be considered, in regard to the question before us, as predicates of fact to which the whole content of the judgment forms the subject. This content itself can be expressed in a form as yet neither affirmative nor negative in the interrogative sentence, and this indeed would take the third place amongst the three qualities of judgment more appropriately than the limitative or infinite judgment, which is supposed to attribute a negative predicate to the subject by a positive copula, and is usually expressed in the formula '*S* is not-*P*.' Much acumen has been expended even in recent times in vindicating this form of judgment, but I can only see in it an unmeaning product of pedantic ingenuity. Aristotle himself saw clearly enough that such expressions as 'not-man' are no concepts; they are not even apprehensible ideas. The truth is that, if 'not-man' means all that it ought logically to mean, that is, everything that is not man, triangle, melancholy, sulphuric acid, as well as brute and angel, it is an utterly impossible feat to hold together this chaotic mass of the most different things in any *one* idea, such as could be applied as a predicate to a subject. Every attempt to affirm this unthinkable not-*P* of *S* will be found by an unsophisticated mind to end in denying the thinkable *P* of the same *S*; instead of saying, 'spirit is not-matter,' we all say, 'spirit is not matter.' Even in cases where in natural thinking we seem really to make a limitative judgment, as e.g. when we say 'doctors are non-combatants,' we are in truth making only a negative one. For this not-*P* has not here the meaning which the limitative sentence would give it; according to that, horses, wagons, triangles, and letters would be non-combatants; what is meant is only human beings who belong to the army but are declared to take no part in fighting. Thus there is never any necessity to the natural mind for forming limitative judgments; every inference which could be drawn from '*S* is not-*P*' can also be drawn from '*S* is not *P*.' It is not worth while to spend more words on this point; obvious vagaries in science must not be propagated even by a too elaborate polemic.

41. Through the forms of *modality* different values are supposed to be given to the relation which is conceived to hold between *S* and *P*; the *problematic* judgment expresses it as merely possible, the *assertorial* as real, the *apodeictic* as necessary. But these new properties are treated quite independently of the way in which judgments have been already determined from the other three points of view. After it has been fixed whether a given judgment *J* connects its elements in categorical, hypothetical, or disjunctive form, after it has been decided whether it affirms or denies the relation conceived in

This, the possibility, reality, or necessity of a relation between *S* and *P*, they express either without any grounds at all, or upon grounds derived from right reflexion upon the facts, which they do not then allow to appear in any way in their logical structure; just for this reason they need additional auxiliary verbs, in order to express independently what does not lie in the form of the judgment itself. In more developed connexions of thought such judgments of course have their value; for what is wanted is often to compress results of previous reflexion into the shape of simple assertions, without perpetually repeating the grounds upon which they rest; here these auxiliary verbs are in place, expressing in the form of a now familiar fact the possibility, reality, and necessity which once had a logical justification. But for the separation of essential forms of judgment and their systematic arrangement, the only modality that could be of value would be one which, instead of going its own way independently of the logical nexus of the other judgments, grew out of that nexus itself, and expressed the claim to possible, real, or necessary validity, which the content of the judgment derives from the mode in which its elements are combined.

42. It would be useless to ask for such a modality, if we could not show the possibility of it. I will therefore anticipate somewhat what I have to say later. The proposition, 'all men must die,' is usually held to be apodeictic; I consider it merely assertorial; for it states only, and does not give grounds for, the necessity of which it speaks; so far as its form goes it does not even decide whether all men die for the same reason, or everyone for a special reason, so that the various conditions agree merely in the fact that they leave no one alive. And yet what we had meant by the sentence was, not only that all men as a matter of fact die, but that the extension of mortality to all has its ground in the universal concept of man, in the nature of humanity; and this thought we do in fact express by the general form of the judgment 'man dies'; for the sense of this judgment, the difference of which from the ordinary universal I shall come back to, is not of course that the universal concept man dies, but that everything dies which is included under it, and for the reason that it is so included. Every hypothetical judgment, again, gives in its protasis the ground for what is stated in its apodosis, and is therefore in my sense an apodeictic form of judgment; the apodosis here is not simply asserted, but asserted conditionally upon the validity of the protasis; but, presupposing that validity, the content of the apodosis is no longer a mere fact, but a necessity, with the same right with which every consequence necessarily follows from its conditions. Similar remarks might be made, if they would not be too long for this preliminary section, about the disjunctive judgment; and thus we should have found in the three forms of relation three forms also of apodeictic modality.

43. I will guard myself against a misunderstanding, though it would be so gross that I am almost ashamed to do so. The form which we give to the content of a judgment can never guarantee its truth to fact; this always depends upon whether the relations between the elements of the content itself are truly such as the form of the judgment, in order to ascribe to them a certain sort of validity, has to presuppose. This holds good of the ordinary modality no less than of that which we would put in its place. In the ordinary form of the apodeictic judgment, '*S* must be *P*,' any nonsense may be expressed without thereby becoming sense; and it is equally open to us to misuse the judgments which I call formally apodeictic, and say 'man is omnipotent,' 'if it rains everything is dry,' 'every triangle is either curved or sweet or hasty-tempered.' These latter forms of judgment, then, do not, any more than the former, make every connexion of concepts which is put into them true or necessary; the significance of them lies merely in showing the formal conditions under which we may ascribe demonstrative certainty to a given content, *if* that content is in itself such as to satisfy them. And here our view of modality differs to its advantage from the ordinary one. The latter merely tells us that there is demonstrative knowledge, and that, if we have got it, we can express it in the form '*S* must be *P*'; but it does not tell us how knowledge must look, and what its internal structure must be, in order to be demonstrative and to justify this expression. Our plan on the other hand does show us this; we find that there are three forms of relation between *S* and *P*, which, when they exist, lead to necessary knowledge; endeavour to bring your ideas into one of these forms; either frame general judgments and look for the *P* which is already implied in the conception of a genus *S*; this *P* then belongs necessarily to every species of *S*: or form hypothetical judgments, and show that the addition to *S* of a condition *X* gives rise to a *P* which would not otherwise be present; then this *P* holds necessarily of every *S* which comes under the same operation of the same conditions: or lastly form disjunctive judgments; as soon as you have brought a question to a definite 'either . . or,' the thing is settled, and all that is now wanted is experience to determine, in each particular instance, which of two predicates, *P* or *Q*, will be true and necessarily true. There are no other ways of arriving at necessary knowledge, and every judgment which we express in the form, '*S* must be *P*,' remains merely an assertion, the matter of which, if it is convincing, has always been originally apprehended in one of those three ways.

44. Thus far I have spoken only of apodeictic judgments: the ambiguity of the ordinary theory of modality is still more striking in the case of *problematic* judgments. The proposition, 'all bodies can be set in motion by adequate forces,' may have any one of the three modalities ascribed to it with about equal right. Firstly, as a statement which does not add the grounds upon which it is made, it is assertorial: but what it states is not a real occurrence, but the possibility of an unreal or only conceived one, and this is enough according to traditional usage to give it the name of problematic: lastly, it may be called apodeictic, because it ascribes a property to all bodies, a property therefore which can be wanting in none and is accordingly necessary to each: in fact, this judgment contains the reality of the necessity of a possibility. From which point of view are we to choose its name? I should be in favour of regarding it as an assertorial judgment, reckoning the necessary possibility as part of the matter asserted. As however the same view may be extended to all problematic judgments of the ordinary form, the question arises whether there is any form of judgment at all which, as such, deserves to be called problematic. Interrogations and prayers have been alleged as instances, for neither of them really asserts anything; the connexion of *S* and *P* which forms their content seems to be presented to the mind as no more than a floating possibility. I doubt however whether they can be considered as specific logical forms at all. For ultimately interrogation must be distinguished from prayer, and the distinction can only lie in the fact that the conscious attitude of the questioner to his question is different from that of the petitioner to his petition. Suppose the import of the question to be, 'I do not know whether *S* is *P*,' and that of the petition, 'I wish that *S* were *P*'; it would of course be very pedantic to say that the speaker himself must always analyse what he says into this bipartite form, but still, if we take his consciousness as a whole, it must contain in both cases two different states, tempers, dispositions, or whatever we call them, which, *if* we wished to express them, could only be expressed in those ways. If this is so, it is clear that both judgments contain a principal sentence of an assertorial form, which says nothing about the content of the judgment, but merely indicates the attitude of the speaker to it; the other and dependent sentence, introduced by the conjunctions 'whether' or 'that,' comprises the whole content, without saying anything about the nature and degree of its validity. It is for this reason that I do not consider the dependent sentence either to be a problematic judgment; for it is not enough that the account of the nature of the import should be merely *absent*; the import ought to be explicitly confined to mere possibility. As to the prayer, it might further be said that it contains the possibility of what is prayed for and nothing else, whereas the question, as it may be a question about possibility itself, does not always do even that: in both moreover the assumption of the possibility of a conceived connexion between *S* and *P* could only be reckoned as a state of the speaker's mind, and would not lie in the logical form of the judgment. I should rather consider this dependent sentence to express without any modality the mere content of a judgment; and it is just because no complete judgment can be expressed without claiming possibility, reality, or necessity for its import, that these sentences void of modality never occur independently, but are always governed by some other independent sentence which asserts one of those modalities of its content.

45. According to our view those judgments only could be called problematic which by their logical form characterise a conceived relation between *S* and *P* as possible and only as possible. This is done by all quantitatively particular and singular judgments. All that is directly expressed by sentences of the form, 'Some *S* are *P*,' 'Some *S* may or must be *P*,' 'This *S* is *P*' or 'may or must be *P*,' is the actual, possible, or necessary occurrence of *P* in certain cases of *S*; they leave it doubtful how the matter stands with the other cases of *S* which are not mentioned; for *S* as such, therefore, it is only the possibility of each of these three relations to *P* which is expressed, and these particular sentences are equivalent to the assertions, '*S* may be *P* possibly,' '*S* may be *P*,' '*S* may be *P* necessarily.' I therefore call particular sentences problematic in respect of the universal *S*; the fact that they are clearly also assertorial in respect of the some *S* of which each speaks, does not at all militate against my view; it only shows us that in fact the only way of recognising a certain relation between *S* and *P* to be merely possible is by observing that the relation does, may, or must hold good of some *S* and not of others. There are therefore certainly no independent problematic judgments, which are not assertorial in respect of a part of their universal subject in so far as they affirm of it a possible, actual, or necessary predicate.

46. Lastly, it is easy to see that, on the one hand, the 'may' and 'must' of the ordinary problematic and apodeictic judgments and the 'is' of the assertorial by no means suffice to express all material differences of importance in the truth of their several contents, and that on the other hand, just for this reason, they lump together very different relations under the same expression. Firstly, what modality have such sentences as these, '*S* will be *P*,' '*S* ought to be *P*,' '*S* may be *P*,' '*S* has been *P*'? No one of them affirms reality, but the unreal which is past in the last is something quite different from that which is permitted, enjoined, or future, in the others: in the third it is possible, in the second its possibility is doubtful, in the first its reality is inevitable, while in the last it is at once irretrievable and unreal. If all these shades of meaning had been taken into account, the forms of modality might have been correspondingly increased in number. On the other hand, how entirely different in meaning are the similarly formed sentences, 'It can rain to-day[1],' 'The parrot can talk,' 'Every quadrangle can be divided into two triangles.' In the first case we have a supposition which is possible because we know no reason to the contrary; next a capacity which exists upon conditions which need not have existed; lastly a necessary result of an operation which we may carry out or not as we please. I will not multiply these instances, as might be done indefinitely; to attempt to analyse them all would be as foolish as to undertake to work out beforehand all possible examples in a mathematical text-book. In practice, indeed, it is just from these material varieties of meaning in the expressions in question that our inferences are drawn; but we have no resource except to observe in each particular instance what we have before us; whether it is a possibility which may be tentatively assumed in the absence of proof to the contrary, or a well-grounded capacity resting securely upon its conditions; whether it is a necessity due to the presence of imperative reasons, or one arising from a command, a purpose, a duty, or lastly one of those combinations of possibility, reality, and necessity which we touched upon above.

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[1] ['Es kann heute regnen; der Papagei kann reden;' in English we say, 'It *may* rain to-day,' so that the difference of meaning is represented by some difference of form.]

The series of the forms of Judgment.

A. *The Impersonal Judgment. The Categorical Judgment. The Principle of Identity.*

47. There can be no doubt that in the series of the forms of judgment the categorical comes before the hypothetical and the disjunctive. We could have no occasion for making the occurrence of a predicate *P* in a subject *S* dependent on a previously fulfilled condition, unless we had already had experiences of the presence of *P* in some *S* and its absence in others. Equally little can we think of prescribing to *S* the necessary choice between different predicates, until previous experiences have established the constant relation of *S* to a more universal predicate, of which the proposed alternatives are specific forms; and these experiences too would find their natural expression in a judgment of the form '*S* is *P*.' The structure, moreover, of the hypothetical and disjunctive judgments exhibits permanent traces of this dependence: however complex they may be in particular cases, the general scheme to which they are reducible is always that of two judgments of the form '*S* is *P*,' combined, either as protasis and apodosis or as mutually exclusive members, so as to form a single complete assertion. But the question may be raised whether a still simpler form must not precede the categorical judgment itself in the systematic series. The sentence '*S* is *P*' cannot be uttered until the current of ideas has informed us of an *S* with a fixed position and recognisable character of its own, to which a *P* can be added in thought as a predicate. Now this will not always be the case; indeed it may be questioned whether the discovery of the definite *S*, which is to serve as subject to a categorical judgment, does not always presuppose experiences of *S* in a less developed form, and their translation into logical equivalents. This question, which relates to the psychological growth of thought, I leave unanswered here; for our present purpose the fact is enough that even our fully developed thought has preserved a form of judgment which performs this simplest of functions, that of giving logical setting to a matter of perception without regarding it as a modification or determination of an already fixed subject. This is the *impersonal* judgment, which, as the first act of judging, I here treat as a preliminary stage to the categorical.

ıg any logical activity. The natural sound which a man who
ering with cold makes when he cowers against another, is a
gn of this sort, which only serves to give tongue to his feeling;
soon as he expresses his discomfort in the sentence 'it is cold,'
undoubtedly performed an act of thought. By giving to the
: of his perception, which in itself is undivided, this bipartite
f a predicate related to a subject by a copula, he expresses that
think of it as a perceived reality in no other form than this. It
that he is not in a position to give the subject an independent
t; he only indicates its empty place and the fact that it requires
either by the indefinite pronoun, or in other languages by the
erson of the verb, which he uses instead of the infinitive: it is
so that the whole content of the perception which he expresses
to the predicate alone: and it is true, lastly, that the copula
ne puts between them has not as yet the sense of a definitely
sible relation; it only keeps formally apart what is substantially
able and interfused. But it is just by this attempt to bring
ın articulation to which the matter of perception will not yet
ıelf, that the impersonal judgment expresses all the more clearly
ıtinct of thought, that everything which is to be matter of
ion must be conceived as a predicate of a known or unknown
.

I will now explain why I have here spoken repeatedly of per-
.[1]. The indefiniteness of the subject in the impersonal judgment
en interpreted to mean that it merely expresses in substantival
hat is expressed in verbal form by the predicate. I do not
hat anyone who is asked what he means by 'it,' when he says
s,' or 'it thunders,' can easily be driven to say, 'the rain rains,'
thunder thunders.' But I believe that in that case his em-
ment makes him say something different from what he really
d by his impersonal judgment. It seems to me to lie in the

the various phenomena attach as predicates or from which they proceed; it indicates the all-embracing thought of reality, which takes now one shape, now another. This has been rightly felt by those who found in the impersonal judgment a judgment of *existence*, and transformed the sentence 'it lightens' into 'the lightning is.' It is only the transformation itself which seems to me unnatural; we never express ourselves in this way; the unsophisticated mind does not think of the phenomenon as if it were already something before it existed, of which we could speak, and of which among other things we could assert reality; on the contrary, it regards the particular reality in question as a phenomenon, a predicate, a consequence, proceeding along with others from an antecedent and permanent, though quite inexpressible, subject. Though however we cannot accept this explanation, it is so far right as that every genuine impersonal judgment expresses an actually present perception, and is therefore as regards its form an *assertorial* judgment. Such genuine judgments are to be distinguished from other modes of expression which begin with the indefinite 'it' as subject, but immediately fix its content by an explanatory sentence, as, e. g., 'it is well that this or that should be done.'

50. The more definitely the mind emphasizes the necessity of the subject to which the predicate is to attach, the less can it rest content with an expression in which this demand is unsatisfied. It is not part of my logical task, as I have already said, to describe the processes of comparison and observation by which our ideas of those subjects are gradually formed, which we require to take the places of the indefinite 'it' in the various impersonal judgments; I have only to point out the logical form in which this requirement is satisfied. Most of the simple instances with which logic usually begins its illustration of the judgment in general, are in the familiar form of the categorical judgment 'S is P,' e.g. 'gold is heavy,' 'the tree is green,' 'the day is windy.' No explanation is needed as regards this form; its structure is perfectly transparent and simple: all that we have to show is, that this apparent clearness conceals a complete enigma, and that the obscurity in which the sense of the copula in the categorical judgment is involved will form a motive that will carry us a long way in our successive modifications of logical activity.

51. A certain embarrassment is at once observable as soon as we ask in what sense S and P are connected in the categorical as distinct from the hypothetical and disjunctive judgments. A common answer

answer is only negatively satisfactory, i. e. so far as it denies of the categorical sentence the idea of a condition and the idea of an opposition between mutually exclusive predicates; but when we know what this form of judgment does not do, the statement that it joins *P* to *S* absolutely gives us no positive information as to what it does do. Such a statement in fact merely expresses the greater simplicity of the categorical copula as compared with that of the hypothetical and disjunctive judgments; but this simpler connexion must still have a determinate and expressible sense of its own, distinguishing it from other conceivable forms of connexion equally simple or more complicated. The necessity of explaining this sense appears most simply from the fact, that, of all connexions of *S* and *P*, the complete identity of the two would be that which most obviously deserved the name of absolute. Yet it is just this which as a rule is *not* intended in the categorical judgment: 'gold is heavy' does not mean that gold and weight are identical; equally little do such sentences as 'the tree is green,' 'the sky is blue,' identify the tree with green and the sky with blue. On the contrary, we are at pains to express our real meaning in such judgments by saying, '*P* is not *S* itself, but only a predicate of *S*,' or '*S* *is* not *P*, it only *has* *P*.' We thus admit that we are thinking of a definite and distinguishable relationship between *S* and *P*, and it only remains to make really clear what constitutes this 'having' which we oppose to 'being,' or, in more logical language, wherein we have to look for that relation of a *subject* to its *predicate* which we wish to distinguish from the relation of identity.

52. Plato was the first to touch this problem; his doctrine, that things owe their properties to participation in the eternal universal concepts of those properties, was rather an inadequate answer to a metaphysical question about the structure of reality, than an explanation of what we have in our mind when we establish a logical relation between subject and predicate. Aristotle made the right treatment of the question possible by observing that the attributes are primarily enunciated of their subjects; this at any rate established the fact that it is a logical operation of the mind which refers the matter of the one concept to that of the other; but more than this name of enunciation, κατηγορεῖν, from which that of the 'categorical' judgment and that of the Latin equivalent 'predicate' are derived, even Aristotle did not discover. He escaped indeed a confusion of later logic; he did not reduce the connexion which he supposed between *S* and *P* from a logical operation to a mere psychical occurrence, thus making the relation between the two consist only in the fact that the idea of *P* is associated in our consciousness with that of *S*: for him the sense of the judgment and the ground for making it was a real relation between the matters of the two ideas. But he did not tell us how precisely *S* is affected by the fact that we enunciate *P* of it; he made the enunciation itself, which can really do nothing but recognise and express this real relation, stand for the very relation which it had to recognise. Now it is easy to see that this fusion is quite inadmissible; it is impossible merely to enunciate the concept 'slave' of Socrates in such a way that the enunciation itself should settle the relationship in which the two concepts stand to one another: what we really mean by a judgment is always, that Socrates is or is not a slave, has or has not slaves, liberates or does not liberate slaves. It is one or other of these possible relations which constitutes *what* is enunciated in each case, and it is only a matter of linguistic usage that, when we speak of enunciating the latter concept of the former, we choose tacitly to understand only the first relation, viz. that Socrates *is* a slave. The relation, therefore, of *S* to *P* in a categorical judgment is not distinguished from other relations by saying that *P* is enunciated of *S*; the truth rather is that the meaning of this enunciation, in itself manifold, is determined by the tacit supposition that *P* is enunciated of *S* as *a predicate of a subject.* It still remains a further question, what constitutes this peculiar relation.

53. We moderns are accustomed on this point to hold to the doctrine of Kant, who represented the relation of a *thing* to its *property*, or of substance to its accident, as the model upon which the mind connects *S* and *P* in the categorical judgment. This statement may have a good meaning in the connexion in which Kant made it, but it does not seem to be available for the logical question before us. I will not here raise the point whether the idea of the relation between substance and attribute is itself so clear and intelligible as to dissipate all obscurity from the categorical judgment; it is enough to remind ourselves that logical judgments do not speak only of what is real, of things; many of them have for their subject a mere matter of thought, something unreal, or even impossible. The relation existing between the real thing as such and its properties obviously cannot be transferred in its full sense to the relation of subjects to their predicates, but only in the metaphorical or, as we may say, symbolical sense. To speak more exactly, the only common element in these two kinds of relation inherence. But in regard to the thing, metaphysic has at any rat exerted itself to show how there can be properties which are not th thing and yet attach to it, and what we are to suppose this attachmer to consist in; whereas in regard to the relation between subject an predicate we find no corresponding account of the sense in which th one inheres in the other. The appeal to the relation between thin and property, therefore, does not help logic at all; the questio repeats itself, How much of this *metaphysical* relation survives as *logical* relation expressible in the categorical judgment, if the thing b replaced by something which is not a thing, and the property b something which is not a property?

54. Without adding any more to these customary but unsuccessf attempts to justify the categorical judgment, I will state the conclusio to which we are driven: this absolute connexion of two concep *S* and *P*, in which the one is unconditionally the other and yet bot stand over against each other as different, is a relation quite impracticable in thought; by means of *this* copula, the simple 'is' of th categorical judgment, two different contents cannot be connected all; they must either fall entirely within one another, or they must remai entirely separate, and the impossible judgment '*S* is *P*' resolves itse into the three others, '*S* is *S*,' '*P* is *P*,' '*S* is not *P*.' We must not stumbl too much at the startling character of this assertion. Our minds are s constantly making categorical judgments of the form '*S* is *P*,' that n doubt what we *mean* by them will eventually justify itself, and we sha soon see how this is possible. But the categorical judgment *requir* such a justification; taken just as it stands it is a contradictory an self-destructive form of expression, in which the mind either represen as solved a hitherto unsolved problem, the determination of the relation between *S* and *P*, or so abbreviates the discovered solution th their connexion is no longer visible. On the other hand we are m by the consciousness that all our thought is subject to a limitation has to conform to a law; by the conviction that in the categoric judgment each constituent can only be conceived as self-same. Thi exactly what sense I shall myself attribute to it in opposition to many of those interpretations. In the case of an ultimate principle, which limits the whole of our thinking, it is obvious that with the application of thought to different groups of objects it will be transformed into a number of special principles, which exhibit its general import in the particular forms in which it applies to the particular characteristics of those groups and has an important bearing upon them. The consequences thus drawn from the principle of identity, some of which are quite unexceptionable while others are by no means so, must be distinguished from the original sense of the principle itself, and do not belong to this part of logic. Thus it is quite useless to expand the expression of the law into the formula, Everything can have at the same moment and in the same part of its whole self only one predicate *A*, and cannot have at the same time a predicate non-*A* contrary or contradictory to *A*. This statement is certainly correct, but it is no more than a particular application of the principle to subjects which have the reality of things, are composed of parts, and are capable of change in time. On the other hand it is incorrect to distinguish, as is often done tacitly and not less often explicitly in formulating this principle, between *consistent* predicates, which can belong at the same time to the same subject, and others which cannot because they are inconsistent with one another and with the nature of the subject. In the applications of thought, of course, this distinction too has its validity, when it has justified itself before the law of identity; but, taken as it stands, that law knows nothing of predicates which, though different from *S*, are still so far consistent with it that they could be combined with it in a categorical judgment; on the contrary, *every* predicate *P* which differs in any way whatever from *S*, however friendly to *S* it might otherwise be conceived to be, is entirely irreconcileable with it; *every* judgment of the form, '*S* is *P*,' is impossible, and in the strictest sense we cannot get further than saying, '*S* is *S*' and '*P* is *P*.' The same interpretation of the principle must also be maintained against other metaphysical inferences which are drawn from it. It may be that in the course of metaphysical enquiry it becomes necessary to make such assertions as, What is contradictory cannot be real, What is must be unchangeable, and the like: but the logical law of identity says only, What is contradictory is contradictory, What is is, What is changeable is changeable: all such judgments as

B. *The Particular Judgment. The Hypothetical Judgment. The Principle of sufficient Reason.*

56. It would be wearisome to stay longer at a point of view in which we could never permanently rest: we will follow thought to the new forms in which it tries to bring its categorical judgments into harmony with the law of identity. Judgments of the form '*S* is *P*' are called synthetical, when *P* is understood to be a mark not already contained in that group of marks which enables us to conceive *S* distinctly; they are called analytical when *P*, though not identical with the whole of *S*, yet belongs essentially to those marks the union of which is necessary to make the concept of *S* complete. In the analytical judgment people have found no difficulty; but the synthetical attracted attention at an early period, and Kant's treatment of it in particular has recently made it conspicuous. He too however was mainly interested in accounting for the possibility of synthetical judgments *a priori*, i.e. such as assert an existing and necessary connexion between *S* and a concept *P* not indispensable to *S*, without the need of appealing to the experience of its actual occurrence: as to synthetic judgments *a posteriori*, which merely state that such a connexion between two not mutually indispensable concepts is found or has been found in experience, he regarded them as simple expressions of facts and therefore free from difficulty. // These distinctions may be fully justified within the circle of enquiry in which Kant moved; but our logical question as to the possibility of categorical judgments extends to all three forms with equal urgency. // The necessity of justification before the principle of identity is only more obvious in the case of the *a priori* synthetical judgment, which formally contradicts that principle; but it holds good of the *a posteriori* also. For a judgment does not simply reproduce the fact like a mirror; it always introduces into the observed elements of the fact the thought of an inner relation, which is not included in the observation. Experience shows us only that *S* and *P* *are* together; but that they are inwardly *connected*, as we imply when we predicate *P* of *S* in the judgment, is only the interpretation which our mind puts upon the fact. How this relation can subsist between subject and predicate in general, and between *S* and *P* in particular, is just as obscure after experience has shown the coexistence to be a fact as when we assert it in anticipation of experience. Lastly, analytical judgments raise the same difficulty. However much yellow may be already contained in the concept of gold, the judgment 'gold is yellow' does not assert merely that the idea of yellow lies in the idea of gold, but ascribes yellowness to gold as its property; gold must therefore have a determinate relation to it, which is not the relation of identity. This relation has to be explained, and the question still remains, What right have we to assign to *S* a *P* which *is* not *S*, as a predicate in a categorical judgment?

57. The only answer can be, that we have no right: the numberless categorical judgments of this form which we make in daily life can only be justified by showing that they *mean* something quite different from what they say, and that, if we emphasize what they mean, they are in fact identical judgments in the full sense required by the principle of identity. The first form in which we get a hint of this in the natural course of thought is that of *quantitative* judgments in general, which I shall in future call shortly *particular*, and consider as the first form of this second group of judgments. Under this title I include not only the traditional forms, such as, 'all *S* are *P*,' 'some *S* are *P*,' this '*S* is *P*,' which have for their subject a number of instances of the general concept *S*, but those also which in various other ways limit to definite cases, and therefore particularise, the universal application of the connexion between *S* and *P*, whether by particles of time (now, often, etc.), or by those of space (here, there, etc.), or again by a past or future tense of the verb, or lastly by any kind of accessory idea, imperfectly expressed or not expressed at all. In the general formula of the categorical judgment, '*S* is *P*,' it looks as if the universal *S* were the subject, the universal *P* its predicate, and the constant, unchangeable, and unlimited connexion of *S* and *P* the import of the whole judgment. If on the other hand we supply explicitly what is suggested, or at any rate is meant, by these particularising accessory ideas, we find that the true subject is not the universal *S*, but Σ, a determinate instance of it; that the true predicate is not the universal *P*, but Π, a particular modification of it; and lastly that the relation asserted is not between *S* and *P*, but between Σ and Π, and that this relation, if the supplementary ideas are correct, is no longer a synthetical, nor even an analytical one, but simply one of identity. A few instances will make this clear.

it is not left to our choice what individuals we will take out of the whole mass of men; our selection, which makes them 'some' men, does not make them black if they are not so without it; we have, then, to choose those men, and we *mean* all along only those men, who are black, in short, negroes; these are the true subject of the judgment. That the predicate is not meant in its universality, that on the contrary only the particular black is meant which is found on human bodies, is at once clear, and I shall follow out this remark later; here I will only observe that it is merely want of inflexion in the German expression which deceives us as to its proper sense; the Latin 'nonnulli homines sunt nigri' shows at once by number and gender that 'homines' has to be supplied to 'nigri.' The full sense, then, of the judgment is, 'some men, by whom however we are only to understand black men, are black men;' as regards its matter it is perfectly identical, and as regards its form it is only synthetical because one and the same subject is expressed from two different points of view, as black men in the predicate, as a fragment of all men in the subject. Again, we say, 'the dog drinks.' But the universal dog does not drink; only a single definite dog, or many, or all single dogs, are the subject of the sentence. In the predicate too we mean something different from what we express: we do not think of the dog as a sort of ever-running syphon; he does not drink simply, always, and unceasingly, but now and then. And this 'now and then' also, though expressed as an indefinite number of moments, is not so meant; the dog drinks only at definite moments, when he is thirsty or at any rate inclined, when he finds something to drink, when nobody stops him; in short, the dog which we mean in this judgment is really only the drinking dog, and the same drinking dog examples, which Kant has made famous. It is said that the judgment, 'a straight line is the shortest way between two points,' is synthetical, for neither in the concept 'straight' nor in that of 'line' is there any suggestion of longitudinal measure. But the actual geometrical judgment does not say of a straight line in general that it is this shortest way, but only of that one which is included between those two points. Now this fact, the fact that its extension is bounded by two points, (and it is only with this qualification that it forms the true subject of the sentence) is the ground, in this case certainly the satisfactory ground, for assigning the predicate to it. It is easy to see that the concept of a straight line *a b* between the points *a* and *b* is perfectly identical with the concept of the distance of the two points; for we cannot give any other idea of what we mean by 'distance in space' than this, that it is the length of the straight line between *a* and *b*. There is not therefore a shorter and a longer distance between *a* and *b*, but only the one distance *a b*, which is always the same. On the other hand, we can speak of shorter and longer *ways* between *a* and *b*; the concept of way implies merely any sort of progression which leads from *a* to *b*; as this requires the getting over of the difference which separates *b* from *a*, there can be no way leading from *a* to *b* which leaves any part of this difference not got over; accordingly, that the shortest of all possible ways is the distance, i.e. the straight line between the given points, is a judgment which, as regards its matter, is perfectly identical, and merely regards the same object from different aspects. Nor again can the arithmetical judgment, $7+5=12$, be synthetical because 12 is not contained in either 7 or 5: the complete subject does not consist in either of these quantities singly, but in the combination of them required by the sign of addition; but in this combination, if the equation is correct, the predicate must be wholly contained; the equation would be false if some unknown quantity had to be added to $7+5$ in order to produce 12. Here too, then, we have a perfectly identical judgment as regards its matter, and it is only synthetical formally because it exhibits the same number 12 first as the sum of two other quantities, and then as determined by its order in the simple series of numbers. I must now add that it is impossible to express everything satisfactorily all at once: what it really means, and how it is possible, that thought should represent the same matter under different forms, we shall very soon have occasion to consider;

59. So far our result seems to be this: categorical judgments of the form '*S* is *P*' are admissible in practice because they are always conceived in the sense which we have called particular, and as such are ultimately identical. No one however will feel satisfied with this conclusion: it will be rightly objected that it does away with the essential character of a judgment, which is that it expresses a coherence between the contents of two ideas. In fact, if, by the supplementary additions which we spoke of, we make our examples into identical judgments, and thus compress their whole content into their subjects, so that *A* means the black man, *B* the drinking dog, *C* Caesar crossing the Rubicon, all that they say, except the barren truth that $A = A$, $B = B$, $C = C$, is reduced to this, that *A* exists as a fact continually, *B* sometimes, and *C* has occurred once in history. In other words, these judgments no longer assert any *mutual relation* between the parts of their *content*, but only that this content as a composite whole is a more or less widely extended *fact*, and this is clearly a relapse to the imperfect stage of the impersonal judgment. The following consideration will make us still more sensible of this defect. I just now described *B* as the concept of the drinking dog, but properly I had no right to do so; for this expression, which joins 'drinking' in the form of a participle to the subject 'dog,' is itself only conceivable and admissible on the assumption that the mark of drinking, *P*, which is not contained in the subject *S*, can really be ascribed to that subject in a categorical judgment, and ascribed to it in the sense of its property or state. Now just this possibility has been done away with by our previous explanation; all that it is now competent to us to do is to understand *B* as the coexistent sum of its marks *a b c d*, and to say, this *a b c d*, which according to the principle of identity is always self-same, has a certain reality, while another aggregate of marks, *a b c e*, has a similar reality on another occasion. But we have no right whatever to regard the common group *a*, *b*, *c*, as something inwardly connected, and more connected in itself than with the varying elements *d* and *e*, still less as something which offers a support to these changing elements as subject to attributes. In language, indeed, we should continue to describe this *a b c* as 'dog,' *a b c d* as 'eating,' and *a b c e* as 'drinking dog'; but these expressions would rest upon no logical ground; none of our judgments could express anything but simple or composite perceptions, and between the several perceptions, or even the several parts of each composite perception, there could be no expressible connexion such as could show their mere coexistence to be due to inner coherence.

60. Against such a complete failure in its logical purpose the mind guards itself, by further transforming the particular judgment in a way which may be primarily considered as a simple denial that the material of our ideas is thus disintegrated into merely isolated coexistent facts. The additions by which we supplemented the subject S expressed in the categorical judgment, were the means by which we helped that judgment to justify itself before the principle of identity; they are now recognised as being also the valid ground of fact which qualifies S for assuming a predicate P, which, so long as it stood alone, would not belong to it. The accessory circumstances, through which S first became the true subject Σ of a then identical judgment, appear now as the *conditions*, by the operation or presence of which S is so influenced that a P, which before was strange to it, now fits and belongs to it consistently with the principle of identity. It is therefore the *hypothetical* judgment which takes its place as the second member in this second group of the forms of judgment; it is compounded of a protasis and an apodosis, which in the simplest typical case have the same subject S, but different predicates, in the protasis a Q which expresses the condition accruing to S, in the apodosis a P which expresses the mark produced in S by that condition. All hypothetical judgments with different subjects in their two members are abbreviated expressions, and can be reduced by easily supplied links to this original form, 'If S is Q, S is P.' If it is further wished to imply that the protasis, which as such is only problematical, is actually true, we get the form, '*Because* S is Q, S is P': and lastly, the assertion that Q is *not* the ground for S's being P gives rise to the last form which we need mention, '*Although* S is Q, yet S is not P.' Logically there is nothing peculiar in these two forms.

61. This short survey is quite sufficient to characterise the external forms of the hypothetical judgment. But an observant reader must ask at this point, what right had we to translate those supplementary additions, to which the true subject Σ of the then identical judgment owed its origin, into *conditions*, which, by operating upon an already existing subject S, *give a ground for* the predication of P. The principle of identity merely asserts the sameness of everything with itself; the only relation in which it places two different things is that of mutual exclusion. If then we supposed various simple elements other element, and the fact of our observing $a\,b\,c\,q$ a second time would not enable us to conclude that p must be there too; any r or s might with equal right take its place. On the other hand, if we make the quite general presupposition that the totality of things thinkable and real is not merely a sum which coexists but a whole which coheres, then the law of identity has wider consequences. The *same* $a\,b\,c\,q$, with which p has once been found in combination, can then according to the law of identity never be found in combination with a non-p, nor can *this* $a\,b\,c\,q$ ever occur without its former predicate p. *How* such a cohesion between different elements is conceivable, we will leave for a moment an open question; but *if* it exists, it must exist in an identical form in every recurrent instance, and (confining ourselves to a combination of three elements) given $a\,b$, c is the only new element which can necessarily accrue, given $a\,c$, b, and given $b\,c$, a; in other words, whichever of these elements occurs first in any case has in the second the sufficient and necessary condition for the possibility and necessity of the accession of the third. That element or group of elements to which we here give the first place, appears to us then logically as the subject; that which we place second, as the condition which operates upon this subject, while the third represents the consequence produced in the subject by the condition. I wish further expressly to point out that this choice of places is quite arbitrary, and in practice is decided by the nature of the object and our interest in it: in itself, every element in such a combination is a function of the rest, and we can pass inferentially from any one to any other. It is usual to conceive of a number of elements which frequently recur together as a subject S, which generally signifies a thing or permanent reality: on the other hand, a single element b, which is absent in some observations of S and present in others, is conceived as the accessory condition Q, and $a\,c$, which always accompanies b, as the consequence P of which Q is the condition. But it is obvious that we may proceed in a different way; and in fact mechanical physics are able to treat the single and uniform force of gravity, b or Q, as a subject, and to investigate the various consequences, P, which accrue to it if the bodies upon which it acts ($a\,m\,n = S$ or $a\,m\,r = S^1$) be regarded as different conditions to whose influence it is liable.

62. In this way the interpretation by which we arrived at hypothetical judgments may be said to be so far justified, as that it has been traced back to the most general assumption of a coherence between the various contents of thought. To prove further than this the admissibility and truth of that assumption itself, cannot be part of our undertaking; any such attempt would obviously imply what had to be proved, for how could we show that it is permissible and necessary to conceive the matter of experience as a web of *reasons* and *consequences*, if we did not base this assertion itself upon a reason of which it was the consequence? This idea of the coherence of the world of thought must therefore either be apprehended with immediate certitude, as the soul of all thinking, or we must give it up and along with it everything that depends upon it. On the other hand, we are justified in desiring further elucidation of the possibility and the meaning of such a coherence of different elements. The possibility of mutual relations between what is different is not really threatened by the principle of identity, according to which each thing is related only to itself; for all that this principle can affirm is the content of the thing itself; it cannot exclude other contents which do not conflict with it. But as regards the meaning of the coherence, we must distinguish two questions. In logic as here conceived we do not trouble ourselves at all as to what the real process may be through which the unknown reality, which we express well or ill through our ideas, reacts upon itself and produces changes in its conditions; to reflect upon the bond of this connexion is the function of metaphysic, and the question should find solution in a theory of the efficient cause. Logic, on the other hand, which includes in its consideration the relations of the merely thinkable which has no real existence in fact, is confined to developing the other principle, that of *sufficient reason*; it has merely to show how, from the combination of two contents of thought, S and Q, the necessity arises of *thinking* a third, P, and this in a definite relation to S; if then it were found in actual experience that such a union of S^1 and Q^1 is an accomplished fact, the particular consequence P^1, which according to the necessity of thought must follow such a combination in distinction from P^2 which could not so follow, could be inferred according to the principle of sufficient reason; but how it comes about that the very P^1, which is required by thought, occurs in reality as well, is a question which would be left to the metaphysical enquiries referred to.

63. The *law of sufficient reason*, with which we now conclude as the third member and the net result of this second group of validity we must seek a ground for its validity, forgets that we cannot seek for that of which we do not know wherein it consists; clearly the first thing that has to be explained is, in what relation reason and consequence stand to each other, and in what sort of thing consequently we may hope to discover the reason of another thing. I shall make my meaning clear in the shortest way, if, on the analogy of the expression of the principle of identity, $A = A$, I at once give the formula $A + B = C$ as the expression of the principle of sufficient reason, adding the following explanation. Taken by themselves, A only $= A$, $B = B$; but there is no reason why a particular combination $A + B$, the very different sense of which in different cases is here represented by the sign of addition, should not be equivalent to, or identical with, the simple content of the new concept C. If we thus call $A + B$ the reason and C the consequence, reason and consequence are completely identical, and the one *is* the other; in this case we must understand by $A + B$ any given subject along with the condition by which it is influenced, and by C, not a new predicate which is the consequence of this subject, but the subject itself in its form as altered by the predicate. In ordinary usage this is expressed differently. Inasmuch as, in speaking of real facts, the one part A is usually already given, while the other B is a subsequent addition, it is customary to describe the *condition* B, which forms only a part of the whole reason $A + B$, as the reason in general which acts upon the passive subject A; by C is then usually understood nothing but the new property conditioned by B, and this is called the consequence; at the same time, however, the property is never thought of as existing on its own account, as if in empty space, but as attaching to the subject A upon which B was supposed to act. Ordinary usage, therefore, though it employs a different nomenclature, means the same as I do. If with the idea of powder, A, we connect the idea of the high temperature of the spark, B, and thus substitute B for the mark of ordinary temperature in A, then $A + B$ really *is* the idea C of exploding powder, not of explosion in general; the ordinary usage makes the high temperature B seem to supervene on the given subject A as a reason from which the explosion C follows; but of course it conceives this consequence, not as a process which takes place anywhere, but as an expansion of the particular powder upon which the spark acted. It is not necessary to continue such simple explanations any further.

64. If we consider the whole of our knowledge, we see at once that the principle of identity cannot be its only source. Taken alone it would isolate every judgment and even every concept, and would not open any way to a progress from the barren self-identity of single elements of thought to their fruitful combination with others. It is a mistake, as is sometimes done, to represent this single principle as the basis of the truths of mathematics; the fact is that here too it is only the principle of sufficient reason which helps to real discovery. From a self-identical major premiss nothing new could flow, unless it were possible in a number of minor premisses to give the same quantity C innumerable equivalent forms, at one time $= A + B$, at another $= M + N$, at another $= N - R$; or, to express the same thing otherwise, unless the nature of numbers were such that we can divide them all in innumerable ways and compound them again in the most manifold combinations; and again, unless the nature of space were such that every line can be inserted as a part or otherwise coherent member in innumerable figures in the most various positions, and that each one of the expressions for it, which flow from these various relations, is the ground for new and manifold consequences. I need hardly mention that mechanics and physics also make the most extensive use of this analysis and composition of given facts, and that the process of thought in discovery in these branches of knowledge rests upon operations which all ultimately come back to the typical formula, $A + B = C$. To Herbart belongs the credit of having brought within the ken of formal logic the importance of a mode of procedure so prominent in all scientific practice.

65. Reserving further illustrations for applied logic, I have another remark to make about the justification of the principle of sufficient reason itself. We were only able to show that an extension of our knowledge is possible *if* there is a principle which allows us to make $A + B = C$. We might accordingly attempt to assert the validity of this principle at once, as an immediate certitude, like the principle of identity. This is what we have done; still there is a noticeable difference between the two principles. The principle of identity expresses of every A an equality with itself which we feel immediately to be necessary, and the opposite of which also we feel with equal conviction to be impossible in thought. The principle of sufficient reason lacks this latter support; we do not by any means feel it

e impossibility of its opposite, the other must be con-
as an assumption which serves the purposes of thought,
of mutual relatedness in thinkable matter the truth of
nteed by the concentrated impression of all experience.
to be misunderstood in this last phrase. In the first
›t mean that it is a comparison of what we experience
ıds the mind to conjecture the validity of such a prin-
ıeral tendency of the logical spirit, to exhibit the co-
›herent, contains in itself the impulse, which, inde-
ı of all actual experience, would lead to the assumption
ı of reasons and consequences. But that this assump-
ned, that thought does come upon such identities or
etween different elements in the thinkable matter which
make, but receives or finds, this is a fortunate fact,
ait in the organisation of the thinkable world, a trait
ılly exist, but has not the same necessity for existing as
›f identity. It is not impossible to conceive a world in
ing should be as incommensurable with every other
is with triangular, and in which therefore there was no
so holding two different things together as to give
hird: it is true that, if such a world existed, the mind
w what to do with it, but it would be obliged to recognise
according to its own judgment. I will add further that,
of a kind of empirical confirmation of the principle of
›n, I do not mean such a confirmation as the whole of
thought, already articulated in accordance with that
ıt find in the fact that *external* reality, so far as it is
›rresponds with this articulation; I am speaking here
ct that the thinkable world, the contents of our ideas
'er their source, we find in our *inner* experience, do
. to the requirement that they should cohere as reasons
ıces. In this stage of logic it is quite indifferent whether
anything which can be called external world or reality
eas which move within our consciousness: like that

66. As to the nature of this responsiveness in the world of thought (if that question is to be raised again here), the shortest way to recall it is to observe that the position occupied in the system by the principle of sufficient reason, as the second law of thought, is analogous to that of the act which[1] we placed second in treating of conception. The possibility of forming general concepts depended on the fact, not in itself a necessity of thought, that every idea is not incommensurable with every other, but that, on the contrary, colours, tones, and shapes group themselves in series of cognisable gradations; that further there are oppositions of varying degree, as well as affinities, in the world of thought, and that opposites cancel one another; and lastly, and most important of all, that there is a system of quantitative determinations enabling us to compare the members of different series, which as such stand in no mutual relation. With this brief indication, we leave the principle of sufficient reason as the conclusion and net result of the second group of the forms of judgment.

C. *The General Judgment. The Disjunctive Judgment. The* Dictum de omni et nullo *and the* Principium exclusi medii.

67. It remains to determine in each particular case, What A, combined in what form with what B, forms the adequate reason of what C. This question of fact logic must leave to experience and the special sciences; but a new question is developed which logic itself must deal with. There would be little result from all the activity of our mind if we were really obliged in *every* particular case to renew the question to experience, What A, B, and C in this instance cohere as reason and consequence? There must be at any rate *a* principle which allows us, when once the one truth $A + B = C$ is given, to apply it to cases of which experience has not yet informed us. What we are here looking for is easy to find, and has been already mentioned incidentally. Whenever we regard $A + B$ as the reason of a consequence C, we necessarily conceive the connexion of the three as a *universal* one; $A + B$ would not be a condition of C, if, in a second case of its occurrence, some casual D instead of C might possibly be found combined with it. The significance of this in its present application is as follows: everywhere, in every subject S in which $A + B$ is contained as a mark along with other marks, NOP, this $A + B$ gives ground for the same consequence C; and this C will either actually occur as a mark of S, or, if it does not occur, it can only be hindered because the other marks, $N + O$ or $N + P$ or $O + P$, formed together the ground for a consequence opposed to and destructive of C; taken by itself, without this hindrance, the power of $A + B$ to condition C never loses its effect. If now we conceive $A + B$ under the title M as a universal concept to which S is subordinate, we may give the following preliminary expression to the principle just discovered, viz. that by right of pure logic and without appeal to experience every subject may have that predicate affirmed of it which is required by the generic concept above it. And it is clear without further explanation that this very idea of the subordination of the individual to the universal is the comprehensive logical instrument, of which we avail ourselves whenever we want to carry further the work of thought upon the material given in experience.

68. The form of judgment, the first of this third group, in which the mind expresses this conviction, is that of the quantitatively undetermined proposition, in which the place of the subject is filled simply by a universal or generic concept M; 'man is mortal,' 'sin is punishable.' I distinguish these as *general* judgments from the universal ones, 'all men are mortal,' 'every sin is punishable.' Although the fact contained in both forms is the same, the logical setting of it in the two cases is quite different. The universal judgment is only a collection of many singular judgments, the sum of whose subjects does as a matter of fact fill up the whole extent of the universal concept; thus the fact that the predicate P holds good of all M follows here only from the fact that it holds good of every single M; it may however hold good of each M for a special reason which has nothing to do with the universal nature of M. Thus the universal proposition, 'all inhabitants of this town are poor,' leaves it quite uncertain whether each inhabitant is made poor by a particular cause, or whether the poverty arises from his being an inhabitant of this town; so too the universal proposition, 'all men are mortal,' leaves it still an open question whether, strictly speaking, they *might* not all live for ever, and whether it is not merely a remarkable concatenation of circumstances, different in every different case, which finally results in the fact that no one remains alive. The general judgment on the other hand, 'man is mortal,' asserts by its form that it lies in the character of mankind that mortality is inseparable from every one who partakes in it. While therefore the universal judg-

ment merely states a universal fact, and is therefore only assertorial, the general judgment lets the reason of its necessary truth be seen through it, and may thus, in the sense laid down above[1], be called apodeictic. This distinction of the two forms of judgment will not lead to any unheard-of discoveries; but in comparison with the many unprofitable distinctions which encumber logic it deserved an incidental mention. It is scarcely necessary to remark that in the general judgment it is not the generic concept M, occupying the place of subject in the sentence, which is the true logical subject of the judgment; it is not the universal man who is mortal, but the individual S who participates in this type, which in itself is immortal. From this we see that the general judgment is properly an abbreviated hypothetical judgment; in its full form it ought to stand, 'If S is M, S is P,' 'If any S is a man, this S is mortal.' And this justifies us in not introducing it in our system until after the hypothetical.

69. But it is no less clear that we must make another step. So long as a universal generic concept M occurs as formally the subject in the general judgment, the predicate P which is joined to it can only be understood with equal universality. If we say, 'man is mortal,' the predicate embraces all conceivable kinds of mortality, and does not determine either the manner or the moment of death; or if we say, 'bodies occupy space,' it remains unexpressed with what degree of density and of resistance each single body realises the universal property of its class. But we saw that it is individual men and individual bodies which are the real subjects of the general judgment; it is therefore quite false to say that P, the mark of their class, is a predicate of the individuals in the same universal sense in which it is joined in thought (and that not as a predicate) to the concept of the class; the truth is that P can only occur in each one of these individuals in one of the definite forms or modifications into which the universal P can be analysed or particularised. The mind corrects this mistake by means of the fresh assertion, 'If any S is an M, this S is either p^1 or p^2 or p^3,' and here $p^1 p^2 p^3$ mean the different kinds of a universal mark P which is contained in the generic concept M. This is the familiar form of the *disjunctive judgment*, the second in this third group, and one which, as such, requires no further explanation. It is usual to mention along with it the *copula-*

form, neither of these has the same logical value as the disjunctive; the first is only a collection of positive, the second of negative, judgments with the same subject and different predicates, which latter are not placed in any logically important relation to each other. The disjunctive judgment alone expresses a special relation between its members: it gives its subject no predicate at all, but prescribes to it the alternative between a definite number of different predicates.

70. The thought expressed by the form of the disjunctive judgment usually finds utterance in two separate laws of thought, the *Dictum de omni et nullo* and the *Principium exclusi tertii inter duo contradictoria*; but the amalgamation of them in a single third law is not only easy but necessary. The careless formulations often given of the first are completely false, e. g. 'What is true of the universal is true also of the particular,' 'What is true of the whole is true also of the parts'; on the contrary, it is self-evident that what holds good of the universal as such or of the whole as such, cannot hold good of the individual as such or of the parts as such. The only correct formula is, *quidquid de omnibus valet valet etiam de quibusdam et de singulis*, and *quidquid de nullo valet nec de quibusdam valet nec de singulis*. But this form of expression (for the history of which see Rehnisch, Fichte's Zeitschrift, lxxvi, 1) is as barren as it is correct; for to hold good of *all* is and means from the very first nothing else than to hold good of each one; if therefore anything worth saying is to take the place of this bare tautology, the nature of the universal concept must certainly be substituted for the mere sum of *all*. But in that case the principle cannot really be accurately expressed except in a form which means precisely the same as the disjunctive judgment; viz. whenever a universal P is a mark in a universal concept M, one of its modifications, $p^1 p^2 p^3$, to the exclusion of the rest, belongs to every S which is a species of M; whenever a universal P is excluded from a concept M, no one of the modifications of P belongs to any S which is a species of M.

71. Of this complete law of thought the ordinary expression of the *dictum de omni et nullo* only regards the one and positive part, which, as we saw, cannot by itself be accurately expressed, the general idea, namely, that the particular is determined by its universal: the other and negative part, which defines the manner of this determination, the idea that the particular admits only one specific form of its generic predicate to the exclusion of the others, has found only a partial expression in the principle of the excluded middle. I think that I can say what I have to say about this most simply as follows. Suppose a subject S subordinate to M, and that this subordination implies that S must choose its own predicate from amongst $p^1\, p^2\, p^3$, the specific forms of P, a universal mark belonging to M, then, if there are more than two of these forms, the affirmation of one of them as predicate of S will involve the negation of all the rest, but the negation of one of them will not involve the affirmation of any particular one of the rest; what is not p^1 has still an open choice between $p^2\, p^3\, p^4$. To predicates of this sort it is usual to ascribe the opposition of *contrariety*. If however there are only two specific forms of P, p^1 and p^2, and S must have a specific form of P for its predicate, then not only does the affirmation of one of them as predicate of S involve the negation of the other, but also the negation of the one involves the definite affirmation of the other; p^1 and p^2 are then opposed to one another *contradictorily*. Thus for the line (S), which must have some direction (P), straight (p^1) and crooked (p^2) are contradictory predicates, and so for man, whose nature it is to have sex, are male and female: for any other subjects, of which it was not yet established whether their concepts contained the universal P at all, these predicates would be only contrary; for such subjects the division of their possible predicates will be always threefold, they are either male, female, or sexless, either straight, crooked, or formless. Now the principle of the excluded middle asserts nothing but what we have just remarked, that of two predicates which are contradictory for a subject S, S always has one to the exclusion of the other, and if it has not the one it necessarily has the other to the exclusion of any third. So regarded, this law is only a particular case of the more universal law of which the disjunctive judgment is the expression, viz. that of all contrary predicates whose universal P is contained in the generic concept M of a subject S, S has always one to the exclusion of the rest, and if it has not any given one, it has only left it the choice between the others; this choice becomes a definite affirmation when it can only fall on one member, i. e. in the extreme case where the number of contrary predicates is only two. Such a case, which is all that is covered by the principle of the excluded middle, is no doubt of peculiar importance in practice, but a system of logic can only treat it as a particular instance of the more universal principle, which we have already mentioned several times and which we will briefly describe as the *disjunctive law of thought*.

72. It is usual to represent this differently. From motives which

roughout (viz. that the given subject S be already understood to
ınd in a necessary relation to the universal predicate P), and to be
owed to speak of two predicates which hold good as contradic-
:ies of any subject whatever. It is soon found that this is only
ıssible, if the aggregate of all conceivable predicates be divided into
definite Q and the sum of all those which are not Q or non-Q; it is
en certain that any subject, whatever it may mean, must be either Q
non-Q, either straight or not-straight, for not-straight will include
ɔt only crooked, but annoying, sweet, future, everything in short
ıich lies outside of straight. On this point I may repeat what I said[1]
ıout the limitative judgment, viz. that non-Q is not a real idea at all,
.ch as can be treated as subject of a predicate; it is only a formula
.pressing a mentally *impracticable* task, the collection of all thinkable
atter that lies outside a given concept into a single other concept.
oreover there is no real reason for propounding this insoluble
·oblem; everything which it is wished to secure by the affirmative
·edicate non-Q is secured by the intelligible negation of Q. I there-
re consider it quite improper to speak of contradictory *concepts*, i. e.
ɔncepts which are of themselves contradictorily opposed and *there-
re* retain that opposition when treated as predicates of one and the
.me subject, whatever that subject may be: if we want a contradic-
ry relation which shall hold good universally, always, and in regard
every subject, it can only exist between two *judgments*, 'S is Q,'
S is not Q.' Accordingly the precise expression of the principle of
.e excluded middle would be, that of every precisely determined
ıbject S either the affirmation or the negation of an equally deter-
inate predicate Q holds good, and no third alternative is possible;
herever it appears to be possible, S or Q or both have either been
ken in more than one sense or in an indefinite sense in the first
stance, or their meaning has been unconsciously or involuntarily
ıanged in the course of reflection.

73. I have one more observation to add. No one doubts that the other predicates $p\,q\,r$ are compatible in the same subject when, as species of quite different universals $P\,Q\,R$, they admit of no comparison. On this point I venture the following reflections. Every predicate p^1 of a subject S must be regarded, in accordance with what we said above and the formula $A+B=C$, as a consequence of a group of marks A^1+B^1 in S, which group tends in all cases (and therefore in the case of S) to produce the same result C^1 (in this case p^1). If now the same S is to have at the same time the predicate p^2, comparable with p^1, it is easy to understand that p^2 must depend on a group of marks A^2+B^2, similarly comparable with A^1+B^1, existing side by side with the latter in S, and in all cases of its occurrence (and therefore in the case of S) giving ground for the result C^2 (in this case p^2). But the consequence of the very comparability of A^1+B^1 and A^2+B^2 must be that, according to a new principle of the general form $A+B=C$, viz. $[A^1+B^1]+[A^2+B^2]=C^3$, their meeting in the same subject S will furnish the sufficient reason for a new consequence C^3, in which the two specific predicates p^1 and p^2 coalesce, and which, as it must resemble both of them, we will call p^3. The only reason, therefore, why two *contrary* and comparable predicates p^1 and p^2 would be irreconcileable, is that they would always give rise to a third and simple p^3; on the other hand, two *disparate* and incomparable predicates p and r, such as sweet and warm, could coexist permanently in S because there is no principle such as $(A+B)+(M+N)=C$ enabling the two disparate grounds $A+B$ and $M+N$, on which the predicates respectively depend, to produce like p^1 and p^2 a third and simple predicate. I will not quarrel with those who find the whole of this exposition superfluous; it seems to me to have some point, when I turn from the examples which logic traditionally employs to others which it would do well not to forget. When anyone says of gold that it is yellow, he has, it is true, no occasion to think of this simple property as a product of two other imperceptible ones, which properly speaking must have been produced separately by two conditions coexisting in gold, but could not remain separate. But when two motive forces contrary or even contradictory in direction act upon a material point, that which in the previous case would have been a needless assumption is now an actual *fact*; we have to conceive both of the condition which tends to produce the motion p^1 and ay appear indifferently red or green: but if the conditions of both ppearances were operating at the same time and with the same orce, it would help us but little to assert, on the principle of exclusion, hat the image of the line cannot have these two contrary properties; *must* present *some* appearance. As however these two conditions re comparable and capable of forming a resultant, a third colour will ppear, the production of which will satisfy the claims of the two conitions, but will at the same time contain the reason why the two ontrary colours, which singly they would have produced, cannot xist separately side by side.

74. The series of judgments concludes here by an inherent neces-ity. The more definitely the disjunctive judgment prescribes to its ubject the choice between different predicates, the less can this ncertainty be final; the choice must be made. But the decision, *hat* p^1 or p^2 belongs to *S*, cannot come from the fact (which is thus ar the only fact) that *S* is subordinate to *M*, for it is just because it s a species of *M* that it is still free to choose: that decision can only ome from the specific difference by which *S*, as *this* species of *M*, is istinguished from other species of it. The proposition '*M* (and very *S* which is *M*) is *P*,' must therefore have added to it a second roposition which brings to light the specific character of *S*, the paricular subject always in question, and shows us *what* species of *M* it s; and from the union of the two propositions must arise a third, nforming us what particular modification *p* of the universal *P* belongs o *this S* because it is, not only a species of *M*, but *this* species. The orm of thought which combines two judgments so as to produce a hird is, speaking generally, *inference*, and it is therefore to the expoition of inference that we have now to pass.

Appendix on immediate inferences.

three. From the *Dictum de omni et nullo* and the principle of the excluded middle we obtain the following results.

75. Between each universal judgment and the particular of like name there is the relation of *subalternation.* Going from the universal to the particular or *ad subalternatam*, we infer the truth of the latter from that of the former, but from the untruth of the universal we cannot infer either the truth or the untruth of the particular. The correctness of the first inference is obvious at once, and it only requires the removal of a misunderstanding to make the impossibility of the second equally so. A person who denies the universal proposition, 'all *S* are *P*,' is usually led to do so by having already observed some *S* which are not *P*; but he will not have included all *S* in this observation. His intention therefore generally is merely to deny the universal application of the proposition to all *S*, while leaving its truth in single cases of *S* undisputed; and thus it is that in ordinary speech expressions such as 'It is not true that all *S* are also *P*,' are actually understood to admit incidentally the truth of the particular proposition, 'some *S* are *P*.' Logic, on the other hand, knows nothing of these unexpressed suggestions in the denial of the universal proposition: it recognises merely what lies in the expressed negation itself. But it is just this which is ambiguous. For the asserted untruth of the proposition, 'all *S* are *P*,' is equally a fact, whether the proposition is true of only some *S* or of none. So long therefore as this ambiguity is not removed by accessory statements, we cannot infer from the negation of the universal proposition either the truth or the untruth of the particular.

76. Going in the opposite direction, from the particular to the universal or *ad subalternantem*, we infer the untruth of the universal judgment from that of the particular, but not the truth. Here, too, the first conclusion is obvious, if we avoid the ambiguity already alluded to. A person who denies the proposition, 'some *S* are *P*,' may, it is true, intend merely to deny that *P* is confined to some *S*, and the effect of his meaning that '*not only* some *S* are *P*' would then be to affirm the universal proposition 'all *S* are *P*.' But just itself; the truth of *P* in the case of some *S* can never prove its truth in all *S*: it is only because this unjustifiable generalisation of single observations is the commonest of logical mistakes, to which science and culture owe most of their errors, that it is worth while to prohibit with especial emphasis this false inference *ad subalternantem.*

77. Universal judgments are contradictorily opposed to particulars of unlike name, *A* to *O* and *E* to *I* and *vice versa;* we infer *ad contradictoriam* both the untruth of the one from the truth of the other and the truth of the one from the untruth of the other. The first inference needs no explanation, the second a brief one. If we deny the proposition *A*, 'all *S* are *P*,' the denial is consistent with both the assumptions *E* and *O*, 'no *S* is *P*,' and 'some *S* are not *P*'; but the second, which is included in the first, is true in any case; consequently the truth of *O* follows certainly from the untruth of *A*. If we further deny *O*, 'some *S* are not *P*,' this means, according to what we said above, 'there is no such thing as some *S* which are not *P*,' and this is equivalent to *A*, 'all *S* are *P*.' If we deny *E*, 'no *S* is *P*,' either all or some *S*, in any case the latter, are *P*, and consequently *I* is true, 'some *S* are *P*': if we deny *I*, this means, 'there is no such thing as some *S* which are *P*,' and is equivalent to the affirmation of *E*, 'no *S* is *P*.'

78. The two universal judgments of unlike names are only *contrariwise* opposed, and we infer the untruth of the one from the truth of the other, but not the truth of the one from the untruth of the other. The first case is obvious: the impossibility of the second follows, after what we said before, from the consideration that, while the negation of a universal judgment allows an inference *ad contradictoriam* to the truth of the particular of unlike name, the truth of the latter does not allow an inference *ad subalternantem* to that of the universal to which it is subordinate. Lastly, the relation between the two particular judgments *I* and *O* is called *subcontrary* opposition. We infer *ad subcontrariam* the truth of the one from the untruth of the other, but not the untruth of the one from the truth of the other. In fact, the two propositions, 'some *S* are not *P*,' and 'some *S* are *P*,' may both be true together; but if one is denied, the truth of the opposite universal follows *ad contradictoriam*, and from this again follows *ad subalternatam* the affirmation of the particular subordinate to it.

79. I may also mention another logical operation which has a kindred object. All observations, which always admit ultimately of being expressed in the form of a judgment '*S* is *P*,' present us only

with that combination of *S* and *P* which actually occurs at the moment of observation: they tell us nothing as to whether *S* and *P* will be separable or not in other cases, whether, in fact, there are *S* which are not *P* or *P* which are not *S*. Now we have a practical interest in this question which is very intelligible: we want to know whether a *P* which has occurred in *S* may be considered as a *mark*, enabling us to determine the nature of the subject in which it occurs: in short, whether everything which has the characteristics of a *P* is also always an *S*. The answers to be expected to this question will accordingly take the form, '*P* is *S*'; and they are therefore called *conversions* of the original judgments which gave rise to them. It is also obvious that we have a special interest in knowing whether *P* points to a subject *S* necessarily and always, or only possibly and sometimes; whether, as it is ordinarily put, all *P*, or only some, are *S*. Hence it is the quantity of the original and the converted judgment to which particular attention is paid, and the conversion is called pure (*conversio pura*) when the quantity of the second is that of the first without any change, and impure (*conversio impura*) when it is different, especially when the universal truth of the original judgment has to be reduced to particular, in order to make it true when converted. The results are as follows.

80. The universal affirmative judgment, 'all *S* are *P*,' understands by *P* either a higher genus in which *S* is contained along with other species, or a universal mark in which *S* partakes along with other subjects. In both cases there is a part of *P* left which has nothing to do with *S*, and only impure conversion can take place into the particular judgment 'some *P* are *S*.' This rule deserves attention, for it is one of the commonest mistakes of carelessness and one of the most favorite means of deception to substitute the universal for the particular inference, and to assert, 'If *P* belongs to all *S*, then *S* belongs to all *P*.' It is true that we do meet with universal affirmative judgments which admit of this pure conversion, those viz. in which the extents of *S* and *P* exactly cover each other, and *P* therefore belongs not only to *all S*, but *only* to all *S*, so that all *P* are also *S*. Such so-called *reciprocal* judgments are, 'all men are naturally capable of language,' 'all equilateral triangles are equiangular'; they can be converted into, 'all that is naturally capable of language is man,' 'every equiangular triangle is an equilateral one.' But it is only knowledge of the matter of fact contained in the judgment in question which can assure us that the relation, upon which this possibility depends, holds good between *S* and *P* in any particular instance.

This complete exclusion of the two concepts from each other clearly holds good reciprocally, and justifies the assertion that 'no *P* is *S*.' The universal negative judgment is therefore convertible into another universal negative.

81. The particular affirmative proposition, 'some *S* are *P*,' obviously yields pure conversion into another particular affirmative, 'some *P* are *S*.' And this inference is satisfactory in all cases in which *P* is a universal predicate in which *S* partakes along with other subjects; thus the assertion, 'some dogs bite,' is rightly converted into 'some things that bite are dogs.' But when *S* is the genus of which *P* is a species, as in the proposition, 'some dogs are pugs,' the only logically admissible conversion, 'some pugs are dogs,' will contrast unfavourably with the actually true one, 'all pugs are dogs.' The former is no doubt true also, but it expresses only a part of the truth, and in a form which appears rather to deny than to affirm the other part, that all other pugs also are dogs. We feel this still more if we start with the judgment, 'all pugs are dogs,' and convert it twice over. From the first conversion, 'some dogs are pugs,' we cannot get back again by the second to the original proposition; and thus the logical operations have here resulted in eliminating a part of the truth. This inconvenience could easily be avoided if the expressions of quantity were regarded, as the sense requires that they should be, as inseparable from their substantives; we should then formulate the proposition, in the first instance as follows, 'all pugs are some dogs'; then by conversion, 'some dogs are all pugs,' and by a second conversion, 'all pugs are some dogs.' But it is not worth the trouble to improve what are after all barren formulae.

The particular negative judgment, 'some *S* are not *P*,' as such pious,' 'some are not Christians,' would yield 'something pious is not man,' 'some Christians are not men,' both inadmissible because piety and Christianity, though not belonging to all men, belong *only* to men. These disadvantages are in general only avoided by joining the negation to the predicate, and then converting the proposition, 'some *S* are non-*P*,' like a particular affirmative into 'some non-*P* are *S*'; e.g. 'something not-black, something not-pious, some non-Christians, are men.'

82. The process necessary in this case has been extended to all judgments under the name of conversion by *contraposition:* in the affirmative judgments the negation of non-*P* takes the place of the affirmation of *P*, in the negative the affirmation of non-*P* takes that of the negation of *P*; the judgments thus changed are then converted according to the ordinary rules. In this way we get the following results; first, for *A*, 'all *S* are *P*,' 'no *S* is non-*P*,' and so 'no non-*P* is *S*'; for *I*, on the other hand, 'some *S* are *P*,' the transformation into, 'some *S* are not non-*P*,' would not, after what has been said above, allow any conversion, and contraposition would therefore be impossible; for *E*, again, 'no *S* is *P*,' we get 'all *S* are non-*P*,' 'some non-*P* are *S*.' To carry out these operations in actual instances would produce unshapely and unnatural forms of expression; the substantial meaning of the four forms of judgment may be given more simply by replacing their quantitative determinations by the equivalent modal ones: even the contraposition of *I*, which in itself is impossible, is thus made available. The conversion of *A* would then mean, 'If the predicate *P* belongs to all individuals of a genus *S*, it is impossible for anything in which this mark is absent to be an *S*:' that of *I* would mean, 'If *P* is only known to belong to some species of *S*, it is not necessary, but only possible, that something in which *P* is absent should not be an *S*:' that of *E*, 'If the mark *P* is universally absent from, or contradictory of, the genus *S*, it is not necessary, but only possible, that something which similarly lacks or is contradicted by *P* should be a species of *S*'; and the same inference applies to *O* also, 'If some *S* are not *P*, something which also is not *P* may be an *S* but need not be so.'

CHAPTER III.

The Theory of Inference and the Systematic Forms.

Preliminary remarks upon the Aristotelian doctrine of syllogism.

I HAVE pointed out the unsolved problem which compels us to advance beyond the disjunctive judgment. Before I follow up this thread of connexion systematically, I think it will be advantageous to state the theory of syllogism in the form which it received from Aristotle. I shall not however follow the original exposition of the great Greek philosopher, but the more convenient form which came into vogue later. The writings of Aristotle are preserved, and anyone who takes an interest in the origin of these doctrines may easily enjoy his masterly development of them: but when we are concerned, not with the history of the thing, but with the thing itself, it would be useless affectation to prefer the inconvenient phraseology of the inventor to those improvements in detail which subsequent ages have placed at our disposal.

83. Following Aristotle, we give the name of inference or syllogism to any combination of two judgments for the production of a third and valid judgment which is not merely the sum of the two first. Such production would be impossible if the contents of the antecedent the *minor*; the conclusion itself is always conceived in the form SP, not in the reverse form PS. This being presupposed, the further differences in the position which the three concepts may assume give rise to the following four arrangements, of which the first three represent the three figures of Aristotle, while the fourth forms that of Galen.

$$(1)\begin{array}{c} MP \\ \underline{SM} \\ SP \end{array} \qquad (2)\begin{array}{c} PM \\ \underline{SM} \\ SP \end{array} \qquad (3)\begin{array}{c} MP \\ \underline{MS} \\ SP \end{array} \qquad (4)\begin{array}{c} PM \\ \underline{MS} \\ SP \end{array}$$

84. If we now ask whether, and under what conditions, these arrangements of premisses, which are in the first instance merely based upon rules of combination, give ground for a valid inference, we find at once that S and P can only be united in the conclusion if the middle concept remains precisely the same; their union is obviously unjustifiable as soon as the M connected with S in the one premiss is different from the M connected with P in the other. Such a division of M would give four concepts in the premisses, instead of the necessary and sufficient three; the avoidance of this *quaternio terminorum*, and the securing of complete identity in the middle term, is therefore the condition of conclusiveness in all figures alike. To fulfil this condition it is first of all necessary in all figures to exclude any ambiguity in the meaning of the word which denotes the middle concept; but besides this there are special precautions for the same purpose, which the peculiar structure of the several figures renders necessary, and which we have now to mention.

85. In the *first figure* S is included in M in the minor premiss, M in P in the major, and therefore S in P in the conclusion. The idea upon which this inference is based is evidently that of subsumption; that which is a predicate of the genus is a predicate of every subject of the genus. This is of itself sufficient to show that the major premiss in the first figure must be universal; for it has to express the rule which is to be applied to the subject of the minor. The necessity that the middle term should be identical leads to the same result. For the S of the minor premiss is always a definite kind or a definite case of M; this however is not expressed in the form of the proposition;

universally of all *M*, thus including the indeterminate cases along with the rest. It is true that in that case the *M* expressed in the major premiss is not identical with the *M* of the minor, which, as predicate of *S*, necessarily signifies only a part of the whole extent of *M*; but this apparent difficulty disappears when we consider that the *M* of the major premiss which is actually *employed* in producing the conclusion is likewise only a part of that which is expressed, that part, namely, which is *intended* in the minor. Further, as the inference in the conclusion depends upon the subordination of *S* to *M*, this subordination must be a fact, in other words, the minor premiss which expresses it must be affirmative; if it were negative, it would simply deny the existence of any ground for the validity of the conclusion. On the other hand it does not affect the logical connexion of the syllogism, but depends merely upon its particular content, whether the major premiss affirms or denies *P* of *M*, and whether the application furnished by the minor of the general rule to an instance embraces all *S* or only some. The quality of the major premiss and the quantity of the minor are therefore unlimited. Lastly, the relation, whether affirmative or negative, in which the major premiss places *M* to *P*, must be transferred unaltered to the unaltered subject, whether universal or particular, of the minor; the conclusion therefore has the quality of the major and the quantity of the minor. If we suppose all the possibilities exhausted for which these rules leave room, we get four valid kinds or moods of the first figure. Their scholastic names *Barbara*, *Celarent*, *Darii*, and *Ferio*, which by the three vowels in order denote (as every one knows) the quantity and quality of the premisses and the conclusion, show at a glance the distinctive feature of the first figure, namely, its capacity to produce conclusions of every kind.

86. The premisses of the *second figure* show us two subjects *S* and *P* in relation to the predicate *M*. If both subjects either have or have not this predicate, i. e. if both premisses are affirmative or both negative, no inference can be drawn from them as to a mutual relation between *S* and *P*. For innumerable subjects may all participate in, or all be excluded from, a mark *M*, without necessarily having any other point in common, and in particular without the one, *S*, being necessarily a species of the other, *P*. Only if the one subject has or has not the mark *M* always or universally, while the other is related to *M* in the opposite way, is there ground for concluding that the second cannot be a species of the first. The premisses in the second figure must therefore be of opposite qualities, and one of them must

be universal. As however it is the tradition that this second subject should be supplied by the minor premiss, the premiss in which the first is mentioned, i. e. the major, must be the universal one. Thus the conditions of the second figure may be summed up as follows: the major premiss is universal, but is not limited as to quality; the minor is of the opposite quality to the major and is not limited as to quantity; the conclusion is always negative, and has the quantity of the minor. The possible moods are *Camestres*, *Baroco*, *Cesare*, *Festino*.

87. The *third figure* brings the same subject *M* into relation to two predicates, *P* and *S*. If *M* has both predicates, i. e. if both premisses are affirmative, the union of *P* and *S* must be *possible*, and the conclusion therefore, according to the usual logical expression of such a possibility, is, 'some *S* are *P*.' The necessary identity of *M* is in this case sufficiently secured by the universality of one premiss, it does not matter which; for it clearly makes no difference whether all *M* have the mark *P* and only some have *S*, or whether all *M* have *S* and only some *P*; in either case there are always some *M* which have both and thereby justify the conclusion, which is always particular, 'some *S* are *P*.' Moreover this case, in which *M* is subject in both premisses, is just one in which its identity might be easily guaranteed by a word of completely individual meaning, the proper name of a person for instance. We often meet with such inferences: in order to prove the compatibility of two actions which seem to be mutually exclusive, we bring forward an instance, e. g. 'Socrates was *P*, and Socrates was also *S*,' consequently 'what is *S* may also be *P*,' or 'some *S* is *P*.' Logic justifies such inferences by attributing to the singular judgment, i. e. one whose subject is not an indefinite part of a universal concept but a perfectly definite and unique individual, the syllogistic value of a universal judgment. Thus this case comes under the above rule, which, where both premisses are affirmative, requires one to be universal, prescribes a particular affirmative conclusion, and admits the moods *Dàrapti*, *Datisi*, and *Disamis*.

88. Again, if the same subject possesses one of the marks but not the other, i. e. if one premiss is affirmative, the other negative, *S* and *P* must be separable, or, according to the ordinary phraseology, the particular negative conclusion follows, 'some *S* are not *P*.' In this case also it is sufficient for the identity of *M* that one premiss, it does not matter which, should be universal, but the minor premiss must be affirmative. For though one of two marks which occurs in a given customarily furnished by the minor premiss, the minor premiss must be affirmative and only the major can be negative. Under this condition mixed premisses yield the moods *Felapton*, *Ferison*, and *Bocardo*, these like the preceding ones having only particular conclusions.

89. Lastly, it is asserted by logic as a universal principle that in the third as in the other figures two negative premisses yield no valid inference. This is incorrect; a conclusion may be drawn from them similar in kind and equal in value to those which are derived from affirmative or mixed premisses. For if the first of these prove that *S* and *P* may exist together, and the second that they may exist apart, two negative premisses prove with equal ground that *S* and *P* are not contradictorily opposed, and that accordingly what is not *S* need not therefore be *P*; in ordinary phraseology, 'some not-*S* are not *P*.' I cannot see why this conclusion should stand lower in value than the two others; the first only says to us, 'when you find *S*, be prepared for the possibility of finding *P*,' the second, 'when you meet with *S* do not reckon upon the existence of *P*,' and similarly the third, 'where you do not observe *S*, beware of inferring for that very reason the presence of *P*.' In life we often meet with such inferences; over and over again, when the necessary presence of some quality has been over-hastily concluded from the absence of some other, we appeal to instances in which neither the one nor the other is found, and so correct an erroneous prejudice by an inference in the third figure from two negative premisses. This conclusion therefore is undoubtedly valid, but it would be an anachronism to invent supplementary names for its various moods.

90. The premisses of the fourth figure, ascribed to Claudius that need air are roses.' On the other hand we cannot by conversion recover from this conclusion in the fourth figure the one which we drew from the same premisses in the first; its conversion only yields the particular proposition, 'some things which are roses need air.' Thus in this case the conclusion in the figure of Galen actually loses a part of the truth which is established by the premisses, a bad recommendation for a process of inference, the function of which is always to conclude from what is given as much new truth as possible. This awkwardness could indeed be avoided, as was shown before, but the inference would not thereby be made more natural. Equally unnatural are *Calemes* and *Dimatis*, the premisses of which will always be transposed by the unsophisticated mind and applied in *Celarent* and *Darii* of the first figure: they do not indeed occasion a loss of truth, since the negative conclusion of *Calemes* admits pure conversion, while that of *Darii* is particular like that of *Dimatis*. It is only *Fesapo* and *Fresiso* which are less readily reducible to the first figure, owing to the negative minor premiss which results in both and the particular major which results in the latter; by pure conversion of their majors they can be transposed into *Felapton* and *Ferison* of the third figure instead, and this change will have the same effect of making the conclusions more natural. In all points, therefore, the fourth figure is a very superfluous addition to the three figures of Aristotle.

91. Aristotle considered the inferences in all the three figures to be valid, but only that in the first to be perfect, because in this figure only does the ground upon which all inference depends for its possibility, the subordination of the particular to the universal, find formal expression in the structure of the premisses. In the other figures too, indeed, (as he held), the inference rests upon the same principle, and the relations of subordination, which are necessary and sufficient for drawing a conclusion according to that principle, are contained in the premisses and do not need supplementing by information from without; but they are not exhibited in the actual structure of the premisses; we have to look for them there. To make good this formal defect in the two latter figures, Aristotle has shown us how, without any change of content, their premisses may be transformed into those of the first figure. To some people this has seemed superfluous, and they have objected that the two other figures also conclude according to prinof the particular to the universal; for to what did those figures apply their principles if not to justify the conclusion by subordinating the content of the premisses to them? Aristotle was therefore right in his general idea of the superiority of the first figure; we may also share the interest which he took in justifying the other figures by these changes of form; but it is true that in practice it is seldom of much use to carry them out; in considering the fourth figure just now we seemed to find such a case; the inferences of the second and third figures are too transparent to need this assistance.

92. It is therefore sufficient to mention that in the names of the moods of the two last figures the scholastic logic has indicated the operations necessary for this purpose by the letters *m s p c.* Thus *m* implies the transposition (*metathesis*) of the premisses: *s* and *p* tell us to convert, purely (*simpliciter*) or impurely (*per accidens*), the proposition whose characteristic vowel they follow: the meaning of *c*, reduction to impossibility (*per impossibile ductio*), is the only one which is not quite so simple, and may be at once illustrated by the case of *Baroco.* The premisses here are, 'all *P* are *M*,' 'some *S* are not *M*,' and the conclusion, 'some *S* are not *P*.' If we suppose this conclusion to be false, it follows *ad contradictoriam* that 'all *S* are *P*.' If this were so, and if this new minor premiss, 'all *S* are *P*,' were subordinated to the given major, 'all *P* are *M*,' it would follow in *Barbara* of the first figure that 'all *S* are *M*.' But this result contradicts the given minor 'some *S* are not *M*'; it was therefore wrong to deny the truth of the conclusion in *Baroco*, and that conclusion, 'some *S* are not *P*,' is right. The other operations scarcely need illustrating. We have lately seen how, by transposition, *m*, of the premisses, and impure conversion, *p*, of the conclusion, which was then drawn in the first figure, *Bamalip* of the fourth is reduced to the first. *Camestres* of the second, 'all *P* are *M*,' 'no *S* is *M*,' 'no *S* requires no further transformation, being immediately identical with that of *Darapti*.

93. Thus far we have conceived of the premisses as categorical judgments of the form '*S* is *P*.' But the course of our thoughts may also suggest them in an hypothetical or disjunctive form. These differences, important as they are for the judgments as such, are not so for the formal connexion of the syllogism; they always belong to its content, and it is only necessary to take note of them, not to alter the ordinary syllogistic rules on their account. This is most obvious where we have two hypothetical premisses, in each of which two of the three *propositions* *M S P* are connected as *protasis* and *apodosis*. Just as with categorical premisses where *M S P* denote three *concepts*, the inference in *Darii* is as follows: '*P* is always true if *M* is true, *M* is sometimes true if *S* is true, therefore *P* is sometimes true if *S* is true'; in *Camestres*, '*M* is always true if *P* is true, *M* is never true if *S* is true, therefore *P* is never true if *S* is true'; in *Disamis*, '*M* is sometimes true if *P* is true, *M* is always true if *S* is true, therefore *P* is sometimes true if *S* is true.'

The cases are more peculiar when the major premiss is hypothetical and connects universally a consequence *F*, expressed in the apodosis, with a condition G, contained in the protasis, while the minor is categorical and affirms or denies either G or *F* of all or some instances of *S*. The simplest way is to class these cases with the immediate inferences from judgments, for condition and consequence are related as *subalternans* to *subalternata*. Firstly, then, the fact that the condition G is not true in certain cases of *S* does not justify us in inferring *ad subalternatam* that the consequence *F* is not true in the same cases, for the same consequence may arise from other and equivalent conditions. But if the condition is true, we infer the truth of the consequence. This gives rise to two syllogisms, since G may imply either that *F* is true or that it is not true; (1) 'If G is true *F* is always true, G is true in all or some cases of *S*, therefore *F* is true in all or some cases of *S*'; this is a *modus ponendo ponens*, which posits the consequence by positing the condition, and it evidently answers to the moods *Barbara* and *Darii* in the first figure: (2) 'If G is true *F* is never true, G is true in all or some cases of *S*, therefore *F* is not true in all or some cases of *S*;' a *modus ponendo tollens*, in so far as it does away with the consequence *F* by positing the condition of position *F* in certain cases of *S* does not prove the truth of the particular condition G on which it was found to depend in other cases, for the same consequence *F* may arise from several equivalent conditions. But the fact that *F* is not true in certain cases of *S* does prove that all conditions upon which it could depend, and therefore the particular condition *G*, are not true. The following syllogisms are therefore admissible: (3) 'If G is true *F* is always true, *F* is not true in all or some cases of *S*, therefore in all or some cases of *S* *G* is not true,' a *modus tollendo tollens*, which by doing away with the consequence does away with the condition which, had it been true, would inevitably have given rise to it; it corresponds clearly to *Camestres* and *Baroco* of the second figure: (4) 'If G is true *F* is never true, *F* is true in all or some cases of *S*, therefore in all or some cases of *S* G is not true,' a *modus ponendo tollens*, which by positing a consequence denies the condition under which it would have been impossible; it repeats *Cesare* and *Festino* of the second figure. Lastly, we may reflect that the fact that *G* is not true may also imply that *F* is or is not true, in which case we get the syllogisms, (5) 'If G is not true *F* also is not ever true, in all or some cases of *S* G is not true, therefore in the same cases *F* is not true,' a *modus tollendo tollens* without any peculiarity, merely translating the *ponendo ponens* into the negative: (6) 'If G is not true *F* is always true, in all or some cases of *S* *F* is not true, therefore in these cases G is true,' a *modus tollendo ponens*, which was wanted to complete the possible combinations of condition and consequence, positive and negative; it posits the truth of a condition by doing away with the consequence which would necessarily follow if it were not true. An easy change in the form of expression shows that these two last cases also belong to the second figure; the latter of them might be put thus, 'If non-*G* is true *F* is always true, *F* is always or sometimes not true, therefore non-*G* is always or sometimes not true.' As this exhausts everything that can be proved from the relation of subalternation, there are no consequences of this kind which could be classed under the third figure.

94. These syllogistic devices are in my mind of less importance than a circumstance which I never find thoroughly considered in connexion with the present subject, the circumstance that all these inferences refer merely to a relation between the *condition* G and its *consequence* *F*, not to that of a *cause* G to its *effect* *F*. It is only in the world of thought that a condition G, if it is once supposed to be true, *always* has the consequence which by a necessity of thought belongs to it; in the real world the cause G, even if it exists and is operative, may always have its effect *F* frustrated by an opposing force *U*. In being transferred to actual events, therefore, all these inferences require to be modified in ways which applied logic will show us: thus it is not allowable to conclude that wherever the cause G operates its effect *F* is necessarily a *fact*, nor to assert that, if G is a cause of hindrance to *F*, where this hindrance exists *F* cannot exist; G also in its turn may be hindered by a *U*, or *F* may be realised in spite of it by a third cause *V*. In pure logic, therefore, it is quite an improper description of the cases which we have been dealing with to say, that their minor premiss expresses the real existence of *G* or *F*; the truth is that these two simple letters stand here for judgments of the form '*S* is *P*'; it is only the logical admissibility or necessity of this connexion of thought between *S* and *P* which the minor premiss asserts in regard to certain cases of *S*, while the major connects it with another similar relation between *S* and *Q*, so as to form an hypothetical judgment of universal validity. I will not pursue this point further here; I have made my exposition somewhat prolix in expression with the view of indicating how the matter really stands.

95. If it is true of a subject *Z* that it is either *P*, *Q*, or *R*, or that it is both *P*, *Q*, and *R*, or that it is neither *P*, *Q*, nor *R*, we first substitute for this triple predicate the simple *U*, and call *U* in the first case disjunctive, in the second positive, in the third negative. If anyone takes the not absolutely necessary trouble to follow the application of such disjunctive, copulative, and remotive premisses in the syllogism, he will find these results. (1) If the major premiss is *Z U*, and in the minor *S Z* an *S* is subordinated to *Z*, the ordinary conclusions *S U* of the first figure follow, and *U* has in them the same meaning as in the major: (2) If the universal major is *Z U*, the minor *S U*, and *U* is in one of them positive or disjunctive, in the other negative, we get the negative conclusions *S Z* of the second figure with the quantity of the minor: (3) from the major *U Z* with a positive or negative *U*, and the minor *U S* with a *U* of the same or the opposite quality, there result the conclusions *S Z*, always particular, of the third figure: (4) in the two latter cases, where *U* having become the middle term disappears from the conclusion, its the particular subject *Z* one of the members of *U*. If the major distinguishes only two alternatives and says, 'all *Z* are either *P* or *Q*,' and the minor 'this *Z* is *P*' or 'this *Z* is not *P*,' it follows that 'this *Z* is not *Q*' or 'this *Z* is *Q*.' These consequences explain themselves from the nature of contradictory opposition; they can be reduced, but without any conceivable advantage, to the first figure; 'every *Z* which is not *P* is *Q*, this *Z* is a *Z* which is not *P*, therefore this *Z* is a *Q*.' The same unfruitful reflexions may be extended to a *U* of more than one member in the major premiss, for we can always make any number that we choose of its members into the subject, and say (with only a bipartite *U*), 'every *Z* which is not *P* and is not *Q* is either *R* or *T*.' Lastly, *polylemmas* (dilemmas, trilemmas) are syllogisms with a disjunctive *U* of many members in the major *Z U*, and the same number of minors, which taken together affirm of each one of the members of *U* the same further consequence *T*. These are not cases of new logical forms but only new applications of old ones, and we may return to them in our applied logic.

96. On the other hand, I have no intention whatever of coming back to the doctrine of *chains of inference*. Every conclusion of a syllogism may conceivably become the major premiss of another syllogism: the first is then called the *prosyllogism* of the second, and each one that follows the *episyllogism* of the one which preceded it. A mere comparison of the names of the moods shows us at once many properties of the chain thus produced. If its last member is to be universal, the whole series of prosyllogisms, and therefore the whole chain, must be in the first two figures; the entrance of any member in the third figure produces a particular conclusion, which never leads back again to universal conclusions. If one of the syllogisms has a negative conclusion, the conclusions of all episyllogisms are negative; and a chain can only end with a conclusion at once positive and universal if it is in *Barbara* through its whole course. It is moreover usual to require, on the analogy of the simple syllogism, that the major premiss of the first prosyllogism should furnish the predicate *P* of the ultimate conclusion, and the minor of the last episyllogism its subject *S*: it would only need patience to find the rules for the formation of such a series, but I cannot see of what use they would be. If the conclusion of a prosyllogism, which is also the major[1] premiss of the episyllogism, is not expressed, the series give rise to the two forms of *Sorites*. The Aristotelian form, '*A* is *B*, *B* is *C*, *C* is *D*, therefore *A* is *D*,' includes each concept in the one which follows; it thus proceeds from the lower to the higher, and is produced by suppressing the conclusions, which we could elicit from each pair of members as follows, '*B* is *C*, *A* is *B*, therefore *A* is *C*,' and then, '*C* is *D*, *A* is *C*, therefore *A* is *D*.' The other form, a late discovery of Professor Goklenius of Marburg (1547–1628) takes the opposite direction; its premisses, '*B* is *A*, *C* is *B*, *D* is *C* . . .,' suppress the conclusion of the two first members, '*C* is *A*,' which as major premiss to the third gives the conclusion of the chain in the first figure, '*D* is *A*.'

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[1] ['*Minor*' premiss, in the Aristotelian *Sorites*. The author's words only apply to the Goklenian form.]

A. *Syllogistic Inference. Inference by Subsumption. Inference by Induction. Inference by Analogy.*

97. The logical truths of which the mind had gradually become conscious in dealing with its ideas were provisionally summed up by the disjunctive judgment as follows: every *S*, which is a specific form of *M*, possesses as its predicate a particular modification of each of the universal predicates of *M* to the exclusion of the rest. The problem which remained was to discover the intellectual operations by which this required specific mark could be determined for a given *S*. This problem is not solved by the Aristotelian syllogisms; they confine themselves to placing the subject of their conclusion in relation merely with the universal form of the predicate mentioned in the major premiss; so that in spite of the manifold development given to them and their possible varieties by the acuteness of earlier logicians, they are merely the expression, formally expanded and completed, of the logical truth already embodied in the disjunctive judgment. Like the impersonal judgment, which, by distinguishing subject and predicate, made formally explicit a division already indicated in the concept, without telling us anything new about the mutual relation of the members, thus produced the Aristotelian syllogism in its first and most perfect figure, to which we mentally refer the others, merely distinguishes in two separate premisses the universal rule and its particular application, which were already similarly related in the disjunctive judgment. Thus the Aristotelian syllogisms, constructed as they all are on the principle of placing one concept within the circuit of another without further defining its position, may be included, under the general name of *inference by subsumption*, and considered as the first and most elementary form of the new group of intellectual operations. We will now attempt to show what is the next step in advance which they compel us to take.

by subsumption is based I choose the mood *Darii*, which
essly brings a particular case in the minor premiss under the
ersal law contained in the major. 'All men are mortal,' says
mood, 'and Caius is a man,' whence it concludes, 'Therefore
s is mortal,' clearly meaning that by this conclusion a truth which
not established before is now made certain by the truth of the
premisses and their relation to one another. But as early as the
icism of antiquity the objection was made, that it is not the
isses which guarantee the truth of the conclusion, but that the
lusion must already hold good in order that the premisses may
o. Where, indeed, would be the truth of the major premiss,
men are mortal,' if it were not already certain that Caius partici-
s in this property? And where would be the truth of the minor
iss, 'Caius is a man,' if it were still doubtful whether among
ther properties of humanity he had that of mortality also, which
najor itself alleges as a universal mark of every man? Instead
of proving the truth of the conclusion by their own independent
, the two premisses themselves are only true on the supposition
truth, and this double circle seems at first to make the syllogism
ally quite inoperative.

). The weight of this objection is not to be got rid of by denying
e will follow out its applications in various cases. If we suppose
najor premiss MP to be an analytical judgment, if, that is, we
ne P to be a fixed mark without which the content of M cannot
ompletely conceived, then certainly the universal validity of the
r is independently established; but then the minor cannot
rdinate an S to M without already attributing to it this indispens-
P, that is, without presupposing the conclusion in which that
ution ought first to find expression. If for instance we reckon
ht in the concept of body, we form the major premiss, 'all
es have weight,' without fear of contradiction; but we cannot go

though at the same time we should be certain, on whatever grounds, that P is always combined with M. The minor premiss would then merely have to show in S the marks which make it an M, and then, and not till then, the conclusion would add the P which belongs to S in virtue of its subordination to M, but which had not before been part of the conception. In the practical employment of subsumptive syllogisms these assumptions are always made. When we assert, 'all men are mortal,' we conceive the physiological character of man to be fully determined by the rest of his known organisation, and regard mortality as a mark which need not be explicitly thought of when we mentally characterise him, because it follows inevitably from the organisation which determines our conception. And thus in the case of Caius it is enough to establish in the minor premiss the fact that he has this essential organisation, in order in the conclusion to ascribe to him its inevitable consequence. This is still more clear if we conceive the major premiss as hypothetical, and think of P as not a fixed and permanent but a fluctuating mark of M, a consequence which follows upon M under a certain condition x, a mark which under this condition M assumes or loses, a state into which it falls, or an effect which it produces. Then we have merely to subordinate S to M in the minor premiss in order to conclude that S also, if the same condition x operates, must exhibit the mark P. And as a matter of fact this is the form to which most of the effectual applications of the syllogism in science are reducible; they almost all show that S, being a species of M, will develop or experience under the condition x the same general effect P as we know in M. But as before with the analytical major premiss the question arose, with what right the minor could be asserted, so here with a synthetical major the question arises, with what right we can affirm the universal validity of this major itself. Mortality is to be a new mark, necessarily accruing to the organisation of man: but this universality can only subsist on the assumption that the conclusion is true, and it falls to the ground if some capricious Caius is found who does not die. It is clear what the answer to this will be: 'of course,' it will be said, 'every universal major premiss is false if there is a single instance in which it is not confirmed, and there is always this danger when the universal in question has been formed only by an unjustifiable generalisation from a number of observed instances: but where the necessary connexion of M and P is into which as yet no contradictory instance has been found: to the physiologist, as a consequence of the known human organisation, it is certainly a matter of settled conviction, but not one which can be proved with the exactness he would wish. But in other cases the universal validity of the synthetical major premiss is guaranteed either by an immediate perception, or by proofs which reduce a given matter to such a perception, and in these cases the syllogism suffices for securing a particular piece of new knowledge; for all that this requires is perfectly practicable, viz. the subordination of an S to an M, which here really fulfils the function of a middle term in connecting S with a previously unconnected P.'

100. I leave it for the present an open question whether, and how far, the immediate perception of the universal truth of a synthetical judgment is possible; for so much is at once clear, that in any case we shall be only very rarely in a position to rest the content of a universal major premiss upon this ground; countless universal judgments are expressed and used for inferences, without the possibility of either themselves passing for immediate perceptions or being reduced to such by any practicable method of proof. This wide field of intellectual activity cannot be simply set aside as invalid, nor can it subsist without logical rules of its validity. These rules we have to look for, and there are two which we want. For the effective use of the syllogism it is, firstly, necessary that we should learn to find universal major premisses, based neither on an immediate certitude nor upon the antecedent experience of their truth in every single instance; it must be possible to assert the universal mortality of men, both before it is understood as the necessary consequence of certain conditions, and also before we have tested every individual man to see whether he is mortal. A second rule is necessitated by the minor

I am here requiring admit (though this is not of essential importance), of being attached to somewhat modified forms of the second and third Aristotelian figures.

101. The problem of all inferential processes is naturally this, from given data or premisses to develop as much new truth as possible; how this is done, is in itself quite immaterial; the method will be determined by the form of the premisses, and these we have to take as experience, internal or external, offers them. Now it often happens that the same predicate occurs or does not occur, not only in two, but in very many different subjects P, S, T, V, W, and the question is, what consequence can be drawn from the premisses, PM, SM, TM, VM, . . . , which belong in form to the second figure of Aristotle. It is clear that in their multiplicity they do not suggest an inference which would connect together any particular two of their subjects; so far as we aim at such an inference, we can only effect it by confining ourselves with Aristotle to two premisses and observing the rules of the second figure. But it is equally open to us to try whether this recurrence of M in such different subjects tells us anything about the significance of M itself, which accordingly would not disappear in the conclusion. Such an experiment is what the natural mind infallibly makes when experience furnishes such premisses, and it is guided in its experiment by the universal principle which dominates all its activity, the principle of translating a given coexistence of ideas into a coherence between their contents. Where we observe the same mark in different subjects, we are predisposed to think that the agreement is not a chance one, and that the different subjects have not therefore stumbled upon the same predicate each through a special circumstance of its own, but are all radically of one common essence, of which their possession of the same mark is the consequence. P, S, T, V will accordingly be different, but still co-ordinate as species under a higher concept Σ; it is not as different individuals, but only as species of the genus Σ, that they bear the common mark M as a necessary mark of that genus. Our conclusion therefore runs as follows, 'all Σ are M'; and in this conclusion Σ stands for the higher universal to which we subordinate the individual subjects, and for the true subject of the M which before appeared as a common attribute of those individuals. Such a process of inference is the simplest case of *Induction*, and under this name forms our *second* member in the group of inferences problem which was set to it, that of producing universal major premisses for subsumptive syllogisms. For everybody agrees in objectng to induction, that if it is complete its information is certain but not new, while, so long as it is incomplete, it is new but not certain. f *P*, *S*, *T*, *V* are all the species of Σ which exist, and if each already has a premiss informing us that it is *M*, the conclusion can only sum up these premisses in a universal judgment, 'All Σ are *M*'; but it cannot even logically be changed into the general judgment, 'Every Σ as such is *M*'; on the contrary, it remains quite uncertain whether he species of Σ merely participate as a fact in the common *M*, and each ultimately for a special reason of its own, or whether the universal nature of Σ really contains the one and selfsame reason which makes *M* a necessity to all its species. If, on the other hand, besides those subjects which are combined with *M* in the premisses, there are other species of Σ of which those premisses say nothing, then the conclusion is an unjustified inference *ad subalternantem* from the truth of a limited number of instances to the truth of all, an inference which may have probability in various degrees, but never reaches certainty.

It appears to me, however, that these observations, right as they are in themselves, confuse the pure meaning of a logical form with he difficulties of its effective application, and that there was the same error in the criticism made upon the value of the Aristotelian syllogism. The leading idea of that syllogism, that every individual derives its right and obligation to the possession of its predicates hrough dependence upon its universal, is without doubt logically a perfectly valid principle, and exhibits in its true light the internal construction of the content of thought in question. It does not lose his logical significance because the truth of the universal includes or, f we prefer it, presupposes its truth in all particular instances; on the contrary, the very meaning of the syllogistic principle is that the two are inseparable. Whatever therefore may be the way by which in practice the mind has *arrived at* the truth of the premisses, when hey are once *found* the first Aristotelian figure does express by its

I have a similar remark to make about *induction;* the logical idea upon which it rests is by no means merely probable, but certain and irrefragable. It consists in the conviction, based upon the principle of identity, that every determinate phenomenon *M* can depend upon only one determinate condition, and accordingly that, where under apparently different circumstances or in different subjects *P*, *S*, *T*, *U* the same *M* occurs, there must inevitably be in them some common element Σ, which is the true identical condition of *M* or the true subject of *M*. It would be quite unjustifiable to object, that as a matter of experience the same consequence *M* is often produced by different equivalent conditions, and the same predicate *M* may occur in extremely different subjects. Such an objection just shows the confusion, which we condemned above, between the logical rule and the conditions of its application. If there are two equivalent conditions for a result *M*, it is not in virtue of that which makes them different, *P* or *S*, but of that which is the ground of their equivalence, that they are really conditions of the same result: so long as we cannot separate this common characteristic in the two, we have not yet found the true Σ of the conclusion, and have not therefore carried out the induction in the way in which it demands to be carried out. Again, if the same *M* is found as predicate in a number of extremely different subjects, and subjects (as is usually the case in practice) the several sums of whose marks are only partially known, we may of course make a great mistake if we combine what is common to the known marks of all of them, and then assume it to be Σ, the true subject of the mark in question *M*. I do not deny that in the practice of induction we are often placed in such unfavourable circumstances; but all these difficulties in carrying out the inductive principle do not alter its universal logical validity when it asserts, that wherever different conditions have the same result *M*, or different subjects the same predicate *M*, there must be discoverable one and only one quite determinate Σ, forming the single invariable condition or the single true subject, to which the predicate or the result *M* is to be universally and necessarily ascribed in a conclusion of the form, 'every Σ is *M*.' We leave it to applied logic to observe the rules by which we may succeed in discovering this Σ.

103. I introduce the third form of this group under the somewhat arbitrary name of the *inference of analogy.* In the third Aristotelian

MP, *MS*, *MT*, *MU*; will show us, in other words, that a number of different marks does or does not occur in the same subject. These data cannot be rejected by the mind, and it employs them to form an inference which is just like the one described above, only in the reverse direction. Here, as there, it is guided by the assumption that the different predicates have not united in the same subject *M* by a number of unconnected chances, but that they must be coherent and owe their coexistence to the presence of *one* condition; they belong to *M* because *M* is a Π, and it is this sum of marks which in its completeness constitutes the nature of Π; and *M*, being a species of Π, has a right to unite them all in itself. Thus from these premisses we form the conclusion, 'M is a Π,' and have so executed our second task of finding for the subsumptive syllogism a minor premiss by which a concept *M* (there called *S*) is subordinated to another concept Π (there called *M*).

104. Yet this task, like the former one, seems to be but badly executed, for analogy, like induction, is liable to the charge that, if complete, it tells us nothing new, and if incomplete, nothing certain. If the premisses already give *M* the marks necessary to make it a Π, we gain nothing in knowledge of fact by actually bringing it under this concept; the change is merely in the form of our apprehension of the given content. But in most cases the premisses give only a part of the predicates necessary to Π, and from the presence of these we conclude without certainty to that of the rest, by which alone the whole of Π is realised in *M*. When we have to do with concrete objects, which in their totality consist of countless marks, in great part unknown to us, in part difficult to observe, this is always the case: from a few properties which we actually observe in an object, we conclude that it is a metal, an animal of a certain kind, an instrument for a certain purpose. It is needless to say that numerous mistakes in the employment of analogy arise from this fact; but here also the difficulty of the application does not infer its further completion and continuance; there is always therefore one and only one Π, which makes legitimate and possible the union of marks given in *M*, and at the same time the addition of others not given. This ideal of thought, which in itself is quite true, only requires, like every form of thought, to be realised in suitable, not unsuitable, matter. It is not any casual pair of predicates in an *M* which suffice for inferring the rest; many such combinations may belong to some other concept Π^1 or Π^2 as well as to Π; in contrast with such unessential marks we shall require essential ones in the premisses, a requirement which is always made in practice, and which it is left to special knowledge of the matter in question to meet. The most important source of inexactness, however, is that all the forms of inference hitherto mentioned give the predicates only a universal form, without indicating their measure, specific modifications, and mutual determination. So long as the premisses only say, '*M* is heavy,' '*M* is yellow,' '*M* is liquefiable,' etc., we certainly find in such data no decisive ground for pronouncing *M* either to be gold or to be sulphur: but this is just why such premisses are only met with in abstract logic; in actual practice attention is always given also to the particular amount, nuance, and combination of the predicates, and from this incipient characterisation its continuity with the completed Π is inferred. It is just this universal practice of the natural mind for which we have to find a theoretical basis in new logical rules, and these we must now go on to consider.

B. Mathematical inferences. Inference by substitution. Inference by proportion. Constitutive equation.

105. I will put together once more, and from different points of view, the motives which impel us to go beyond the syllogisms and look for new forms of thought, and for this purpose I will first touch upon the nature of the judgments which the ordinary theory conceives of as members of the syllogism. In judgments of the form '*S* is *P*,' as I have already observed, language expresses the predicate with a universality with which it does *not* belong to its real subject, and logic usually concedes this when it asserts that not only does the predicate contribute to the determination of the subject, but the subject also to that of the predicate. When we *say*, 'this rose is accurately still, the precise red of *this* rose. If then we wished to express our thought exactly, we should have to say, 'this rose is red with the redness of this rose.' In this apparently quite barren proposition the logical activity would show itself in the fact, that the perceived property of the rose is no longer apprehended as an isolated thing, without any other home in the world; in regarding it as a kind of red in general, which occurs elsewhere and holds good independently of this instance, the mind, as we said before[1], objectifies its perception; it gives to what is perceived a definite place in the world, which makes it something on its own account, and not a merely subjective excitation of the percipient at the moment. In this lies the logical gain which always results when the particular content of a perception is replaced in the judgment by the universal of which it is an instance. But at the same time of course there will be a logical loss, if we get no further than the expression of this universal, and if the other part of the perception does not get its due by addition of the particularisation which is necessary to make the universal *named* equivalent to the individual *intended*. This loss is sustained by all ordinary judgments of the form just mentioned, and the Aristotelian syllogisms too confine themselves to dealing with the universal M or the universal P.

106. In this way they leave unsolved the particular problem which the disjunctive judgment suggested, and fail generally to satisfy the practical needs of thought as a living process. For already in the disjunctive judgment it was asserted, that it is not the universal predicate of its genus which belongs to the individual, but a definite modification of it, p, to the exclusion of the rest. What this p is, ought to have been made out by the syllogism; and it could only have done so by supplying to the major premiss, which connects the genus with the universal P, a minor bringing out the peculiarity in virtue of which S is this particular species of the genus and no other, and must therefore have for predicate this and no other modification of P. This has not been done; the minor premiss also only mentioned generally the subordination of the individual to the genus, but not its specific difference from other species of it; hence the conclusion could only say what belongs to the individual as *a* species of its genus, not as *this* species. It hardly needs to be further explained that this falls short of what the actual processes of thinking demand. If we argue, 'heat expands all bodies, iron is a body, therefore heat expands iron,' or, 'all men are mortal, Caius is a man, therefore

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[1] [Above, § 3.]

Caius is mortal,' everyone will feel the barrenness of this procedure, and will reply, 'Undoubtedly heat expands all bodies, but each body in a different degree; undoubtedly all men die, but the liability to die in one man is different from that in another; what we want to know for technical purposes or for administering a life-insurance company is, how iron expands in distinction from lead, or how the mortality of Caius is to be estimated in distinction from that of other men.' This then is what the new forms have to do; they have to make the individual felt as a definite species of the universal, and so enable us to argue from its distinctive difference to its distinctive predicate.

107. From another point of view we may notice the fact, that in logic it has been too exclusively the custom to use categorical judgments as illustrations, and therefore also to represent the inclusion of one concept within another as the most frequent and most important of logical operations. In the living exercise of thought this is by no means the case; we are seldom concerned in practice to determine a mark which belongs to a concept once for all, or in the circuit of which the concept is to be classed; most frequently we want to know what variable mark P will occur in a concept S if S is subjected to the condition x. Questions of this kind are being raised at every moment by life, science, and art. We must admit that the ordinary syllogistic method does not entirely overlook such cases; but it is only an imperfect way of dealing with them to make P the universal result of the coexistence of x with M in a major premiss, and then to ascribe P to S, again only universally, by subordinating S to M or Mx. What good is it to say, 'if a man is offended he gets angry, Caius is a man, therefore if he is offended he will get angry'? What we want to know is, *how* Caius, being the person he is, will get angry, and consequently how far we may go with him. The subordination of Caius to the concept of humanity helps but little to answer this question; we must look for the special characteristics which distinguish Caius from other persons, and must then have the means of calculating the effect which offence will have upon these characteristics. This may be briefly expressed thus: our inferences cannot be derived from extensive relations between the given concepts, but only from their content; without making the unprofitable circuit through the universal genus, we have to determine directly from the given marks of a bination of certain marks in an S the necessary presence, absence, and mode of attachment of other marks in the same subject. We may doubt indeed whether such inferences from content to content, from mark to mark, are possible on merely logical grounds, and whether the few which really are possible are not already anticipated by the familiar logical doctrines of the compatibility of disparate predicates, the incompatibility of contraries, and the necessary choice between contradictories: statements such as, 'where p is there q must be,' will after all (it may be said) be supplied by experience alone, with the single exception, with which we wish to have no more to do here, when q is already included in the content of p or p in the extent of q. In itself this doubt is right; all assertions about the necessary connexion or incompatibility of two predicates, with the exception of the cases last mentioned, can never be based upon any evidence but that of observation; but it is still a question whether logic, with the means hitherto at its disposal, has made even these necessarily presupposed facts yield all the consequences which they might be made to yield: that it has not done so, I can show more shortly by exhibiting the actual forms of inference to which I refer: in the natural use of the mind they are current and familiar, and all that is done here is to give them the place which belongs to them in a system of logic.

109. Let us leave to the major premiss of our new figure the form, 'all M are P' or $M = P$; to the minor however we will give, not the indefinite form, 'S is an M in general,' but the definite one $S = sM$; that is, S is that species of M which we get if we conceive the whole structure of the marks in M as determined or modified by the influence of a specific condition s. The conclusion will then have to be, 'S is σP,' and it will assert that S, so far as it is this distinctive species of M characterised by s, possesses, not the universal mark P, but that specific impression of it, σP, which the influence of s must produce in the structure of M. To avoid misunderstanding, how P is changed by the influence of s upon M. This remains impracticable so long as we produce M merely in this simple form of a universal concept provided with a name: in order to know how s influences M, we must analyse the content of M into its several parts, and observe in what manner they combine. Nobody, for instance, will undertake to judge how the working of a machine will change under the influence of a force s, so long as he merely has the machine before his eyes as a simple object of perception, M, a steam-engine in general; he must first get to know the inner structure, the connexion of the parts, the position of a possible point of action for the force s, and the reaction of its initial effect upon the parts contiguous to that point. Accordingly, it is only by *substituting* for the condensed expression or concept M the developed sum of all its constituent parts, with attention to their mutual determinations, that we can hope to follow the influence of s, and so determine, firstly, what is the whole nature of S which $= s M$, and, as a consequence, what is the modification σP of the predicate P which belongs to this S. As a matter of fact, this second part of the problem is always included in the first; the specific modification of a particular predicate for S cannot possibly be found without first finding the total change produced in M by s, on which the modification depends; for if P were part of a different concept N, the effect on it of the same condition s would not be the same as when it is a part of M. For this reason I shall take no more notice of the inference to σP, but shall consider the problem of the new form to be to determine $s M$, and give it therefore the form,

$$\begin{array}{ll}
\text{Major premiss}: & M = a \pm bx \pm cx^2 \ldots \\
\text{Minor premiss}: & S \ = sM. \\
\hline
\text{Conclusion}: & S \ = s(a \pm bx \pm cx^2 \ldots)
\end{array}$$

from which, in regard to single predicates, e.g. b, there would follow the definite conclusion, 'S is $s . bx$,' instead of the indefinite one, 'S is bx.'

110. There is always a danger in expressing very different and yet connected cases by the simplest possible symbols; to avoid misunderstanding, therefore, I add the following observations. By a, b, c, x I wish to be understood, speaking generally, different marks of a con-suffice for a full expression even in a case where M does not mean a conceptual content of qualitatively different marks, but a mere whole of quantity composed of the commensurable quantitative parts a, b, c, x. The only symbol of a more exhaustive kind would be that of the mathematical function in general, which we used before, $M = F(a, b, c, x \ldots)$; but this would have the disadvantage of merely calling up to thought *all* modes of connexion between the parts, without giving a sensuous illustration of any. The form of the series $a + bx + cx^2$ is also an arbitrary symbol; the x only indicates a possible difference of value in the marks, one of which, x, leaves only one other, a, entirely free, while it accompanies the rest as a determining condition. The s of the minor premiss and conclusion appears here as a multiplying factor; this is similarly intended to represent to sense, by the simplest and most familiar form in which one quantity can influence another, the countless different ways in which any concrete condition may act upon the manifold content of any given subject. If we express by a letter placed underneath on the right any kind of change produced by a condition in any kind of given matter, and represent M as a function of a, b, c, x (i. e. $M = \phi(a, b, c, x)$), we should in general only be able to represent the conclusion by $S = \phi_s(a_s, b_s, c_s, x_s)$, not by $S = \phi(a_s, b_s, c_s, x_s)$; for it is obvious that the effect of s may not always be merely to change the single marks, retaining their general connexion (as expressed by the second formula), but also (as expressed by the first) to change this connexion itself; in fact, a condition may so transform the whole structure of a concept that in its new shape it has to be subsumed under a different concept M^1 or N instead of the previous M. The admission which I have now to add makes it unnecessary for me to go further into this point.

111. The advantage which we anticipate from this figure of *syllogism by substitution*, the first of this second group, depends ultimately upon nature of numbers, each one of which has an expressible relation to every other, allows us to disclose the hidden content of *M*, by substituting its quantitative parts, in such a way that the condition *s* can really operate upon it, and that by applying the various rules of calculation, by cancelling incompatible and compounding compatible elements, the change which *s* necessitates in *M* can be really carried out and the form of the new result exhibited. On the other hand, if we replace commensurable quantitative parts by incommensurably different marks of a concept, these advantages disappear again; the content of *M* is only imperfectly disclosed by such a method of substitution; for we have no rule here, as we have in the case of numbers, by which to measure the effect of a condition acting upon these heterogeneous elements. It is true that even in such cases we apply the general idea of substitution: if we want to know how a condition *s* will act upon a thing, of which we have only the concept *M* which its natural history supplies, we analyse *M* into its marks; but the calculation of the effect which *s* will have on each and all of them, is based merely upon more or less indefinite analogies, suggested by experience or some chance feeling of probability.

112. The fact that the use of the syllogism by substitution is confined to mathematics, cannot hinder us from giving it a place in the systematic series of forms of thought. For in the first place we must not forget that calculation in any case belongs to the logical activities, and that it is only their practical separation in education which has concealed the full claim of mathematics to a home in the universal realm of logic. But it is not only because they are indispensable to a part of the work of thought, that these forms have their place here; even in those cases where their demands cannot be realised, they are still the ideals of our logical effort. For if they can be applied directly to none but quantitative relations, it is true on the other side that wherever we are quite unable to reduce the object of our investigation to those relations, our knowledge of it remains defective, and that no other logical form can then help us to the answer which a mathematical treatment of the question, if it were practicable, would give us. It is hardly necessary in our days to draw attention to the fact, that natural science owes its existence to mathematics; in other fields also we have learnt to prize the important aid of quantitative statistics in discovering the laws which govern the combinations of crime by substitution into its several elements, and from $s M$, i.e. by calculating the particular values of the single elements of the crime in this instance, and so the particular value of the whole, it would deduce σP, i. e. the kind and amount of punishment which the particular instance deserves.

113. There are other things however besides pure mathematics, and science has certainly succeeded in establishing links of connexion, even between incommensurable phenomena or attributes, which allow us to infer from one to another. For logic on its part the next problem must be, to look for the forms in which such inference is possible, and so to supplement the imperfection of the substitutive syllogism. It would partly seem indeed that science has only succeeded in thus bridging the incommensurable by doing away with the incommensurability, and showing that two facts, a and b, which at first appear to our perception entirely different in quality, really depend upon quantitative differences between commensurable circumstances: I may recal how physics has reduced the qualitative differences of our sensations of colour, tone, and heat to merely mathematical differences in commensurable motions of commensurable elements. If however we look more closely at these cases, we find the fact to be, not that our sensations, a and b, are reduced to motions, α and β, commensurable with one another and with the sensations, but merely that the occurrence of α or β and its effect upon us is represented as the condition upon which the sensation a or b necessarily arises. The perceived colour a remains just as incommensurable as ever with the vibration of ether, α, by which its origin is explained; and if experience did not teach us that a is the consequence of α, we should have no logical means of divining has informed us of the coherence of two such elements, a and α, then thought concludes that this coherence will be maintained even in the event of their both changing, and that therefore a definite change of α into α^1 must always be answered by one and only one definite change of a into a^1. Again, these changes themselves, $a-a^1$ and $\alpha-\alpha^1$, are not directly commensurable, either in kind or amount: if the number of vibrations of the sound-wave is increased by the amount $\delta=\alpha-\alpha^1$, it is true that a definite increase, $d=a-a^1$, in the heard tone depends upon it; but this change in the pitch is a process quite different in kind from the increase in the number of vibrations, and cannot be compared with it; each quantity can still only be measured by a standard of its own, and their mutual coherence can be expressed as a fact and nothing more. But the changes in pitch are commensurable with one another, and so are the changes in the number of vibrations; and if we refer these changes to d and δ as their respective units, we may ask, By how many units m of the kind d does the pitch change, if the number of vibrations changes by μ units of the kind δ? m and μ then stand in a purely numerical relation. This relation may be infinitely various; but, as before, I shall not indicate the possible variety any further in the form which I give to this inference. I choose for its name and scheme the simplest form of proportion, $E:e=T:t$, which, though it only illustrates the case in which $m:\mu$ is a constant quantity, still sufficiently symbolises the logical idea implied in the process.

114. I will illustrate that idea once more by a very elementary example. Two angles E and e are commensurable; so are two segments of a circle T and t; but an angle and a segment are incommensurable and cannot be directly measured by any common standard: so too the difference of two angles, which again represents an angle, is incommensurable with the difference of two curves, which again forms a curve. Nevertheless, if it is once established that a certain length of curve t belongs to an angle e at the centre of a circle of a given diameter, and if we form the angle E by m times e and the corresponding curve T by n times t, then the pure numbers m and n are commensurable, which tell us how many times the two intrinsically incommensurable units t and e have to be multiplied in order to find two corresponding members in the two series of angles and curves. For the circle geometry tells us that $m=n$. Given therefore the two units, e and t, we only require to know a definite number E

Major premiss : $E : e = T : t$.
Minor premiss : $E = F(e)$.
Conclusion : $T = \frac{F(e) . t}{e}$.

115. I need hardly point out that upon this *inference by proportion*, in the simple scheme of which I include all more complex relations between *m* and *n*, rests ultimately the whole possibility of bringing qualitatively different occurrences into such mutual dependence as allows us to calculate one from another. It is also scarcely necessary to observe that we can only expect this figure to be fully effective, so far as we succeed in reducing the relations of things to terms of pure quantity: we should justify this limitation in the same way as we did the similar limitation of the syllogism by substitution. In a more lax way we are constantly judging of things, even in ordinary life, on the ground of inexact proportions, which mostly pass into mere comparisons: a general likeness is found between the relation of *a* to *b* and that of *α* to *β*, but the equal exponent of both is not precisely specified, and so the inferences drawn generally carry little conviction; e.g. 'If one of these relations under a certain condition *c* has a certain result *γ*, the other will have a generally similar result under the same condition.'

I have only one more remark to add, in repetition of what I have already said, viz. that the form of proportion indicates a limit of knowledge. We find in it the interdependence of two members *F* and *T* merely expressed as a fact, and as such utilised for further purposes; on the other hand, the question remains unasked and unanswered, in what way, by what means, through what mechanism, so to say, the one member *E* sets about bringing the other *T* into any sort of dependence upon itself, especially into this particular sort. Of course there are a great many composite phenomena, in the case of which this question too can be answered: scientific investigation, as we said, has reduced many pairs of apparently disparate properties or occurrences to merely ~~qualitative~~ differences of commensurable terms, Quan and we are then able to see how it comes about that *T* must be connected with *E*, and a particular increase of the one with a particular increase of the other. But there is a limit to this possibility: the ultimate discoverable laws of phenomena will always be found to involve determinate relations between disparate elements, which we can only accept as facts and utilise in the form of proportion, without being able to show the reason why the two elements must be proportionals. We refer many phenomena to the law of gravitation, the intensity of which is reversely as the square of the distance; but hitherto at any rate no attempt has succeeded in showing how the distance contrives to weaken the force. We show how the sensible pitch increases with the increasing number of vibrations, and how our sensations in general, and in fact all our mental activities, change proportionally to physical motions in our organs; and yet after all, tones and vibrations, mental functions and physical motions, remain for ever intrinsically incommensurable, nor do we ever experience how the one contrive to compel the others to corresponding changes. From one disparate thing to another our thought has no means of transition; all our explanation of the connexion of things goes no further back than to laws which admit of being expressed in the form of proportion; and these laws make no attempt to fuse the two elements into an undiscoverable third, but leave them both in their full difference, and merely point out that, in spite of their mutual impenetrability, they come as a fact under a common law by which they mutually determine one another.

116. In the actual application of the inferences from proportions another defect, hitherto only briefly indicated, is tacitly supplemented by attending to an idea which necessarily accompanies them; this supplementary idea we have now explicitly to recognise as having a place of its own in the systematic series of intellectual operations, the last place in the present group. In the above scheme the proportion between the changes of two marks E and T was represented as if it always subsisted between the two marks as such, it being indifferent in what subject they occur. Now there are, it is true, predicates which upon logical grounds, on account of their contrary or contradictory opposition, or because the one in any case includes the other, must be either present together or absent together in every subject: but there are no marks whose quantities and quantitative changes must always stand in the same proportion to one another, whatever be the nature of the subject in which they are united. On the contrary, it is just this nature which determines the exponent of their proportion; and the same universally expressed marks E and T, which in one S can only coexist in the ratio $n:m$, are in another S^1 only possible in another ratio $n^1:m^1$. Heat expands all bodies, but the ratios of the degree of expansion to an equal increase of temperature are different in different bodies. In practice, where we he limitation can we talk of using proportions. ing but the specific character of a given subject, in obedience to which all its marks mutually determine one another, justifies us in concluding from a known value of one of them to the corresponding value of another according to a proportion which holds good for this subject only. This merely brings us back to the idea which lay at the root of ana-ogy; for it was only on the strength of the coherence of all mutually determined marks in a concept, that we felt justified in inferring from a imited group of them to the necessary presence or absence of others, as we might infer from the beginning of a pattern to its continuance. This tacitly assumed condition must therefore be added in order to complete the expression of the proportional syllogism, and its major premiss ought to stand thus, 'If S is an M, for this S it is always rue that $T : t = F : e$.' And the problem which logic presents to us would not be merely to establish this major premiss through experience, in order then to bring a particular case under it in the minor, 'S is M'; it would rather be to show how a concept M can be found at all, such that the proportions required between every two of its marks can be derived from it.

117. The means for the discovery of such an authoritative or constitutive concept have already been indicated; they lie in the fact that every mark is determined throughout by every other, though in very various ways. The effect of this variety will be that, while in certain cases the presence of a single proportion between any two marks is sufficient to determine the rest, in others the knowledge of certain essential marks is necessary in order to deduce the unessential from hem, but knowledge of the unessential is not enough to establish with certainty the whole content of the concept. But I shall be clearer if I preface these reflexions by an instance of the actual measure of alteration in its curvature at every point in it, the direction of its concavity or convexity, the area which it contains between any given limits. It is in view of these developments (the further course of which is too simple to need mentioning here) that we give the name of *inference from constitutive equations* to the method in question. The method itself is not confined to these geometrical problems; but the other and in some ways much more interesting examples supplied by other branches of mathematics, especially the calculation of variations, cannot be so easily represented with the simplicity requisite to symbolise the form of thought which we are considering. Natural science also could furnish approximations at any rate to what we are looking for. Chemistry would possess constitutive equations for analogously compounded bodies, in which the different chemical elements take the place of co-ordinates and constants, if it could succeed in expressing by its formulae not only the quantitative proportions of the elements, but also, more exactly than its symbols at present do, the rule for the grouping of atoms and the general character of their interaction.

118. Admitting the objection to the whole of this method, that like the preceding one it is not fully effective except in mathematics, we rebut in the same way as we did before the censure which it seeks to convey, and only examine it more in detail with a view of finding new ways to supplement what is still defective in the method. It is true that the apparent wealth of development from geometrical equations is, from a logical point of view, more specious than real. In order to determine the form of the curve we give one of the co-ordinates x arbitrary values, calculate the corresponding values of y from the equation, and then connect the extremities of the perpendiculars (y) erected upon the extremities of the abscissae (x) so as to form a continuous line; the curve is therefore only the geometrical locus in which the countless results of a countlessly repeated proportion between different values of the co-ordinates are combined. As for all the new properties which we proceed to deduce, concavity, uniform or varying curvature, closedness or openness, falling or rising of the curve to this or that side, though at first they look like new marks, they also are really nothing but relations of magnitude and position between spatial constructions, relations, it is true, between different elements, but otherwise of the same nature as those assumed between the co-ordinates. Starting with a proportion between two eption has followed in reducing spatial relations to the universal of arithmetical quantities. These inferences therefore are far from ting our requirement. The case is very different when we have to , not with mere spatial magnitudes, but with concrete objects, in :h a number of qualitatively incommensurable marks are united, and hich moreover science is unable to explain these primarily incom- surable elements as merely different combinations of commensur- ones; in the face of these difficulties, thought will still have to look . form which promises, approximately at any rate, the same advan- s as those which mathematics with their easier problem offers in full.

l9. The group of mathematical forms of inference ends naturally , with the emphatic recognition of the fact that the point which ; not admit of being dealt with mathematically, the disparateness ıarks, is precisely the point which we cannot avoid considering. place of the equation will be taken externally by the form of ıition, for this combines a number of heterogeneous marks into a le, but distinguishes in them a group of essential from another of ssential ones; the former are regarded as containing the law for combination of the whole, the latter as dependent on and deter- ed by the former in accordance with that law. Lastly, this ileged group of essential marks can only be found by a com- son of the given concept with those which resemble it, and thus are driven to systematic forms of grouping different things, and, ıarily, to *classification*.

'. *The Systematic Forms. Classification. Explanatory Theory. The Dialectic ideal of Thought.*

20. When we began the account[1] of the formation of our con- s, we were already at the opening of the road which we have now ravel. We already recognised the matter of an idea to be a totality

to account in the later developments of logical activity. Judgments and syllogisms based on subsumption have only required us to consider the one relation which obtains between a concept S and its proximate higher universal M; there was no occasion for following up the relations of M itself to the higher grades of the series of concepts above it. For our only object was to make sure that a predicate P, which, for whatever reason, belonged to an M, must also belong to every S that falls within that M, and for this purpose the logical structure of M itself was to a great extent a matter of indifference. As middle term it bore the name of concept, but the character of a concept was in no respect essential to it; any simple mark, any sum of marks, whether combined under a definite rule, or merely brought together anyhow in thought, was good enough to constitute such a middle concept. It was only our concluding reflexions, which I shall not recapitulate here, that drew our attention to the necessity that the middle term should be a *concept* as we understood it at first, if we are to derive from it the right and obligation of a subject to possess the marks that it displays; for it is only when thus understood that the concept really forms the complete rule under which the whole content presented by the subject coheres and is organised.

121. In saying this we are not simply returning to an earlier standpoint. In considering the most primary and simplest forms of thought, the logician can as a rule only elucidate their results by the use of examples which contain more logical work than he means them to illustrate. For these examples must be drawn from language; and language is not the expression of a thought which has stood still where it began, but of the developed thought which has advanced by a multitude of successive steps beyond the imperfect results of its earliest endeavours, and which now conceals the recollection of them under the more elaborate setting which it has now given to its objects. And it may therefore seem as if our present problem, the formation of an essential concept, had been solved already in the above-mentioned passage; but it needed more than the logical acts which were then under discussion to generate the ideas which were there employed as instances; such ideas could only arise by help of the processes which have now, familiar as they are, to be considered in their place in our system. Thought, in that earlier stage, met the lue of its results on one condition, namely, that the attention which ecuted it should be directed to a number of objects *S*, *R*, *T*, whose mmon element really consisted in the pervading law of their whole ructure, and not to a number of others *U*, *V*, *W*, differing in all reects except the possession in common of a limited group of marks. hen, in the beginnings of thought, there was no logical rule for is selective guidance of the attention; on the other hand, it was en then most effectively secured by the psychical mechanism, which akes those compound ideas reproduce one another predominantly memory which are similar in the whole form of their connexion, d specially commends them to the attention, to the exclusion of ose whose structure is dissimilar and whose agreement is confined isolated groups of marks.

122. In the actual course of its development, therefore, thought first directed to those universal concepts which really contain the w for the complete formation of the individuals for which they are quired; it is not until it has some special motive in investigation that frames universals in which things otherwise unlike are grouped nder a fraction of similar elements. Thus when we were speaking f the first formation of concepts, the current instances of suborination, e.g. of Caius and Titus to the concept of man, or of the oak d beech to that of plant, seemed to us quite natural and intelligible; was as if the mere direction to grasp the common element in the dividuals was enough to put us upon the track of these really uthoritative concepts *M*. And yet the same direction might equally ell have led us to invent for negroes, coal, and black chalk a common ame *N*, expressing the union of blackness, extension, divisibility, eight, and resistance: only the tendencies of the psychical mehanism favoured the first and hindered the second of these appli rise to different results; we saw that the truth of the particular followed from that of the universal, but not that of the universal from that of the particular; and that while we could always infer from a definite reason to a definite consequence, a given consequence need not always lead back to only one reason, but might lead to several equivalent ones. Applying this to the organisation of a concept, we find in it certain marks $a\,b\,c$ the presence of which has a determining influence upon the presence, absence, or modification of others, while the presence of these others, $\alpha\,\beta\,\gamma$, does not necessarily affect the former, but is equally compatible with different ones, $p\,q\,r$. This is the ground for the difference already mentioned between *essential* marks, $a\,b\,c$, and *unessential*, $\alpha\beta\gamma$; it is only in the union of the former that we could expect to find the authoritative concept for the individuals compared, for it is only this union which determines the other marks and therefore includes none but those individuals which are of kindred structure throughout; the latter group of marks, on the contrary, would leave the former undetermined, and would therefore, if conceived as a universal, comprise a number of individuals otherwise entirely different.

124. Our problem accordingly would be, to distinguish the essential marks from the unessential. This is easy so long as we have to do with objects which we can observe in different circumstances; in that case the variable properties, which come and go as the conditions change, contrast of themselves with the permanence of what is essential. It is different when there is no possibility of such observation, and where, in the absence of varying circumstances, our object is to separate the essential from the unessential in permanent and invariable marks of the same concept: we have then to substitute comparison of different instances for observation of changes. Suppose $a\,b\,c\,d$ to be the group of marks in one case of a given concept; then, if in a second case of it d is wanting or is replaced by a quite different δ, it follows, on the assumption that all the parts of the concept cohere, that the remaining marks also experience a change; I denote the second case by $a^1\,b^1\,c^1\,\delta$, to indicate that the alteration of d to δ does not cause the entire disappearance of any one of the marks in their universal sense, but only the transition of each from one of its possible modifications into another, the form of their combination remaining the same. In this case d does not belong to

e very different; it is possible that only $A B$ or $A C$ or $B C$
ı the real law for the formation of the whole, while the third
is merely a necessary sequel or allowable addition to the other
As the mind is not yet in a position to investigate the actual
with all the appliances of science, its only method of deciding
ɔubtful question is to continue the same process. We must
re $A B C$ also with instances of the form $A B T$; if the differ-
f the last mark is here too accompanied by no more than the
us deviation in the others, and the connexion of the whole
ıs the same, the coexistence and relation of A and B will be
minant rule for the original $a b c d$, or will represent that union of
al marks which makes the presence of the rest possible or
ary, or at any rate determines their amount, connexion, and
n to the whole. If we conceive this process continued, we
urselves on the way to *classification.* We can now no longer
ɜ our consideration to the individual if we would determine its
ɔt; that can only be done in this first of the *systematic forms,*
, by investigating its nature in its relation to others, and judging
ts position in an ordered series what degree of formative in-
ɜ its several marks exercise upon its whole nature and be-
r. The authoritative principle of its formation will appear to
ie in that inner circle of marks which, when we ascend through
xt universal to higher and higher degrees of universality, re-
together the longest and unchanged in its general form; and
ıly way to conceive completely the nature of the particular is
nk of this supreme formative principle as being specialised
.lly, in the reverse order to the grades of universality, by new
ons which come within the influence of its reaction.

The desire to get an explanation of the inner structure of
mposite object by this systematic arrangement, lies at the root
scientific classification, but is not equally satisfied by every
ɔf it : before going on to consider the only form which will

Then, on the principle of the disjunctive judgment, every species of M must possess one modification of each of the universal marks of M to the exclusion of the rest. If for the sake of simplicity we confine ourselves to two marks, of which the one, A, falls by disjunction into only two members, a and b, the other, B, into three, α, β and γ, the binary combinations arrived at in the ordinary way, $a\,\alpha$, $a\,\beta$, $a\,\gamma$, $b\,\alpha$, $b\,\beta$, $b\,\gamma$, will comprise all conceivable species of M. Lastly, it makes the collective survey of them more easy if we place the modifications of the particular mark which forms the basis of classification before the other marks, as was done above, or in the form $M = a(\alpha+\beta+\gamma) + b\,(\alpha+\beta+\gamma)$. The simplest instance of this classification is the arrangement of dictionaries; the fixed order of the letters in the alphabet here gives the basis of division, not only in the first instance, but also for the numerous subordinate combinations contained under the head of each letter. The obvious advantage of this lexicographical classification is, that it gives a survey of the material, not only embracing all the words of the language, that is, all members of the object to be divided, but also making them easy to find, and this first advantage it shares with all successful attempts at artificial classification; but when we go beyond this we find that the degrees in which they contribute to the real knowledge of their objects are very various.

126. We observe firstly that this method of combination only takes account of the marks of the given concept in their isolation, not in that mutual interdependence in which alone they really constitute the concept. Thus it is true that the sum of the combinations discovered includes all species of M, but it may also include others besides them, which would be true species if the concept were merely the sum of its marks, but are not true because it implies their union in a certain definite form which these other species contradict. The concept of a triangle does not consist in the fact that we think three angles *and* three sides, but in the fact that three sides intersect one another so as completely to bound a plane space and by this very fact produce the angles. It is this connexion of the sides and angles which makes equiangular unequilateral and rectangular equilateral triangles impossible: in a classification by mere combination these the reasons which make the valid kinds possible and the invalid impossible. If on the other hand M is a generic concept derived from experience, the inner organisation of which can only be represented imperfectly by description, not exactly by construction, the species which we have not actually observed but should have been led to infer by the method of combination, remain doubtful; further observation may discover them, further knowledge of facts may show them to be impossible; the use of assuming them provisionally may here also be to stimulate advance in one of these two directions.

127. If the method of combination, when applied to objects of experience, is liable to the uncertainty whether its results do not include more than the facts, it is true on the other side that, as ordinarily practised, it gives no guarantee that they exhaust the facts. It is beyond the power of human imagination to anticipate completely all the modifications to which a mark may be subject; our attention will always be confined to those, $p^1 p^2 p^3$, which we happen to have observed; another modification, p^m, which does not come within the circle of our experience, will be missing in our classification along with all the species in which it may possibly occur, and this gap will not be filled up until our experience has grown. This is the ground for a logical rule, which is valuable when the decision of a question involves exhaustive knowledge of all the possible cases of some object Z; the rule is to go on dividing and classifying them by simple contradictory opposition. The sum of all possible cases of Z is always of the nature Q or of the opposite non-Q; the cases of the form Q are always either R or non-R, those of non-Q always either S or non-S; so that at whatever point the division is broken off, all possible cases are included by it. Such form the principal division by its modifications, and any other may be subordinated to it as basis of a subdivision. The obvious disadvantages of this uncertainty are avoided in practice by concomitant reflexion and an estimate of the different values of the marks, based upon a knowledge of the facts or a right feeling, often merely upon an instinctive taste: all that logic can contribute to these precautions is the general direction not to choose as bases of division *notiones communes*, i.e. marks which are known to occur in the most different objects without exercising any recognisable influence upon the rest of their nature. The positive direction answering to this prohibition, viz. how to find the decisive bases of division, logic leaves entirely to be given by special knowledge of the matter in question. And as regards complex concrete objects at any rate, so long as fundamental divisions were based upon single marks, the specialist has always been open to the criticism that he sometimes removes closely related species to different and often very distant parts of the system, while he brings others which are totally and strikingly unlike into surprising proximity. This is quite intelligible when we consider the different influence which the marks have on the structure of the whole concept. There is no reason, for instance, why the mark B, so long as it occurs in the modified form b, should not conspicuously affect the formation of the whole, and in that case all the species under the head of b will remain connected in form; but the same mark may entirely lose this influence as soon as it enters into the group of marks in the modified form β; then the species under the head of β follow all the variations due to the now influential difference of the other elements $A\ C\ D$, and examples of M, otherwise most unlike, now find themselves in the closest proximity. This is what happened to the Linnæan system, which selected the number of stamens as the basis of division; the result of this view was, that in the cases where the whole organisation of the plant made the stamens of importance, the related species were brought together; where this was not the case, they were separated, and different species were united. An instructed taste will partially obviate this evil also, by selecting different bases of division for different sections of the whole system. Nothing but an unseasonable logical pedantry could require that a system which had begun by dividing its whole object-matter according to the modifications $a\ b\ c$

view approaches by that means, and by that means only, to the essence of the thing. The risk which such a method runs of discovering all the species completely, must be avoided in s other way; classification does not create the complete material, assumes its completeness to be guaranteed elsewhere.

129. Classifications would belong entirely to applied logic if aimed at nothing more than complete summarisation, such a required either when we wish to deal with a subject practicall when we are just beginning to consider it logically. But they do than thus merely prepare the ground; they themselves represe logical ideal, which has its necessary place in the systematic seri the forms of thought; the very fact that a manifold material has brought into the connexion of a classified system, is of itself supp to tell us something as to the nature of each and all of its mem and not to be a mere preliminary to future enquiry. This ap in the objections which we make to forced classifications; we not require the lines along which we must look, in order to fi particular species, to be precisely laid down beforehand in a seri concepts, but we expect the actual places in which the several sp are found to correspond in position to the affinities of the sp themselves. For practical purposes any order will serve that is h for the person who is going to use it, but the order which demands must be true to the facts. Now if we wish to form a plete idea of any composite object, it does not matter with of its parts we begin, provided only that the order in which we each new part is adapted to the particular point with which we chosen to start: any idea of a given content so arranged for *concept* of it, sufficient to distinguish it from others and to show it is itself. Amongst these various concepts of the same M th one distinguished from the rest by having for its starting-point th which determines the order of all the other marks, and this is th

130. It will help us to realise what has just been said if we mention here two incidental notions which always attach themselves readily to this search for the idea of an object, conspicuously in the attempt of naturalists to improve the artificial classifications of plants and animals by reference to their natural affinities. In these cases we are prone to regard the universal idea of animal or plant as a living and operative force, whose unvarying and consistent activity gives rise to a series of different forms, accordingly as external conditions determine one or more of its points of incidence and oblige it to change correspondingly the whole course of its action. Another way in which we are equally prone to regard it is as an unvarying end, which regulates its modes of operation according to the relations in which it finds itself placed, and in the different forms which it is thereby compelled to assume realises one and the same purpose in various ways or with various degrees of completeness. From this point of view the different species classified together express the result of the interaction between the universal idea and the particular relations, with which as universal it has nothing to do. It will be admitted that these ways of looking at the matter place it before us in a clear and vivid light, but it will also be objected that they are both quite foreign to logic. The objection is unanswerable; our intention however is not to turn the ideas of active tendency and purpose to account for the benefit of logic, but to show that even in their proper place they only have meaning on the assumption of a purely logical notion, which we will now explain. If it is to be possible for the same end to be fulfilled under changing circumstances, it must also be possible to express its content by a group of ideas, Z, in which these different forms of fulfilment cohere as possible species, and from which they necessarily result if each one of the marks of Z and each of their mutual relations is successively subjected to all the changes of which, as parts of Z, they are respectively capable. If, again, an active tendency is to change its activity under varying conditions and to manifest itself in new results, the combination of forces in which it consists must be expressible by equations, from which all these new formations necessarily follow as soon as we give the quantities entering into the equations all the values successively which their natures allow. Activity, then, whether intentional or unintentional, never produces anything but what is abstractedly possible to thought, and this fulfilment, nor as an active force which causes its results, but merely as the conceived or conceivable reason, the consequences of which under certain conditions are the same in thought as those which must follow in reality, under the like conditions, from an intelligent purpose or a causative force. Keeping this in mind, we may tolerate a phraseology which imports into logic the idea of an end or of a tendency to development: it will nevertheless be better to avoid these expressions, and not to use what is found only in the real world as a name for the mere reason upon which in thought the reality rests.

131. Another point which logic cannot neglect may be introduced here as a sequel to these accessory notions. We are not surprised in a self-realising tendency if, under certain conditions, it fails in its endeavour; and we find it intelligible that an end should be attained under different circumstances with different degrees of completeness. Thus both these notions very naturally give rise to the assumption that different realisations or examples of the formative idea are of different values, and that they are not merely co-ordinated in a general way as species under the universal concept of their idea, but form within this co-ordination an ascending or descending scale in which each one has its uninterchangeable place between certain others. The attempts at natural classification, which endeavour to satisfy our modern requirements, are dominated throughout by this thought; and it remains to show that this familiar tendency to pass from classification by mere combination to classification in the form of a developing series, is justified on general logical grounds, and that this is the place to justify it.

If, as is too often the case at the beginning of logic, we regard a concept M merely as a sum of marks universally expressed, there is no sense in rating one of its species higher than another. Every S either contains all the marks of its universal M, and in that case it is a species of it, or it does not contain one or other of them, and then it is, not an imperfect species, but no species at all of M. But living thought in actual practice is far from acquiescing in this hard antithesis; it distinguishes species which correspond or are adequate to their generic concept in various degrees. The possibility of making and standing to the others in all sorts of relations. By 'simple marks' here I mean, not only sensible properties such as red, sweet, hot, but others also like heavy, extended, irritable, which, though no doubt they contain the result of previous observations of complex modes of behaviour, contain it in so simple a shape that our logical imagination has long accustomed itself to attach them to their subjects as stable and simple predicates. To all these elements of the concept quantitative differences extend. No mark of any one of its parts is conceivable without a definite degree of its specific kind of intensity, and the degrees may vary infinitely; the number of the parts themselves can, like every number, be increased or diminished, and every part moreover can alter its logical value by expanding the simplicity which belongs to it as a member of the genus into a complex organisation of its own inner nature; and lastly, every relation between the various constituents of the concept varies in value according to the value of those constituents, or admits of greater or less closeness according to some standard of its own. The joint effect of all these possibilities of variation is to produce a number of species noticeably different. If we suppose that when a mark P of the generic concept M assumes the value p, the influence which it always exercises upon the other marks is so intensified as entirely to change the form of the whole content of M, the resulting species will no longer be a species of M, but of some other genus N. And those values of P which approach this decisive limit but do not reach it, will produce forms which still fall under the genus M, but approximate gradually to the structure which is characteristic of N. It is upon this that the difference is based between species which are more and less appropriate or adequate to their common generic concept; each species is in a certain respect more perfect the farther it is from passing over into another genus, and that is the logically most perfect whose divergences from all proximate genera make up the greatest total amount.

132. I believe I am justified in saying that this point of view belongs entirely to logic, and is independent of the views which we may form on other and material grounds as to the value, meaning, and function of anything which has the law of its existence in a generic concept. I will therefore illustrate it by examples which are not affected by these incidental considerations. The equation of the ellipse, $a^2y^2 + b^2x^2 = a^2b^2$, leaves the two axes a and b to be chosen at pleasure, and the formula claims that it will always produce an from that of the ellipse. But this example shows at the same time, what we did not choose to assert universally above, that the extreme species of a genus M, when produced in this way, not only must belong to a new genus N, but may also continue to come under the former genus M. It is true that the central equation of the ellipse can tell us nothing about this case when $b = 0$, because it then ceases to indicate a curve. But there is another expression of the essential formation of an ellipse which is still valid; namely the rule that the sum of the *radii vectores*, drawn from two fixed points on the major axis to one and the same point on the periphery, is constant and equal to the major axis. In the present case where the ellipse has shrunk into a straight line the two extremities of the line are identified with those two fixed points, the *foci* of the ellipse, and for every intermediate point c we have the sum of the distances $ac + cb$, that is, the sum of the two *radii vectores*, equal to the length ab of the straight line.

If a heavy rod of the fixed length ab stands with one end a on a perfectly smooth horizontal surface, and with the other b leans against a perfectly smooth vertical wall, the pressure of its weight makes equilibrium impossible and it falls. An easy calculation shows that the path described during its fall by any point C in its length is an ellipse. At the same time it is clear that the end b must slide down the wall in a straight line perpendicularly, while the point a must move away upon the smooth surface in a straight line horizontally. As then every point in the line is affected by the same group of conditions, these rectilinear motions also must be regarded as specific forms of the elliptical path required generically by those conditions. They are in fact the two extreme cases which we get if we make first one and then the other axis $= 0$; the end of the rod then moves in a straight line in the other axis. The middle point of the rod supplies another singular case; the axes of its elliptical path are equal, and thus it describes the arc of a circle. The nature of the problem before us compels us therefore to conceive the circle as a species of ellipse, is, strictly speaking, determined; in the present state of our knowledge this question is decided upon incidental grounds of various kinds.

133. On the other hand, these examples leave an ambiguity which must be removed in regard to the standard by which we measure the degree of perfection, or, to put it shortly, the height of each species. Mathematical figures have no history telling of their life and growth; being merely legitimate possibilities of thought without real existence, they can be produced for our imagination in the most various ways, and it is in the abstract indifferent, and in any particular case depends on the nature of the problem in question, from what point we begin their construction, or under what generic concept, what universal rule of construction, we bring them. If we look at them, not geometrically, but aesthetically, I mean if we attend to the total impression of the figure as it is, not to the way in which it came into being, circles and straight lines contrast decidedly with ellipses. In the *impression* of the ellipse as we perceive it the inequality of axes is a necessary element; on the other hand it is true that the greater this inequality is, the more does the curve approach the extreme forms which we wish to exclude, that of the two straight lines which coincide with one or the other axis. The characteristic impression of the genus would be best produced by an ellipse equally removed from the equation $a - b = 0$, that of the circle, and from the equation $a - b = a$, that of the straight line. By combining both equations we might define the condition of this impression by saying that one axis must be double the other, and this would be tolerably correct; only that a thing cannot be mathematically determined which does not depend simply on mathematical laws. Our logical imagination is dominated in every direction by similar tendencies. Nothing is commoner than for a person who speaks of a quadrangle to mean really a parallelogram, or often even a square; and this inexactness in expression is very natural; the imagination wants to realise the concept in perception, but can only hold one image at a time, and it therefore chooses the image which is logically most perfect; and it is the fact that the parallelogram, by increasing inequality either of the sides or of the angles, continually approximates to the ultimate form of the straight line, in which all the four sides coalesce. The observation of natural objects evinces the same tendency; we always regard as

l here repeat an observation which I made before. I
d that anyone will criticise this mode of estimating the
it of species on the ground that it has nothing to do
s defect is rather that it starts from inadequate logical
. does not adapt itself sufficiently to the nature of its
put it shortly; that the highest perfection of a species
ı the equilibrium of its marks as described above, is the
hich we must come on purely logical grounds, so long
no positive knowledge to supply us with some *other*
neasurement based upon the essential characteristics of
question. It may lie in the nature of things that a genus
naintain this equilibrium of marks, but is destined by
one and intensifying another to pass over into another
that case its species will be more perfect in proportion
ach more nearly to this point of transition at which they
ong to their own genus. We find that the most im-
ıpts at natural classification are deeply imbued with this
tination to be attained, which is constantly impelling the
ra to advance beyond themselves; I therefore introduce
tionally, in order to notice its significance for logic, with
elf it has nothing to do. We have already[1] separated
productive activity from the concept of tendency, and the
ose from the concept of end; we must in the same way
e the idea of *obligation* from the concept of destination.
l see that the effect of this separation is to do away with
ıaracteristic in the meaning of these three concepts; but
vhat we are aiming at. It is not the concept of des-
f which we are importing into logic, but merely that of
elation upon which it is essentially based, and of which

speak of a 'destination' to which the several genera have to approach. Hitherto we have looked upon the generic concept *M* as the ultimate authoritative principle which regulates the series of its species, and that species therefore as the highest which exhibits this concept in the most perfect equilibrium of its marks; now we are reminded by a consideration originally foreign to logic, that the case may be different, and that the formation of the series of species in *M* need not really depend on anything in the generic type of *M* itself, such as could be discovered by merely examining its own constituent marks; that, on the contrary, the formation of this genus is not rightly explained until we compare it with another genus *N* into which it passes, and with a third *L* from which it came by a similar transition, and these again with those which went before and came after them; not till this comparison has been made do we get the direction in which the progress towards perfection takes place within a higher genus *Z*, of which *L M N* are species; then, in the series of species in any particular genus *M*, those species will be the highest which have advanced the farthest in the direction in which *M* as a whole is developing towards the most perfect expression of the higher *Z* which includes it. It remains to show that this line of thought, to which we were originally led by an extraneous suggestion, has its necessary place here in the internal economy of logic.

135. It is scarcely needful, however, to show this. We have seen that we could only produce the universal concept, which includes a number of individuals under it, by uniting their permanent and common marks; then we saw that this constant group of marks might contain elements of very different values, and in order to separate those which are not only constant but contain the rule to which the rest must conform on joining them, we had to compare the universal already found with other universals, and species with species; that which still cohered in this wider field of change we regarded as the true essence of a genus *M*, the species of which were to be ranked higher or lower in proportion as they realised it more or less perfectly. But this process has no natural ending; the same questions continually recur; the marks which constitute *M* will themselves differ in value, and the only way to distinguish the essen-

M L N, as well as that of their several species, by the degree in which they realise this law *Z*, instead of by the degree in which each species expresses the more special law of its own proximate genus. This progress might go on to infinity, or to the point at which we succeeded in finding a highest ideal *A*, exhibiting the mode of connexion to which all kinds of existence, real and thinkable, must conform: from this *A* a classification might be derived in the form of a development which evolved from itself the whole content of the universe, and this development, if it were possible, would give the only logical security that every species had a place in the series of cognate species answering to the degree of essence which it expressed. Thus the problem of natural classification leads of itself beyond the isolated treatment of a particular problem to the systematic organisation of the whole world of thought. And this tendency has in fact guided the most important attempts at such a classification. Those who have wished to exhibit the development of plants or animals in an ascending scale, or the events of history (for this form of thought claims to apply to processes also), have always been obliged to justify their selection of a particular standard for measuring the increase in value of the several members of the series; this justification they have always had ultimately to find in certain general views as to the meaning of all being and process, views which are either formally expressed at the very beginning of the enquiry, or make themselves tacitly felt throughout it as a guiding principle.

136. Natural classification, then (to sum up under the traditional name the procedure just described), differs from combinatory or artificial classification in taking account of the mutual determination of marks which in the latter received only subordinate attention, while in its result it is distinguished by its *serial* form, in which the members are not merely placed side by side, but follow each other in a definite order leading from the province comprehended or dominated by one species into that of another: this order begins with those members which answer least to the logical destination of the whole system, and ends with those which express in the most complete and pregnant way the fulfilment of that destination. But the simplest case here supposed, that in which the series has only one direction, is not necessarily the only one. In the first place it is conceivable that single marks in each species may vary without altering the characteristic structure of the species at all, so far at least as we can see: in that case the different instances of this species are equal in value, and the series may thus be increased in breadth by co-ordinated members without growing in length. It is also possible that, owing to different or opposite variations in several marks, a species *M* may not only pass over into one proximate species *N*, but branch out into several, *N*, *O*, *Q*, with which it has equal affinity and which contribute equally to carry out the general development; these will then become starting-points for new series, which either continue side by side or subsequently coalesce again somehow with the central series. Thus the form of natural classification in general is that of a web or system of series; even the culminating point of the system need not be a strict unity, for the most perfect attainment of the logical destination is compatible with a variety of precisely equivalent forms.

137. As the occasion suggests it, I will mention two more concepts in frequent use, which may find a logical explanation here. The new kind of value which each species acquires in proportion as it approaches the end to which they are all developing, does not exclude the other kind which we mentioned earlier, depending on the equilibrium which it exhibits in the marks of its proximate genus. The two values subsist side by side, though the one impairs the other. We feel the conflict between them in our aesthetic judgment of phenomena. Every species which expresses its genus in the stable equilibrium of its marks, impresses us as perfect, relatively or absolutely: such a species forms the *type* of the genus, that type which is the indispensable though not the sole condition of beauty in the beautiful, and which gives even to what is abstractedly ugly the formal right to a subsidiary place of its own in artistic representation. On the other hand, species in which this equilibrium is disturbed by approximation to an end higher than can be attained within the limits of the genus, give us the ambiguous impression which we call 'interesting,' like dissonances in music, which do not satisfy us but prepare us for a higher satisfaction. *Ideal* as opposed to *type* would mean a phenomenon in which the equilibrium of marks required to make it typical coincides happily with the highest development in regard to its logical destination; logic does not exclude the possibility of such a coincidence, and art may perhaps find it realised or be able to realise it in a phenomenon in repose, though more probably only in some situation of the phenomenon.

the particular object or the universe at large. To this we can only answer, that by way of mere logic it is quite impossible to arrive at such a certainty. The form of classification by development, like all logical forms, is itself an ideal, an ideal which is demanded by thought, but which can only be realised, so far as it can be realised at all, by the growth of knowledge. Nor indeed is this an exceptional condition, such as would lay this first of our systematic forms under a disadvantage. The judgment also enjoins a connexion of subject and predicate which thought has to make if it wishes to come into contact with its object in its own way; the hypothetical judgment, for instance, tells us, that only by annexing a condition to the subject S is it possible to ascribe to it a predicate P which is not already contained in the concept of S; but logic does not tell us what condition x is necessary in order to secure this particular P for this particular S; it waits for special knowledge to put its injunctions into practice. The theory of the syllogism also teaches us how to draw conclusions when the premisses are given, but it does not give us the premisses, nor does it guarantee their truth, except so far as they may themselves be conclusions deducible from other premisses; these latter then serve as the material given to thought, and lead back finally to some truth which is no longer logically deducible. Similarly all that the theory of natural classification asserts is, that every group of complex and coherent objects, and therefore (since everything coheres) the whole realm of the real and the thinkable, must be regarded as a system of series in which concept follows concept in a determinate direction; but the discovery of the direction itself, and of the supreme directing principle, it leaves to positive knowledge to make as best it can.

139. It is not this objection, but a difficulty of another kind, which obliges us to continue our enquiry. The difficulty will be most easily understood by reflecting on the place which classification occupies in our system. As a certain arrangement of concepts, it answers primarily to our first main section, the theory of the concept itself; but we were obliged to pass on from the concept to the judgment, for we found changes in the content of thought which could not be apprehended by conception alone; on the contrary, the concept presupposed relations between its marks which it needed the judgment to interpret clearly. Classification answers moreover to the first form of judgments, the categorical; as in these the subject simply had, assumed, or lost its predicates, so here the supreme authoritative concept appears by itself as the sole producer of all its judgment met the categorical with the objection that a single subject *S* cannot by itself give rise to any multiplicity; and, similarly, all theories of emanation will have to ask themselves the question, what second condition it is which makes their first principle develope at all, and whence come the data in reaction against which it is obliged to expand into these particular forms and no others. A corresponding advance is called for here; and it will prepare the way if we consider it in still closer connexion with the characteristics of classification described above. We made it an objection to artificial classification that it may lead to impossible instances, while in classification by development we gave proportionately more attention to the mutual determination of marks; we assumed that a change in one mark reacts upon the rest, that through this change one concept passes into another, and that one species answers better than another to its concept. This clearly implies that in the formation of its species the concept depends, not only on itself, or, in figurative language, on its own purpose, but also on another power which determines what kinds of realisation of that purpose are possible or impossible, adequate or inadequate. This power we have to investigate.

140. The problems of thought are not completely solved until it has developed forms for the apprehension of everything which perception offers to it as an object and stimulus of its activity. This requirement, that *all* thinkable matter should be included, is not satisfied by classifications. Their natural objects are always those stationary generic forms with stereotyped marks, which we believe ourselves to have before us in perception as fixed points for manifold relations, but which are far from constituting the whole of what we really perceive. The several genera are not found in reality arranged in the system in which classification exhibits them; as they actually appear they are always realised in numberless individual instances, separated in time and space, and subject to continual change both in their own conditions and in their relations to one another. Even if we admit that the nature of each generic concept contains the law which every instance of it will obey *if* it occurs under certain circum-

that species of itself, must also be taken account of as an
l part in the organisation of the thinkable world as a whole.

These considerations are not disproved by the fact that, as we
d before, classification by development may extend, not only
ric forms of the real and the thinkable at rest, but also to
sive processes. For when it is attempted to represent history
evelopment, the question what it is which makes process
, the coming of one state into being out of another, equally
the grasp of logic. When they are reflecting on the past or
ing the future, these speculators may picture to themselves
situations as temporary states of equilibrium, which they
to follow one another on the stream of events in a fixed and
ry order; but how the transition from one to another actually
ıbout, they cannot tell us. Nor could they do so even if they
ok the endless task of dividing the interval between two such
f equilibrium into an infinite number of stages; they would be
show that the concept of each stage, when it is reached, is pre-
y to the concept of the next, but they could not show how the
which this concept expresses brings the reality expressed by
ier in its train. We must reflect moreover that in the real
ure concepts do not occur or develop themselves, but only par-
examples of them, each with all its marks specifically modified
y which its concept allows but does not necessitate. Not only
re does the process of becoming remain a mystery which
ation cannot explain, but the result of the process results,
m the concept of the stage preceding it, but from that par-
realisation of the concept of which also classification takes no
t. All the attempts both of ancient and modern times to
the world by way of emanation from an original concept,
ject to the same defect. If their original concept is really
but the pure thought of a relation which certain elements
named necessarily imply, all that they can derive from it
—certain forms likewise universal in the shape of possi-

itself with the impulse to development which those elements do not supply, in the shape of an inherent restlessness which drives it to evolve its consequences, this is only to admit that the complete form of each new stage of development does not depend only on the concept of the preceding stage, but on the special form in which, as a fact, but without any reason, that concept had already realised itself. It is to admit, in other words, that alongside of their categorical development by emanation of the concept out of itself, another power is also at work; this power, which their theory entirely disregards, consists of a sum of authoritative hypothetical relations, which ordain that if the marks in a given concept have as a fact a certain value, and if certain conditions act upon these marks, the form of the new resulting concept, the new stage of emanation, is then, but also not till then, completely determined. Lastly, if we compare the theory of emanation with the method of the inferences by subsumption, we may say shortly that what it lacks is the *second premiss*, by which alone they produce from the universal major the comparatively more special conclusion. These subsidiary ideas, which are here only tacitly presupposed, logic has to supply explicitly: it cannot stop at a classification based upon *concepts*, but must point out also the legitimate connexion of the *judgments* which express the power of a mark already in existence to determine another which is to come into existence out of it. //

142. But it is not necessary to confine ourselves to that side of classification where it fails to give a complete solution of the problem of thought; the attainment of its own more limited end implies the same tacit assumptions. Each of the generic concepts classified is necessarily composed of marks which occur in other concepts as well. It would be lost labour to construct a scale of genera L M N, if L had marks which were heard of nowhere else in the world, and M and N were distinguished by similar uniqueness. The marks must rather be looked upon as building-stones lying about ready for use; they have to be cut differently according to their different positions, but they are all of commensurable material, and it is only the different ways of using it which give rise to concepts of different structure. Now in classification by development the marks united in the same generic concept M are spoken of as mutually determining each other; a change in one is followed by changes in another; and

the marks themselves hold good beyond the limits
concept *M*, this relation also must be independent
mation, therefore, of the several species of *M*, their
ossibility, and ultimately the possibility or impossi-
all entirely depend on what is allowed or not allowed
l laws of connexion between the marks. Accordingly,
of concepts cannot fulfil even its own proper function
sing a system of judgments or universal laws regu-
ibility, mode of connexion, and mutual determination
h are to be united in this or that generic concept.

ention here an apparent contradiction, the removal
nclude these preliminary considerations. We have
ıg of the form of proportion, spoken of the necessity
terdependence of marks; we there corrected ourselves
vhen a constant relation exists between two marks, the
interaction is not found in the marks as such, but in
ıe whole in which they occur or in the concept of
seem here to be retracting this statement, but we are
ng it. For the very point which we have now made
content of the concept, to which we there transferred
ıence, is nothing but a number of marks, each extend-
concept itself, and all connected in it in a definite
hese marks, as we saw, different relations are possible;
that the idea of one involves that of another; in that
ect which has the first will have the second also;
hat two marks exclude each other as contrary and
mbers of a common element, and in that case there is
ubject in which they can exist together: between these

different species. Thus it is true that the concept determines for its subordinate species the proportion in which each pair of marks condition one another; but it only does this in virtue of the *ordered sum* of its other marks, and so far as these are known to have definite values. Our method, in fact, has always been based upon this supposition. In proposing to classify a generic concept by developing its species out of it, we have always had to assume that certain of its universal marks are already defined by their places in the series; not till then could the rest acquire that definite character which was necessary to complete the distinction of one species from another. In the concept itself the existence of this primary definiteness, of which the rest was a consequence, was only a possibility; its realisation was assumed in thought independently of the concept.

144. If we sum up these considerations, we may say that every individual and every species of a genus is what it is through the cooperation of the complete sum of its conditions; these conditions consist in the fact that a number of elements or marks, which might also exist in separation, are as a fact given in a certain combination, which might conceivably be different, and each with a certain quantitative value, which is one amongst other possible values. From this given union of conditions, according to universal laws which hold good beyond the limits of these elements, this perfectly definite result follows. Every such result, when it is once there, can be compared with others, and co-ordinated with them as species with species or subordinated to them as species to genus; but these concepts, which hitherto we are considering as the key to the understanding of the structure of their subordinates, must not be credited with any mysterious and authoritative power, beyond the fact that they are condensed expressions for a definite union of separable elements, which act and react upon each other according to constant and universal laws, and give rise in one combination to one set of results, in another to another.

145. It is evident what a revolution these considerations cause in the whole view of logic: we see it in the logical form of *explanatory theory* which modern science opposes to that of classification, by inkable matter, proceeding by the mere impulse of a plan of evelopment contained in the point from which it starts, without the id of any other conditions; the form of science becomes essentially *ypothetical.* It does not describe what is and what comes to be; it efines what must be and come to be *if* certain conditions are given; e question whether, and in what order and connexion, these conitions occur, is excluded from the province of logic and left to e answered by experience, which will bring the facts to illustrate the pplication of the theory. Nor will I here raise the question, how his theory gets at those universal laws by which it decides, that herever a particular group of conditions is given, one particular esult and no other must occur; it is sufficient at present to observe hat it does start with this conception of a *law* which fixes the articular result of a particular condition *universally.* This means, hat wherever the condition $a + b$ is found, only c follows from it, nd the nature of the object in which $a + b$ is found has no power to ive this condition directly any other result than c; it can only do so hen other conditions, $a + d$, are present in it as well as $a + b$, and he former co-operating with the latter oblige c to change into γ; and his co-operation also takes place by a universal necessity quite independent of the nature of the particular object and equally binding pon all others. And in the new result γ the law which connected c ith $a + b$ is not eliminated, but continues to operate concomitantly; or $a + d$ alone would not have produced γ, but δ.

From these universal laws arises that mechanical character, of hich the adherents of these theories make a boast, and their logical ntagonists a reproach. The tendency to derive a series of phenomena 'organically,' as the phrase is, from the meaning of a conception which develops itself in them, is met by the assertion that a mere meaning which wants to develop itself does not produce anything, but that everything exists, and exists only, when the complete sum of conditions is given from which it follows necessarily by universal laws; it must be regarded as the result of these conditions alone to the special characteristics which distinguish one concept from others, and not to the prescription of a universal authority which is equally binding upon everything thinkable and everything real. It is their pride not to need generic concepts and their arrangement in a system of classes, but to show that, whatever the context from which a phenomenon gets its meaning, we know all about it as soon as we know the sum of related points combined in it; for whatever is, is merely an example of what must come to be when the universal laws are applied to this or that particular group of given elements. Even the position which is sometimes taken up as the utmost that can be conceded on the other side, does not satisfy the demands of these theories, the position that everything obeys universal laws, but each domain of reality its own, and that the laws of living and spiritual existences are different from those of lifeless and material ones. It is indeed obvious that those special laws to which any given phenomena are immediately subordinate, and with which therefore they are most closely connected in matter and form, vary with the varieties of the subjects which they express; but there could not be two worlds depending on two supreme and independent laws, unless they had nothing to do with each other and no effects from the one were ever felt within the limits of the other: anyone who speaks of one world, embracing those different groups of self-developing things and events, must start with a single law valid for all reality, or a single unbroken circle of law, of which all the special laws of different domains are particular cases, and from which they arise as soon as it is supplied, in a succession of minor premisses, with the different conditions which differentiate the several domains of active existence.

146. In accordance with my plan of dividing the problems of logic, I have omitted from the preceding account of explanation all mention of the means which the theory employs, partly for discovering the universal laws which it assumes each coherent group of existence to obey, partly for detecting in the manifold variety of experience those inner coherences themselves which the subordination of different elements to the same common principles admits or requires. I have reserved to applied logic the utmost freedom to follow evailing conviction of the present day. Explanatory theory is t the only form in which the scientific activity of our time ts itself; the consciousness (so late in making itself felt) of inciple which that theory has to follow, strongly separates all n science from that of antiquity and the middle ages, and the ds of investigation developed in consequence of it form the us treasure which places the modern art of discovery far above f ancient philosophy. Yet the opposition so unremittingly to this form of thought, when it claims exclusive dominion over inkable world, shows that the belief that it leaves nothing more h for is not universal. If we consider first the familiar forms that opposition assumes in our collective view of the world, we e able to disengage from it the purely logical residuum of feeling the explanatory theories fail to satisfy.

. The assertion that all existence is subject only to universal and that every individual is nothing more than it must become ding to those laws, if conditions, which might have been com- differently, have as a fact combined in a certain form, is most usly distasteful on aesthetic grounds and to artistic natures. y, it is felt, cannot be understood upon such a view; it only of value, and to be really itself, if the ultimate form which s our admiration is the result of a single power, a result which eed inevitable, but which, besides being inevitable, is also the ent and manifestation of a living impulse: it would appear lligible, if it were merely a lucky case of harmony between lly coincident elements. I have tried elsewhere to show that esthetic objection is wrong, if it goes on to deny the universal ty of the explanatory or mechanical theory. As understood by heory, the meeting of the various conditions is never a matter of e, but always the necessary consequence of the past states of orld. If we follow out this thought, it leads us back to some state, there is no reason why we should not take a section, as it were, of the world's course at any point of time that we choose, and suppose the combination of all the forces then acting simultaneously, just because it is that combination and not any other equally conceivable, to be the one and sufficient reason of all individual beauties. Such a supposition would give room for everything which our aesthetic feeling considers necessary to maintain the dignity of beauty; it would merely have somewhat changed the place of the single impelling power; this power would no longer lie self-centred in the individual beautiful thing; it would continue to be active in the individual, but only as the after-effect of a universal which permeates all individualities. By thus putting back the origin of beauty we do not run counter to aesthetic requirements; on the other hand, the mechanical theory, obliged as it is to assume some existing state of things in which the continuity of development according to universal laws is exhibited, has no motive for conceiving that state as meaningless rather than full of meaning, as irrational rather than rational, as the source of caprice in the world's course rather than of consistent purpose. There is however one point which the requirements of aesthetic feeling and the admissions of scientific explanation equally imply, namely, that the secondary premisses, which we bring under the universal laws and by which we denote the facts to which the laws apply, cannot have the casual origin which they doubtless seem to us to have when we are absorbed in some particular field of enquiry and have taken them out of their mutual connexion. They must themselves be systematised and form parts of a whole, that whole which comprehends all real objects to which the universal laws apply. The minor premisses to our general view of the world must not be conceptions of a number of disconnected possibilities in hypothetical form, each of which, *if* it occurred, would lead by universal laws to a definite result; they ought to distinguish categorically each possibility which occurs from those which do not occur, and exhibit it as a legitimate member with a place of its own in the universal order of reality.

148. This requirement is partly supported, partly modified, by metaphysical considerations. For what would be the meaning of assuming on the one side a realm of universal laws, and on the other a sum of reality which conforms to them, if no further relation existed between the two and made this subjection intelligible? And in what could the subjection consist if not in the fact that the behaviour prescribed by the laws is from the very first an actual pro-

of saying, to a given content, unless the content itself answers
; every application is merely the recognition that what we wish
ply is the very nature of that to which it is to be applied. Now
ited number of observations enables us to discover that every-
real exhibits certain constant characteristics, and these charac-
ics then take the shape in our mind of expectations which
be confirmed, and which we bring with us when we make further
rvations; thus we easily come to regard them as something
h exists independently in fact as well as in our thoughts, and is
to the object in which we shall find fresh confirmation of it;
e all that strange phraseology which regards universal laws as
rs ruling on their own account, to which everything real, what-
its origin and whatever its nature, is subsequently obliged to
it. If we avoid this wrong conception, and connect that which
ubstitute for it with that to which our aesthetic requirements
rise, the one and undivided object in which our thought now
s satisfaction is a being, which, not in consequence of a still
er law but because it is what it is, is the ground both of the
ersal laws to which it will always conform, and of the series of
idual realities which will subsequently appear to us to submit to
e laws. I have no intention of exhausting this subject here, and
ss over many difficulties which we shall have to notice later,
of them in the course of our present logical enquiries, others in
metaphysical context: it is enough here to follow out the
al form of thought which the mind must look for if it tries to
fy the want just described.

19. This form will no longer be quite that of inference as de-
ed above. The universal law, to which the major premiss there gave
irst place, instead of standing out from the other elements as their
ntial condition, will now accompany them as a latent idea, always
rstood but not expressed; its former place is taken by the universal

upon its particular position or the particular point to which it has thus far developed, not through extraneous influences but through its own movement. It would be possible, but would only lead to prolixity, to express the essential truth in this kind of idea without importing into it the conception of motion; we should then find ourselves requiring an idea which includes in the system of its species and sub-species the whole of reality; but the differences and the order of these species would not be determined independently of the idea by pre-existing marks and their modifications; the idea itself would contain the reason for the presence of the marks, for their possible divisions, and for the arrangement of the resulting varieties according to their value, in fact the whole reason for its own classification. We may formulate our requirement most shortly as follows: the form of thought for which we are looking must have only *one* major premiss for all its conclusions, and this premiss must express the movement of the world as a whole; its minor premisses must not be given to it from elsewhere, but it must produce them from itself in the form of necessary and exhaustive varieties of its meaning, and thus must evolve in an infinite series of conclusions the developed reality which it had conceived as a principle capable of development in the major premiss.

150. It cannot be said that the impulse to organise the whole world of thought upon this pattern is foreign to the mind when left to itself; it has been at work at all times, and whenever a view of the world more or less like the theory of mechanical explanation has developed itself, this impulse has met it with the reiterated demand that the world and all things in it should be regarded as a *living* development. For it is in the phenomenon of life that we believe ourselves to see these claims of the mind completely satisfied; as there the original type of the organism is made into the efficient power which produces the incentives and conditions for its own consistent development, so we would have the world as a whole evolve from itself the occasions which are the necessary conditions of its gradual self-realisation. We need not here notice the errors in this belief in the independent development of the individual organism; it is enough that it *appears* to be a graphic instance of what we are looking for. The same image has also been a constant favorite with the theory which, for the last time in our day, avowedly aspired to a vision of the

ing, bringing independent minor premisses under universal
rs, that the Hegelian philosophy even wished to derive the from its single principle: it only proposed to look on and see he development followed from the inherent impulse of the idea. or this intellectual vision, this '*speculative*' thinking in the original of the word, it believed itself to have found a guide in the tical method, a guide which enables the spectator to follow the ourse of the self-realising development. I shall still keep to my ple of saying nothing in this survey of logical forms about the cal rules for securing their application to the matter of thought, herefore leave for a later occasion what is to be said about this od as a method; but I shall appropriate the antithesis between lation and explanatory theory for the purpose of describing the shape which we aim at giving to all thinkable matter, and ne *form of speculative thought* this third member, with which the of comprehensive and systematic forms comes to an end.

1. And yet I feel that I must not conclude quite so shortly; I return once more to an observation which I have already made. orms of thought which we are considering are ideals; they in- e the final shapes which thought wishes to give, or to be able to to the matter, great or small, which it has before it, in order to y its own inherent impulse by showing the coherence of all that sts. Nor is the validity of these ideals at all impaired by the that human knowledge is not able to apply them to every instance. It may be that we are not always in a position to ver the universal laws which govern a particular circle of phe- na; and it may be that, if we had discovered them, we should ucceed in bringing all particular cases under them so completely the necessity of any given result was at once apparent. But we d not push forward our enquiries in this direction so untiringly, were not convinced that the principle of the explanatory theory is rsally valid, and that its validity is independent of our present ability rify it in every conceivable instance. Perhaps the form of spe-

without proof, for it leaves no elements remaining in mere unconnected juxtaposition, but exhibits everything in that coherence which had been all along the aim of thought. At the same time it points beyond the province of logic. From the point of view of the explanatory theory it might still seem as though the universal laws, which thought produces from itself alone, gave a right to decide *a priori* what reality will be like; speculation does not deny this right, but by making the *content* of a supreme principle the one and only ultimate ground of everything, both of the power of these universal laws themselves, of the direction in which the world as a whole develops, and of the individual forms which in consequence reality assumes at each moment, it indicates that the final fulfilment of all logical aspiration could not be attained by new logical *forms*, but only by material *knowledge* of that supreme self-developing principle which speculation presupposes.

In concluding this account I am conscious how much its method deviates from those which are in vogue at the present day. We are so accustomed to being told the history of things, and to feel our curiosity satisfied when we have discovered or invented an origin for them, that even logic is flooded with psychological explanations and derivations of its doctrines: on the other hand it strikes us as antiquated, odd, and unmeaning if anyone attempts to arrange the forms of thought in a progressive series according to the nature of its problems, instead of following the order in which the mental activities necessary to their solution develop in the individual soul. I am content that this should be so, and hope that in the form of my exposition my readers will recognise the premonitory influence of the idealistic philosophy to which it is intended to lead: I have no fear that by choosing this form I have distorted the substance of truths which, on any view of logic, must be equally regarded as established.

BOOK II.

APPLIED LOGIC.

PREFATORY REMARKS.

152. WE are so much accustomed to oppose the world of our oughts to an external reality, that as soon as we speak of an object which the forms of our thinking are to be applied, it seems as if we n mean thereby nothing but this external reality. When we call to ind the natural sciences, which occupy so large a portion of the ld of science at the present day, we are confirmed in this opinion; n the other hand, when we think of mathematics and jurisprudence e are likely to be shaken. The external reality supplies neither the jects with which the mathematician deals nor the methods by which e deals with them. That which it yields does but give him an occaon to turn his investigations in this or that direction. The true jects of his enquiry are always nothing but the forms which our tuition or our thinking finds in itself or creates, and of which the pearances of the outer world remind us, without ever perfectly corsponding to them. And his business is, in accordance with laws of asoning, which at any rate are not derived from any external exrience, to develop the countless necessary conclusions which follow om the various possible combinations of these forms. Nor is this evelopment speedily achieved: these consequences do not unfold emselves in such a way that we need but to look on and watch: on free choice, are yet the necessary consequences of ideas of right and justice, consequences of a truth that ought to be, which has its home nowhere but in our own minds. And so logical acumen is just as constantly employed here also in setting forth ever more precisely and irrefragably the connexion of the several conclusions already drawn both with one another and with the highest principles from which they flow.

Thus both these branches of science show that logic need not go to the external reality to find objects for its application,—that it finds fully work enough in investigating the connexion of that which is possible in thought and necessary in thought,—that finally the inner world of our conceptions is wide enough to contain unknown regions, still to be discovered by means of systematic enquiry.

153. Keeping to this line of thought we may now turn to the natural sciences. Even the external world which we assume is after all an object of our enquiry only so far as (in some way or other which does not here concern us) it has become a world of conceptions in us; we survey, dissect, and investigate not that invisible something which we suppose to lie outside us, but the visible picture of it that is formed in our consciousness. We may believe that we are compelled, as the result of prolonged labour, to accept certain connexions according to law between the unknown parts of this unknown external something; but all these assertions (whatever they may be) are after all grounded solely upon the relations which prevail either persistently or in succession between the contents of our thoughts. Whatever may be the causes which produce this succession, the laws by which it is regulated can only be known by itself, i. e. by the order in which certain thoughts follow certain others in our minds, by the constant union of some thoughts, and the impossibility of uniting others. It is enough then even for the treatment of the external world to regard it in the first instance as a world of thought set up somehow or other in us; whether the appearances which surround us correspond to a real world of external things, or whether they be products of a creative faculty of imagination in us, guided by unknown impulses, the discovery of the connexion between them will always necessitate the same methods of enquiry.

t aside.

CHAPTER I.

The forms of Definition.

154. INNER states, sensations and ideas, feelings and impulses, cannot be conveyed like material things, which may be separated from their original possessor and passed on as they are from hand to hand. We can communicate them only by subjecting our neighbour to conditions under which he will be compelled to experience them or to beget them anew in himself.

If we had to communicate for the first time something yet unknown, which was too simple to be created by thinking, or too complex to be exhausted by it, our only resource would be to produce the *external* conditions of perception. If our neighbour had never seen light, or heard sounds, or felt bodily pain, our only course would be to put his eye within reach of a source of light, to bring waves of sound to act upon his ear, and by the application of a stimulus to his body to let him experience that feeling of pain with which we ourselves had made acquaintance in precisely the same way. If we wish to enable him to recognise a person whom he as yet does not know, the description of the countless little marks which distinguish that person from others will never make sure, but by pointing with the finger we can show him precisely whom we mean. We need do no more than thus barely mention the fact that wherever it is applicable this direct reference to the object itself or to some likeness of it is always useful. But in view of the questions which here concern us we further presuppose two things,—first a large stock of past experiences common to the persons who are to communicate with each other, and secondly a language intelligible to both parties, to the several words of which each attaches (to a large extent at least) the same ideas. Then by a series of spoken words we call to our neighbour's recollection the ideas conjoined with them in that order which is for him the *internal* condition of his creating or experiencing in his own consciousness that which we wish to communicate.

155. This form of communication also includes much else that our logical enquiry can only take note of by the way. Both poetry and eloquence aim by this method at something more than imparting ideas: they count upon the attachment to the images thus called up of feelings of pleasure and pain, of approval and disapproval, of exaltation and aversion. The effects which they thus produce are powerful but uncertain. Different minds are indeed pretty uniformly organised for the mere apprehension of matters of fact, and their general habits of perception do not change; but in estimating the degrees of emotion which we annex to what we perceive we must allow not only for original differences of temperament, but also for the changefulness of the mood of the moment, which depends upon what we have just gone through. Thus different persons are very differently receptive even of actual facts; still less can we hope by the imperfect recollection of such facts, which is all that speech can rouse, to create in others precisely the same emotion which they produced in ourselves. How much may be done by skilful guidance of the train of ideas and by well-measured expressions to lessen the uncertainty of the result is a question for the art of poetry and rhetoric. Our own problem is narrower and is limited to the communication of that which has been already refined from a state in which we are acted upon into an idea which we apprehend,—i. e. of thoughts, not of feelings and moods.

156. The certainty even of this kind of communication seems to be imperilled by the fact that after all the same words do not always have the same meaning for the speaker and the hearer. It must be allowed that, apart from subsequent confusion of originally different roots, there are in every language many words which denote several very different things,—in consequence no doubt of a resemblance which these things bear to one another, but still of a resemblance which is not always so obvious now to him who uses the traditional words as it was to the first inventor of these metaphorical expressions. And even when a word denotes the same thing for all, that does not ensure that all have the same conception of the thing denoted. The special circumstances under which each individual became acquainted with the thing, the peculiar point of view from which he first regarded it, the connexion in which he found it and from which he had to detach it, give a peculiar colouring to his picture of it, and dispose him to other conclusions than those anticipated by the speaker when he named the common word, hoping thereby to give some particular turn to the course of his hearer's thoughts. It is impossible to deny these facts, dangerous to disregard them altogether, yet foolish to press them too far: the intercourse of daily life sufficiently proves to how large an extent speech enables us in spite of them perfectly to understand each other's thoughts about the most various matters. There will certainly remain ideas which it is hard to communicate with precision; but were there no such difficulties there would be no good in seeking rules for helping us by the appropriate use of unequivocal words to remove the ambiguity of others and to fix their meaning so that all who wish to converse may use them in the same sense. It must be left to the unfettered acumen of the speaker to determine what words may be accepted as precise enough to explain other words; but however far we may feel constrained to go back along this line and to remove all ambiguity from the instruments of communication which we wish to use before we use them, there will still be only two possible ways for us, abstraction and construction.

157. We explain a conception, which we will call M, by *abstraction*, when we first refer to a number of known instances, in each of which M forms a part of the notion, and then bid the hearer separate from these instances that which does not belong to the conception M which we wish to communicate. This is the way in which all our general conceptions[1] and general ideas[2] were originally formed; in the case of a general idea that which was common to a number of impressions comes of itself to stand out as the object of a new separate idea; in the case of a general conception this process is consciously directed by attention and reflexion. And when we are at a loss we all come back to this same way. The man of no logical training does so when to the question what he understands by M he replies, in the fashion which the Platonic Socrates so often complains of, only by giving examples which contain M, leaving to his questioner the trouble of separating the common element which he wants to get at from that which is foreign to it. But the logically trained thinker also proceeds really in the same way: however carefully he may choose his terms so as to express the universal itself without any reference to particular instances, yet this expression is only obtained by a tacit comparison of a number of cases. It is only by such a comparison that we learn what marks of M must be precisely fixed in order that the expression may exclude all that is foreign to M, what other marks must be left undetermined in order to include in M everything that is properly an instance of it. And lastly, only by the fact that instances

.o instances. Wherever our aim is to fix some very simple ı which underlies a whole group of kindred ideas, it is the od possible. Such a conception can only be pointed out away from known instances of it all that does not belong to ı never put it together out of its component parts, for it has he labour expended upon this impossible aim always ends ıs circle, since among the materials that are to be used in uction the very thing that was to be constructed is taken for vhole and entire, however much it may be concealed under xpressions. Thus, for example, in our idea of *becoming* the of being and not-being are no doubt united as two conints of relation; but if we should try to characterise becomunity of the two we should not attain our object. In the : we should be bound to fix the precise sense to be here to the expression 'unity' which in itself is very ambiguous. mean the mere co-existence in the same consciousness of leas of being and not-being, for obviously becoming is the f a relation that exists between the contents of these two ut if we try to unite being and not-being as predicates : at the same time and in the same manner to one and the g, we do not arrive at becoming, but simply find ourselves d by the impossibility of actually executing in thought a task olves such a contradiction. Suppose then that we separate being and the not-being of this thing and say that the one is applicable to it when the other is not:—even by this change t get hold of becoming; it falls between the two moments of is to be found in neither. We shall have therefore to bring ether once more: but as long as they are separate from one ecoming will lie outside of them, we can only get hold of it abstraction from the instances in which it forms a part of the thought, not to be created by the putting together of ideas which as yet do not contain it. Precisely the same considerations hold with respect to the equally simple conceptions of being, acting, thinking, affirming, denying; and the geometry of Euclid follows precisely the same method in determining the surface as the limit of the space occupied by a body, the line as the limit of the surface, the point as the limit of the line,—in each case teaching the learner to get the simpler conception, which is harder to grasp, by abstracting what does not belong to it from the more complex conception which lies nearer to sense or which has just been determined.

159. The opposite method would fully deserve the name of *construction* only if it enabled us completely to put together the idea to be conveyed out of a definite number of unequivocal parts by a series of acts of thought which we were required in unambiguous language to execute upon those parts. Almost the only conceptions that really admit of this treatment are the mathematical conceptions and some others that arise out of the applications of mathematics,—conceptions which as creations of our thought contain only what our thought has combined in them. They admit of it because the several ideas which make up the whole conception can be completely enumerated, and because not only each of these ideas but each of the ways in which they are to be joined together is such that we can state the characteristic quantity by which it is distinguishable from others of its kind, as well as the special quality which distinguishes it from those of another kind. Here then nothing remains indeterminate that should be determined; he who follows the directions given must see the picture he is desired to form rise before his mind's eye with just that degree of individuality or generality which the speaker wished to give it.

If on the other hand we wish to convey a notion of some really existing thing we are met by well-known difficulties. Our mental picture of a real thing is not made up of a limited number of points of relation which are to be brought into combinations also limited in number, but is compounded of a countless number of ideas. And of these component ideas those that belong to different senses cannot be compared with one another, while even those of the same sense can only be designated by general names, and scarcely admit of precise *escription.* In describing we try, if we understand our business, first o fix the main outlines of the whole idea, whether this be done by a imple construction, or by taking as illustrations similar things already nown and proceeding by alteration and transposition, by the removal f some features and the addition of others, to elicit from them the eading lines of the picture we wish to convey. Then we fill in the nass of details, never completely, for they are usually inexhaustible, ut skilfully selecting those by the mention of which we may hope hat the hearer's attention will be at once stimulated to supply from is own memory those that are not mentioned. We need but remind he reader of the wonderful effects which the poet produces in this nanner, bringing a whole picture before us with a touch; though the uncertainty of the result is equally manifest. The way in which each man supplies what is not mentioned varies according to his nature: were it possible to bring to view in detail the different pictures which the same description calls up in different hearers, their variations would show what an inadequate basis a description must be for the support of definite conclusions. For scientific purposes therefore description needs a regulation of its method, and this it finds in the rules of definition.

160. For the *definition* of a conception M it is usual to require a statement of the next higher generic conception G (the *genus proximum*), and of the characteristic mark d (the *differentia specifica*) by which M is distinguished from other kinds of G. By requiring the generic conception G we set bounds to the arbitrary and capricious course of description. In describing you were free to begin at any point whatever, and then gradually to add the remaining points in any line that you pleased, so long as you could be sure of producing in the end a clear picture of what you meant. But even in a description

If we are told that a creature we have never yet seen is a bird, this general conception gives us at once a clear picture of a number of members united in a characteristic manner, and at the same time of the peculiar kind of locomotion and vital action to which they are instrumental. The further special characteristics are easily added to this outline, for it indicates of itself the places to which they severally belong. We should never get such a clear idea of the unknown creature if we had to put it together out of its primary components. It would be an endless task to enumerate all the variously-coloured spots on its body with their position and the extent to which they may be displaced, so as to give a notion even of what it looks like. Still more endless would it be to add to this the peculiarities of life and habit, which all belong at any rate to our idea of the animal in question if not strictly to our mental picture of it.

We see then the value of the abbreviation effected by starting from a general conception that can be assumed as known: we understand also that we must choose for starting-point not merely any higher universal, but expressly the *genus proximum*, which in its characteristics and in the mode of their combination comes closest to the conception to be defined, and so clearly prescribes the point at which and the manner in which we are to add each of the last characteristics by which the conception is finally determined. By starting from a higher universal than this we should not only lengthen again the rest of our task, which definition was intended to shorten, but we should run a risk of failure. For we should then have to add a whole series of further characteristics in order to exclude everything foreign in the long descent from that less determinate universal to the particular species in question: and each new characteristic would open a new source of error; for it is hardly possible to determine quite precisely the mode and manner in which each is to be added to those that have preceded it without appealing to a picture which it may be assumed that each man already has in his mind. The notion of that *genus proximum* therefore would not by this method be produced afresh with that definiteness and certainty with which it could be recalled to the memory at once by the mention of its name, and which it must have if it is to serve as an outline for the filling in of the final characteristics of the conception which we desire to convey. All that we could get by this method would be more or less of a riddle. For when we propound a riddle what we do is this,—we tell our hearers e completely the modified forms p^1 q^1 r^1 assumed in the case P Q R the general predicates of the genus G. Instead of characteristics the usual rule for definition requires us to set ly one characteristic d, the specific difference, by which M is shed from all other species of the genus G. Definition thus re limited and therefore a more practicable aim than descrip- stead of setting forth positively the whole content of M it to state the mark by which M may be separated from all t M. This is the origin of the terms *definitio* and ὁρισμός, which imply only the marking off of one thing from another. act the general aim of definition must be thus limited. As advances we feel no doubt the need not only to distinguish, now completely what we have distinguished; then we make lemands upon definition; then we refuse to admit as a lifference anything but one of those characteristics that really species, i. e. one whose occurrence decisively modifies the ssumed in M (the thing to be defined) by all the other cha- cs of the genus G which are not mentioned in the definition. eavy demands however can be completely satisfied only at lusion of an enquiry which has made us perfectly acquainted nature of M, and which thus enables us to solve the problem emains, of fixing a final and classical expression for that

esides this there are other no less pressing problems. We e to begin a speculative enquiry, which has to find a number nknown propositions that are true of M; or in a practical e may have to determine what is the proper consequence of situation M: in either case it is of the utmost importance M, to which the propositions we are going to assert or the presence of this obscure characteristic *d*, or will show that they are valid over a wider or narrower field than this, so that *d* is not the proper characteristic of their subject. In the other case, that of a practical matter, the exact meaning of a legal situation to which a law is to apply must be completely considered beforehand while the question is still *de lege ferenda*; but he who has to carry out the *lex lata* rightly demands that this previous consideration shall have given the law the form of a definition which distinguishes, not by the most profound but by the most obvious mark, the cases to which a decision shall apply from those to which it shall not. These are problems which applied logic cannot decline, and we overlook them when we think too disparagingly of this traditional form of definition. We misunderstand the sound sense of many such definitions in practical philosophy and jurisprudence when instead of the marks of *M*, which they intend to give and do give completely, we see in them nothing but an inadequate statement of the whole nature of *M*, which it is not their purpose to give at all.

162. It will be convenient to notice in this context the distinction which is commonly drawn, but not always in the same sense, between *nominal* and *real* definitions. We may utter a name or replace it by another; but we can never define anything but its meaning, i. e. our idea of that which it is intended to signify: the thing itself again is not in our mind, but only the picture we have formed of it. These two kinds of definition therefore seem to be identical; and they are in fact identical for everything that exists only in our minds, and whose whole nature therefore is exhausted by our idea of it. There is no real definition of a geometrical figure that can be distinguished from its nominal definition; any correct definition that we give of it expresses at once the whole nature of the thing in question, and the whole meaning of the name.

In other cases however the distinction between these two modes of definition is one that it is worth while to make. If we call the soul the subject of consciousness, of thinking, feeling, and willing, this may be appropriately called a nominal definition; it specifies a condition which a real thing must satisfy if it is to be entitled to the name of a soul. But who or what this thing is whose peculiar nature enables it to satisfy this condition, is still quite an open question; we have not fixed the real definition of the soul till we have got a theory which proves either that only a supersensuous and indivisible being, or that only a connected system of material elements can be the vehicle of consciousness and its various manifestations. It was a to be found not in the conformity of the beautiful object with some conception, not in its capacity to satisfy a desire in us, but in the fact that it pleases directly and without reference to any interest. The real definition of beauty would have to point out the precise relations between various things or components which enable every object in which they occur to produce this pleasing effect. And so we may say in general terms, when experience shows us a group of characteristics $p\ q\ r$ often occurring and continuing together, or when in the course of our investigations we light upon a coincidence which induces us to put them together and to regard the group as a subject for further enquiry, we proceed in the first instance to form for the group a conception M, of which a nominal definition can always be given, because it has only to set forth the predicates which led us to invent the name, or the effects which we expect from the thing to which the name is applied. But a real definition cannot always be given: for there is no assurance that we have not combined in M characteristics whose union we thought ourselves justified for some reason or other in assuming or desiring, when there is in fact nothing to be found in which they really are or can be united. It is a common error to mistake this mere indication of a problem we should like to solve for the solution itself; and on this account the distinction between these two kinds of definition is useful as a warning.

163. We have to beware of three faults which vitiate a definition.

In the first place its assertion $M = Z$ must be no tautology; but it becomes a tautology whenever M itself is explicitly or implicitly assumed among the ideas combined in Z by which M is to be explained. This fault (called *circulus in definiendo*) is often committed through carelessness which no rules can prevent; but we are almost of also be true that every $Z = M$: whenever therefore further reflexion or fresh experience shows that some Z are not M, we know that the definition $M = Z$ was too *wide* (*definiendo latior*), and included some non-M which it ought to have excluded.

To point out how to avoid these faults would be more useful than thus merely to name them; all we can do in that way however is to indicate their usual source, viz. the limited range of our observation, which as a rule opens to each individual only one and the same fragment of the entire field covered by a conception, and further the one-sidedness into which our thinking is apt to lapse if it does not constantly receive fresh stimulus from without. In the temperate zone the way in which plants awake in summer and sleep in winter makes a strong impression upon our feelings; animal life, with its continuous activity, seems to offer a complete contrast. Now we certainly should not base upon this a scientific distinction between animal and plant; yet countless comparisons, employed by poet and orator, show that we are accustomed to consider this yearly alternation as the essential characteristic of the plant. But a definition which expressed this would be at once too narrow and too wide: it would exclude tropical plants whose life is an uninterrupted growth, and would include hibernating animals, which in this climate easily escape our attention, directed as that is mainly to the domestic animals. It may easily happen that one who wishes to establish on a new basis the rights and duties, both political and social, of all the members of the state, thinks only of the male world to which the conduct of these transactions is usually confined, and then his proposals will be too wide, in as much as he demands for all what he intends for men only, or too narrow, in as much as he expressly enacts for men only what must obviously apply to all. From this we may draw a lesson of universal application: we should never attempt to treat a problem off-hand, when it is possible to extend the limits of our own experience by converse with others or by taking count of views which are already recorded in the literature of the subject. Learning is not in itself inventive, but like any other training and discipline, it makes us more secure against extreme errors than if we proceed by the mere light of nature.

164. We further require in a definition elegance and brevity, which I will illustrate by a simple instance. If we define a circle as a curved line all the points of which are equidistant from its centre, we first of all make an actual mistake in giving too wide a definition.

crosses and recrosses a great circle of the sphere making equal curves on either side, all the points of this line are equidistant from the centre of the sphere.

If further the line, in returning to its origin in the great circle, describes an uneven number of these double curves, it will consist of an infinite number of pairs of points, forming the opposite extremities of so many diameters of the sphere. The centre of the sphere therefore bisects the rectilinear distance between the two points of each pair; and so, in every sense which can here be given to the word, it would also be the centre of the sum of all these pairs, i. e. of this line, which nevertheless would not be a circle. We ought therefore to have said that a circle is a curved line *in one plane* which fulfils the above condition.

But elegance further demands that a definition shall not contain more ideas than are indispensable for the complete determination of the given conception. So we may be called upon to speak not of a curved line but of a line simply: if a line fulfils the annexed condition it follows without more ado that it cannot be straight. The condition itself however is not correctly expressed. A definition should not employ among its instruments of explanation ideas which are themselves unintelligible without the conception to be defined. In this case the idea of the centre is certainly such an idea. If we had not yet got the idea of a circle (and in fact there is nothing in this case at least to suggest this idea to us, after we have omitted the characteristic of curvature from our definition) we could at first think of the centre of a line only as the point of bisection, and we should not discover our error till we attempted to construct a circle on that understanding. Instead therefore of this sense of the term centre which common usage suggests, and which compelled us to be so painfully discursive just now in speaking of our serpentine line, the definition requires the precise statement in general terms of the meaning which the word is to bear for all figures whatsoever. This statement can easily be given, but I may omit it, as it follows therefrom that *if* there be a point in a plane which is equidistant from all the points of a line in that plane, that point is the centre of the line. But if we now introduce this definition of centre into our definition of a circle, the statement of the further condition under which the line in one plane becomes a circle comes to be a mere tautology, and the meaning of the whole definition is evidently nothing more than that a circle is a line in one plane for which there is a point in the same plane from which all its points are equidistant. The definition is substantially correct; yet fault may be found with its form. For now after omitting the term centre we remember that it was only the presence of that term that forced us to look for the equidistant point in the same plane. Not this actual centre only, but any point in an axis drawn through it at right angles to the plane of the line fulfils the condition of being equidistant from all points of the line. It is enough therefore to say that a circle is a line in one plane such that a point may be found from which all its points are equidistant. It is needless to mention that there are several such points and to say where they lie: the attempt to construct the line according to this direction will at once teach us both. But once more even in this form the definition is not quite all that can be desired. It does indeed say that all the points of a circle are equidistant from one and the same point, but it does not formally state whether or no all points that are equidistant from this point are points in the circle. They are so in fact provided they lie in the same plane, and thus in order to express this along with the rest we may finally say that a circle is a line which contains all the points in one plane which are equidistant from any point.

165. Different opinions may be entertained as to the requirements of definition which I have just illustrated by the example of the circle. Every one will allow that it is a serious fault to employ ideas which (like centre in this case) though a meaning may be given them apart from the conception to be defined, yet are not fully intelligible without it, except perhaps in the context of a scientific treatise. But it may be thought that the addition of superfluous characteristics is unobjectionable, since it makes the definition easier to understand without impairing its correctness. Nevertheless it should be avoided. For the addition of some characteristic z that might be dispensed with, is apt, as we are not told that we might dispense with it, to make us think that it is inserted in order to distinguish the M we are defining from a non-M to which everything in the definition is applicable excepting only z. If we say a circle is a curved line in one plane such that there is a point from which all its points are equidistant, the form of the statement suggests that there are also straight lines which satisfy that condition. It matters little in so simple a case as this; but in more complex cases serious disadvantage may be the result of this apparently harmless addition of superfluous matter. At the least it hampers us in the drawing of conclusions, which after all was our sole purpose in laying down the definition. It may happen, for instance, that it has been quite clearly established, perhaps in bringing Q under M and actually drawing the conclusion which that would justify. And so we may say generally that it is right to demand that a definition shall contain only those terms that are indispensable for the specification of the object, but shall exclude all merely descriptive elements: if it does not enable us very readily to form a picture of the thing, this will be atoned for by the certainty of the conclusions we can draw from it.

166. Hitherto we have been considering the usual form of definition by the proximate genus and the specific difference as the only valid form. But the untrained intellect is wont, to the annoyance of the logicians, to use another mode of definition, and to say, for instance, in its familiar uncouth way, sickness is when something pains me. Such a phrase certainly needs to be amended, yet not exactly in the way which logicians rather intolerantly require, but rather in the way in which physical science actually defines many of its conceptions. The ordinary form is properly adapted only for defining the meaning of a substantive: when we have to do with adjectives and verbs it is not only shorter but more correct to give them their proper place in the grammatical structure of the definition, and to let them bear plain reference to their subject, seeing that it is only as expressing states or properties of a subject that they have any meaning. It is quite right therefore to define adjectives like sick or elastic by such propositions as 'a living organism is *sick* when its functions depart from a certain course;' 'a body which on the cessation of external constraint resumes its original shape is *elastic*.' And in defining the meanings of the verbs *to live* and *to sin* it would be quite proper to name first the subjects to which they can be applied, an organic body and a spirit that is seriously coming back in the course of its enquiry to their real subjects, though it is only as properties, states, or activities of these that they exist, and though their apparent development is every moment bound up with the real development of these subjects.

167. Under the head of conceptions to be defined we have hitherto considered only comparatively simple ones, conceptions of figures, things, properties, and easily intelligible relations: but among the words used in speech, every one of which may under certain circumstances call for a definition, we often find very complex relations between a great variety of points of attachment comprehended in one simple expression. No one who was not hide-bound by prejudice would require that the explanation of such conceptions should take the regular form of a simple definition; and to find special names for all the other very various methods which may be employed would be nothing but useless pedantry. The universal principle of applied logic is simply that all ways are allowable which lead to the goal; it hopes for no more than to remove our doubts as to which way is passable right up to the end, and which not, by pointing out that which has long ago been tested: it never forbids our seeking new ways to satisfy new needs. It is always allowable therefore to begin with a preliminary description, with comparisons and analogies, with discussions of any kind, in order to familiarise the hearers with the meaning of the subsidiary ideas we wish to employ and the peculiar combinations we wish to establish among them, and having thus prepared the way to proceed to set forth what we wished to explain in a formula which is brief and intelligible, though it presupposes what has gone before and cannot be separated from it.

This reminds us however of another twofold division of all definitions. We may characterise M by the aggregate of marks displayed by the conception when it is present to our minds in its completeness: this kind of definition, which we illustrated just now in the case of the circle, may be called *descriptive* definition: we have recourse to it mainly in the case of actual things which we only know from the outside and whose definition therefore is in fact nothing but a

. The circle as such is not made at all: but supposing a par- r circle such as we draw to have been already made in some or other, we may certainly form a mental picture of it in the way ated by this definition. But we may form that mental picture lly well by supposing the length of the two axes of an ellipse to till both are equal to *r*; or by supposing a cone to be intersected plane at right angles to its axis. And thus an idea, whose ent has in itself no genesis, may admit not only of one, but of so genetic definitions as there are ways of forming the idea of this ent by the manipulation of other ideas. Among these genetic itions then, using the term in a somewhat extended sense, we include the above-mentioned miscellaneous methods: they try by ect means to make us form a mental picture of *M*, when it is ssible or inconvenient to say directly what *M* is.

8. Strictly speaking, whenever we undertake to define a con- on *M*, our aim is to give it a higher degree of definiteness it yet has. But in fact the problem usually narrows itself to the formation of a *clear* idea (*clara perceptio*) which we already have , into a *distinct* one (*distincta*), or of a *mere mental picture*, which but comprehend *M* in a loose general way as a connected whole up of parts which are familiar, into a real *conception* of *M*. e two expressions may be regarded as equivalent. For accord- o old established usage we are justified in saying we have a clear of anything when we think of it as one, and as a connected e, and lastly as distinguished from others with precision enough oid confusion; but it does not become distinct till to this is d the general law which regulates the connexion of the parts, further the characteristics which it has in common with other

though their meaning has never been precisely fixed. Thus we speak of virtue and sin, of good and the highest good, of appearance and reality, with a full conviction that we mean something very definite by these names, and ready to draw important inferences from them in reference to that to which we apply them. But at last the difficulties in which we entangle ourselves convince us that strictly speaking we did not know precisely what we meant, that we had not completely fixed the conditions which must be satisfied in order to justify the application of these names, that we had in short trusted to hazy ideas, the clearing up of which is of the very first importance. This we try to effect in a very simple way. If we were entirely ignorant of the meaning which M was intended to bear, we should have no means of finding it out; but also it would never have occurred to us to apply this name had not some part of its meaning (say a) been fixed beyond a doubt—that very part namely which now impels us to use the term the rest of whose meaning is still hazy. This a we first take tentatively as a complete definition of M, and consider whether a corresponds to what we *mean* by M. It is a matter of common experience that in cases where we are not in a position to express the meaning of M in positive terms we may yet see whether an idea a that is offered as a definition of it is adequate or not. Thus when we are trying in vain to recollect a name we can yet pronounce with perfect certainty that a suggested name is not the right one; and further any resemblance it may have to the right one makes an impression on us, and sometimes reminds us at once of what we want,—at any rate it helps to make plain the other points in which the right name differs from the suggested one. We are in the same case here: a is not utterly wrong and incapable of comparison with M: the comparison of the two therefore does not lead to the bare negation of their identity, but puts us on the track of a supplementary b which must be added to a, or an alteration b which must be effected in a in order to make it answer exactly to M. Now putting M down as equal to $a + b$ we make a second attempt and repeat the same course of comparing and supplementing by fresh terms c and d, till at last we get a definition $M = a + b + c + d$ which in its expanded sum of characteristics exactly coincides with what we meant by M. In this very simple process of thought rather than in

CHAPTER II.

Of the limitation of Conceptions.

.69. IN the course of an investigation we may be led by a definite
pose to trace a group of characteristics *i k l* through all the other-
e different objects in which it occurs, and to ask what influence is
rcised by its presence upon the rest of their characteristics. The
ılt of this comparison then will itself teach us whether the other
racteristics which each of these subjects has in virtue of the genus
which it belongs are modified by the presence of *i k l* in any re-
rkable and particularly in any constant manner. If this is the case
often form out of *i k l* and out of the idea of a more or less pre-
ely determined subject a new generic conception *M*, treating all the
as in which *i k l* occurs as species of *M*. But whenever this is not
case (and not seldom too when it is) we content ourselves with
ting the presence of *i k l* as one of the countless variable con-
ons, which affect other ideas so far as to necessitate certain altera-
ıs in them, but do not themselves form a generic conception under
ich the several instances in which they occur could be arranged as
cies. Now a living language is believed by those who use it to
'e already sufficiently distinguished in the coinage of its words the
) kinds of cases in which these two methods are severally appro-
ate. Of course they will allow that enquiry, as it goes deeper and
per, will discover many a new group of characteristics *i k l* having
h a decisive influence upon the whole bearings of every concep-
n that contains it as to make it worth while to erect this group
a separate generic conception *M* and to mark it by a name; and

170. These conceptions which our inherited language supplies are the tools with which our thought must work—and that not merely because we have no means of communication except the words which have been invented to express them: in this store of words is treasured up the concentrated result of the thought which the human mind has from the earliest times bestowed upon the world to which it has access, and we may suppose that the same impulses which led it to fix its conceptions in this form would also in the first instance assert themselves in us were we to go through the same labour.

But that these impulses, however natural they may be to man, yet leave room for doubt is shown by the divergence that constantly occurs in the application of the conceptions thus formed. When the question arises whether some predicate P is to be affirmed or denied of a subject S, one maintains that S is a kind of M and therefore is a P; another objects that S is no M and therefore no P; a third allows that S is indeed no M, but an N, but declares that this does not matter, and that what is true of M holds good of N also, while a fourth insists that the difference between M and N establishes a difference between the two in respect of P.

The divergence that here shows itself culminates in two opposite tendencies, dominating the whole of our thought. The one is a tendency to exaggerate every difference that presents itself into absolute difference, and with the familiar formula 'this is something quite different' to resist all argument from one case a to another case b which resembles a but is not exactly like it: this tendency becomes in life and in science the spirit of the pedant and the philistine. The other is a tendency to ignore the fact that a difference which is not absolute difference may yet have a qualified value, and with the barren phrase 'all is one at bottom' to obliterate all the fixed boundaries which define the province of each conception, thereby destroying the only grounds upon which certain predicates are attached to certain subjects and to no others: this becomes in thought and action the principle of a no less ruinous libertinism. A glance at the momentous consequences of these confusions makes us alive to the necessity of clearly understanding what reasons there are to justify us in dividing the whole extent of the intelligible world into definite conceptions, where the boundaries of their several provinces are to be drawn, and what value is to be assigned to this demarcation.

171. We are led to very various issues by the attempt to answer these questions even where they are easiest and least pressing, viz. in regard to the simple contents of sensuous impressions. We have the difference between A and B as greater or less than that between A and C or between B and C. We find these relations, or rather this lack of any assignable relation, between A B and C if A stand for colour, B for sound, and C for smell. We may keep the old name and call them *disparate* or incomparable.

This conclusion will not be affected by various secondary considerations which may be urged. It may be pointed out, for instance, that all three exist only as states of our consciousness. To this we reply that they all are indeed sensations, and may be called, according to the usage of logic, species of sensation; but that the conception of sensation in general cannot serve here as a generic conception in the sense of supplying a law for formation. When we think of the shape of an obtuse-angled triangle as subordinated to the general conception of a triangle, we have in the latter a constructive formula, whose application has but to be varied within its own limits in order to show us that there are right-angled and acute triangles besides that one species from which we started. But the subsumption of colour under the general idea of sensation (for it is only subsumption that is possible here, not subordination) can never enable us to conclude from this general idea that there are such sensations as sounds and smells besides colours. Although these three then are, to use the ordinary phrase, kinds of sensation, yet within the limits of this universal they remain quite disparate the one from the other.

Again as states, as motions or affections of the soul, these various kinds of sensation may produce certain secondary effects that are comparable with one another, and it is certainly allowable on that account to compare a certain colour a^1 with a certain sound b^1 or a certain smell c^1: but still that which produces these comparable after-effects remains itself quite incomparable. And we must make the same reply to the physicist and the physiologist, when the processes account must have different organs, are themselves homogeneous and distinguished only by secondary sensations dependent upon the position, shape and action of their respective organs. But it is not the business of logic to decide this question: all we need do here is to warn the reader when he has a direct perception that two modes of consciousness *are* incomparable, never to allow this to be overborne by sophistic arguments based upon the similarity of their *antecedents* or *consequents*.

172. The other question, not as to our right to separate *A* and *B*, but as to our right to join together all that we comprehend under *A*, calls for a similar remark. For a long time people tried to dazzle the public with the stupid paradox that black and white were no colours because they did not like the prismatic colours depend upon a definite number of undulations of light. The progress made of late in the physiology of vision have completely cut away this ground; but even if this had not been done, no one could have had the right to override language in this fashion. Long before we knew anything about the exciting causes of our sensations, language had invented the name of colour for a group of sensations which by a homogeneous quality directly perceived and undeniable, viz. by *shining* or whatever else we like to call it, are at once bound together and separated from tones that *ring* or resound and scents that are *smelt*. Granted that the name shining is only appropriate to white and not to black, still the fact that the fundamental quality thus imperfectly designated is shared by both in common with the other colours admits only of a verbal not of a real denial, and the common usage of the term colour so as to include both was therefore completely justified against the unsupported objections of the savants.

accompanied by visible light and sensible heat, which permanently altered the constitution of a body: a glowing iron rod therefore was not said to burn, because no lasting alteration was found in it when cooled: but also such a permanent change would not have entitled he process which produced it to the name of burning, in the absence of the sensible development of flame and heat. The notion of burning then by no means coincides with that of oxidation: many substances are oxidized without burning, and on the other hand, when heated antimony is immersed in gaseous chlorine and combines with chlorine, throwing out flames the while, this process is undoubtedly one of burning though not oxidation. Geometers, again, knew ages ago that any system conceived in abstract terms, i.e. arithmetically, provided that not more than three scales be required for the arrangement of its various elements, may be presented to our perceptions by means of spatial constructions. Now there is nothing to prevent a mathematician from conceiving systems based upon any number of scales greater than three, only it is plain that such systems can no longer be envisaged in space, and that the name 'dimensions' which could be applied to the scales in its ordinary sense of dimensions of space so long as they were only three, can now bear only the more abstract sense which I tried to express by calling them scales. As space therefore means for us nothing but a system that we envisage in this peculiar way which certainly cannot be derived from any considerations of mere number, to continue to speak of a system of four or five dimensions as space is but to make sport of logical distinctions. Let us be on our guard against all such attempts: they are nothing but scientific freaks, which intimidate the popular consciousness by utterly useless paradoxes and make it doubt its well established rights in drawing the boundaries of its conceptions.

173. When we now ask how the several coherent members of one of those disparate kinds of content A B and C are related to one another, we find that these relations are peculiar and not always of the same kind. No one has yet succeeded in reducing the several kinds of taste to a satisfactory system: but the path which common usage takes in naming them, incomplete though that nomenclature be, seems to me the right path. Certain primary forms are distinguished by names of their own, as sweet μ, sour ν, bitter π, and the others such as sour-sweet $\nu\mu$, bitter-sweet $\mu\pi$, are regarded as com-here, i.e. that sour and sweet may be united in one impression in a manner that we can scarcely describe but easily feel, while sweet and red cannot, distinguishes the relation of the several tastes to one another from that of the disparate groups $A\ B\ C$.

It may be objected that in the sour-sweet the difference between the sour and the sweet is only present potentially not actually; that there may easily be a third impression ω, itself simple and in no way compound, yet forming a connecting link between μ and ν; and that this then, on account of its resemblance to both, is designated in speech by the two limits μ and ν between which it falls, without implying that it actually is a mixture of the two. This objection I should not consider sound unless there were present in ω besides that in which it resembles μ and ν an independent remainder that could not be accounted for by the combination of μ and ν; where this is not the case this third impression ω will not merely be called a mixture $\mu\,\nu$ by an arbitrary freak of fancy, but will in fact be that and nothing else. But the primary forms $\mu\,\nu\,\pi$ and all mixtures of these, though made one group by the fact that they all alike appeal to the sense of taste C, yet within those limits can only be regarded as *disparate* from one another. A man who had tasted nothing but sweet could never by any conceivable modification of the feeling it gave him discover the peculiar nature of sour or bitter that he had not yet experienced. There is then no transition from μ to ν or π through independent connecting links, but we must first know $\mu\,\nu$ and π and then get the intermediate links by various mixtures of these.

We find the same relations between colours, and I took occasion in an earlier passage[1] to justify the common usage of speech in always distinguishing a limited number of primary colours, and inserting the rest as mixed colours between them. It is of course over in the simple nature of red anything which could possibly be nodified, heightened, or cooled down in such a way as to lead him to magine what blue is : he would have to learn what blue is before he ould mix the two extremes together so as to arrive at the internediate violet. The modifications of which the several primary olours are capable must also be regarded in the same way. We ndoubtedly have the right to consider bright blue and dark blue s kinds of the same blue : but these kinds also are produced by the nixture of white or black with a pure blue that is always the same hough never visible in its purity. Only I would once more briefly emind the reader that all that I have hitherto said refers only to the ature of our sensations after they have arisen in our consciousness, nd has nothing to do with the physical or psychical conditions of he act of sensation.

174. With sounds the case is essentially different. After a comarison of several sounds we distinguish first of all three predicates. The peculiar tone of the instrument which is sounding, whatever he physical antecedents may be, is for our feeling a simple proerty which defies further analysis, more analogous to a taste than to nything else. However strongly we may be moved by the secondary ffects of this peculiar tone the essential nature of the note seems to s to be quite independent of this, and also of its second property, iz. its loudness or strength: we regard both only as ways of producing the same note, the distinctive nature of which lies in its pitch. But in this third aspect sounds do not like colours fall into a number f distinct stages, such that one can pass into another only by mixture, ut they rather form a continuous series, in which the difference etween two more distant members is only a multiplication of the difference between two adjacent members. It is impossible to make proportion in which red shall stand to blue as yellow to any fourth olour: but the difference between two notes can always be stated as together, but are on a footing of perfect equality, as original members of the series, with those members between which we place them.

And lastly the whole series is endless: it is not possible, in addition to the colours known by experience, to imagine a new colour of which we can have an idea though it happens that our eyes never saw it; the scale of sounds, on the contrary, may be continued *ad infinitum* because each is generated out of its predecessor by a heightening which is felt to be homogeneous. It is not unmeaning to talk of sounds higher or lower than any that can ever come within our experience, because we have here (what we could not have if we tried to imagine new colours) a distinct idea of the way in which these sounds would differentiate themselves *if* they were audible.

175. With some modifications, which I leave the reader to make, these remarks apply also to the series of our sensations of heat: but at the same time the latter exhibit a new feature. The living body's own need for warmth gives a peculiar significance to certain sections of the series; we distinguish cold, cool, lukewarm, warm, hot, and fancy that these terms have a definite meaning; but not only would it be impossible to draw a hard and fast line where cool ends for everybody and lukewarm begins, but even if we interrogate our own feelings merely we are obliged to confess that there must be a certain caprice in choosing the one name or the other. We may connect with this contrast of heat and cold, and of high and low sounds, a great number of other pairs of ideas, the content of which is not so directly derived from sensations, e.g. great and small, strong and weak, many and few, old and young, and many more of the same sort.

However decided a contrast is intended by the two terms of these taken from *a* there is left a positive remainder δ. And it is the same with the other examples: these adjectives are all derived not from the apprehension of one idea but from the comparison of several, and denote relations which have no fixed value or meaning apart from a second point of comparison. These adjectives therefore are indefinite in the positive; only their comparatives have an unambiguous meaning. Where the positive form is used in speech it means that the comparative term may be applied to the thing denoted when compared with an unexpressed standard, which either in the estimation of the speaker or in common opinion is the normal or usual state of the thing in question.

176. There is one more point to consider in connexion with sound and sensations of heat. Sounds being in themselves of perfectly equal value we have no inducement to select some few of them as fixed points and to give them prominence by naming them. But on aesthetic grounds we want to articulate the whole series. As the simple sensation of a note is undefinable we characterise it by stating the cause which will at any moment produce precisely that note, i. e. the frequency of the vibrations upon which it depends. But there is no reason for preferring one number to another, and as every member of the series may be defined with equal ease in the way named, the musical scale has in fact no absolute starting-point. It is true that other circumstances, viz. the harmonic relations of notes, which I must here pass over in spite of the interest which they have even for the logician, lead us to arrange the series in octaves; but even this arrangement has no fixed starting-point; we may begin at any height we please.

Our sensations of heat do not admit of such a simple definition by their causes; we are obliged to have recourse to the other observable effects of their unknown cause, viz. the expansion and contraction of bodies. To take the melting-point of ice as the point from which the degrees of temperature should be measured in an ascending and descending scale was to choose a quite arbitrary zero to reckon from, great as the difference between 0° or x and 6° (which is equal to $x + 6$ units).

The reader may see by this simple illustration that though a series or a complex system cannot be articulated and arranged in a regular order unless there be a corresponding regularity in its own relations, yet thought frequently has to take a quite arbitrary starting-point and an arbitrary standard in order to master and make use of this regularity; and that such an arbitrary arrangement, though admitted by the nature of the object and justified in its results, yet must not be looked upon as a property inherent in the object itself.

177. Practical life offers many illustrations of this remark. We here have to do with qualities which either attach to various persons and things in very varying degrees, or which in one and the same subject take successively a continuous series of values, from which proportionate effects are expected. But it is only in nature that effects vary continuously in accordance with the conditions: where the result does not follow till it is produced by human action, the exact observance of the desired proportion is generally prevented by the fact that the labour required would be out of all relation to the end in view. We have to content ourselves with breaking up the whole series of values into sections and acting as if the conditions were the same throughout each section, fixing the result at an average amount, which will be too great for the first and too small for the last members of that section of the series. Thus for the purposes of taxation we divide the series of properties, from absolute poverty up to the highest pitch of wealth that is likely to be found, into a number of classes; in calculating the premium to be paid on a life-insurance we reckon age by years or at lowest by some considerable fraction of a year; in calculating interest we keep to a day as an indivisible unit. Again it may happen that a quality gradually attains a certain pitch to the

But for all that the needs of social life require that a definite time be fixed; so the law has to fix it summarily, and attaches to the completion of certain days and hours the beginning of certain rights and duties, though no one supposes that the capacity and the obligation which were absent yesterday have actually sprung up in the course of the night. But though this proceeding is summary it is not without reason: the choice is limited to times which correspond without any appreciable difference in accuracy to the requirements of the situation; all that is arbitrary is the preference of one out of a number that would all do equally well.

There are other cases in which we are still further from finding any precise standard in the nature of that which has to be settled, and must look for it in the further ends whose attainment is to be facilitated by the settlement. Such are the fixed periods within which certain conditions must be satisfied in order to establish some legal claim or to avoid some legal obligation: though the outlines of these arrangements are determined by the object specified, their details aim at nothing but logical precision. This our ancestors effected by not measuring the more important periods by entire units of time of the larger kind, but adding to such units some fraction of them, some days to a week, some hours to a day; by these means they narrowed the period within which (to use a common but rather loose phrase) a man might have fancied that he was satisfying the law. The police again are quite right when in order to prevent disturbance of the peace they summarily fix the number of persons that shall be held to constitute a forbidden assembly at three or five, thereby barring disputes like those the old sophists used to raise when they asked how many grains of corn are required to make a heap, or how many hairs must be lost to make a bald-head.

ness and badness, and no one can persuade us that no acts are indifferent till he has artificially limited the conception of an act. But it really is no use to try to fend off the threatened confusion of good and bad by first dividing all actions into those which can be judged morally and those which cannot, and then proceeding confidently to divide the former into two absolutely opposed groups, the good and the bad. We thereby do but move our doubts a step further back; for the question now is where is the line to be drawn between that which calls for a moral judgment and that which does not; and this line as before will seem to vanish in a perpetual passing of the one into the other.

Again the relation of the pleasant to the beautiful and the good, though a less pressing question, is one of great interest on aesthetic grounds. To the man without a theory they seem to arrange themselves in an ascending series, not merely according to their value but according to the meaning of their content; not of course in the sense that by mere intensification what is extremely pleasant would become beautiful, or the highest beauty pass into the lowest grade of goodness, but in the sense that there are kinds of the pleasant, distinct in quality, which begin to have a right to the name of beautiful, and forms of beauty which produce an aesthetic impression akin to moral approbation. But those who theorise upon morality and upon art alike resist this admission; they deem the beautiful falsified if it has anything to do with the good, the good degraded if it has anything in common with the beautiful, and through this with the pleasant. Here too, with regard to beauty at least, people have been found to deny all differences of degree, and to maintain that what is beautiful at all is entirely beautiful, and that if you allow there is anything more beautiful, you cannot think this really beautiful at all.

179. Let us, in order to settle these doubts, look around for other particular line which is accurately known; however near it may come to a straight line it is yet quite undeniably curved, so long as the radius of curvature has any finite magnitude.

Again a curve may in one portion of its course be concave to an axis to which it is convex in a further portion; if it makes this change of direction in an uninterrupted sweep without any angle that breaks the continuity, there is no doubt that its tangent at the turning-point, and therefore the element of the line itself, is parallel to the axis in question, and so neither concave nor convex; but although both directions thus visibly meet in one zero point of indifference that belongs to neither, yet the opposition between them is thereby neither altered nor removed; on this side of that point the curve remains entirely concave, on that side entirely convex. Take a simple instance: between 1 and 2 we may insert countless fractions rising gradually in value from 1 to 2; between full daylight and midnight darkness countless degrees of illumination not only are conceivable but actually occur; between pleasure and pain there lies an uninterrupted series of feelings which connect the one with the other: but 1 does not on that account become equal to 2, nor do darkness and pain cease to form a perfect contrast to light and pleasure; and at the same time each member of these pairs, by itself and without reference to the other member, is something so definite that we never mistake the one for the other. These illustrations are sufficient to explain the statement that the existence of countless degrees through which two opposites A and B pass till they meet in a common zero-point of indifference, does not destroy the difference or opposition between the meanings of A and B themselves.

180. And so even if the moral philosophers had succeeded (and it is their business and not ours at present) in determining what they which the terms are applied partake more or less fully of the character of one or the other of these opposites. But the zero-point of indifference can still less contribute to the confusion of the two, for its meaning is not that both are true at this point, but that neither is true; it is therefore merely a point of separation,—on this side is only good, on that side only bad.

On the one hand then the maintenance of the distinction between good and bad is no reason why people should deny that there are degrees of good and bad; on the other hand we must insist upon an explicit admission of the fact that there are degrees. To deny it, to repeat the old Stoic paradox *omnia peccata esse aequalia*, or to go on preaching that even the smallest error is still not truth but error and nothing else, is but to waste time in tedious assertions which as they contain only half-truths may on this very principle be called errors and nothing else. It is not true that a curve is once for all a curve, so that the degree of its convexity or concavity is quite a secondary consideration, which has nothing to do with its character as a curve; the fact is that one curved line is actually more curved than another, and so realises more intensely the character common to both. Similarly the good or bad intention out of which an action springs can not only be measured in a secondary way by the importance of the interests affected by the act or of the circumstances under which it is done, but can itself be estimated according to its degree of goodness or badness; for such an intention is by no means a mere form which is alike in all cases; it is an inner process which not only must reach a certain degree of intensity in order to generate the impulse which every act requires or to overcome certain obstacles, but has also a certain degree of value according to the amount of the good or evil which it consciously aims at producing. Error again is not merely not-truth; that would not distinguish it from doubt; it is a departure from truth, and has therefore a measurable magnitude, indeed is inconceivable without it; a man whose thoughts are occupied with real problems therefore will not be so silly as to reject in identical terms, as mere errors, two assumptions of which the one is so far from the truth that it leads to no knowledge at all, and the other so near that it leads to nearly all the knowledge of the subject that can be expected.

181. It may be that the series of the pleasant, the beautiful, and the good (the further consideration of which I leave to the reader) has already suggested another relation that can exist between a series of conceptions, which I will first of all illustrate from geometry.

:he generating law of each of these solids, with reference to mon axis of all three and the position of the apex in that y be stated in a formula, which would have to be compared general conception of *A B* and *C* respectively. It would)ear that in the *A* series there is one member that also satis- ·equirements of *B*; and therefore that as to this member it is of doubt or of indifference whether it is to be classed under eption *A* or *B*,—not because it satisfies neither, but because :tly satisfies both at once. But with the exception of this .r case all the instances of *A*, all the other planes by which pound solid thus formed could be intersected, would belong :ly either to *A* or to *B*. The same would be true of the mmon to *B* and *C*.

:se cases then it is due to the very nature of the essentially conceptions that certain members of the series which they characterise become ambiguous, so that by themselves and taking count of some secondary point, such as the manner origin and development, it is not safe to ascribe them ex- to any one of these conceptions, though apart from these r cases there is no doubt at all about the difference of the ons. We have here named *A B C* and so expressed them as ons, leaving the particular cases unnamed. But the purposes h may sometimes suggest the opposite procedure. We may ıd fix certain conceptions *M N O* which have quite unam- and distinct meanings only in particular cases, which we may ɔ ourselves as salient points, as *maxima* or *minima*, in a con- :ries. We shall then find the reverse of what we found just comparison at once. With such a conception no doubt every instance agrees which in each of these respects is found to have the appropriate mark; but the applicability of the conception becomes doubtful in many other cases, which from one point of view would certainly be included under it, but from another which must also be considered would certainly not. Various thoughts thus cross one another in the conception of illness. Illness is certainly above all things a departure of the bodily condition from a supposed fixed standard. But a malformation, which departs considerably from the natural structure of the body, still cannot be called an illness, so long as it does not impair the vital functions, nor so long as it remains constant and runs no natural course through various stages. A wound always in some degree alters structure and function, and also runs a natural course; but a slight wound is not called an illness, plainly because it does not involve danger nor make the body unserviceable for any important purposes of life; but again a very severe wound is also not called an illness though it does both; its origin is too sudden and too entirely due to external violence,—and now we observe that when we spoke of illness, we thought of a state which, though dependent upon some external cause for its origin, yet takes its definite shape from the peculiar interaction of the internal forces. But now a cold is such a reaction of the internal forces against an external stimulus: but a cold is scarcely called an illness so long as the element of danger is absent: and just as we here help ourselves out with the milder phrase 'unwell,' so we use the term health with a certain latitude, allowing room for the slow advance of a number of disturbances connected with individual idiosyncrasies.

It is not difficult to say what is the right course here. It is impossible in such cases to find a definition which shall be in harmony at once with the requirements of science and with these strange caprices of language: if we want to determine the conception, we must disregard usage and fix it arbitrarily. In the instance we have chosen this is scarcely needed, for pathology gets on very well without any unimpeachable definition of the nature of illness in general; and the physician has absolutely no need for logical generalities which yield no guidance in practice.

But in other cases it is not so. In our conception of crime all sorts of considerations cross one another,—we consider whether it was deliberate or precipitate, what was the degree of evil intention, whether it was attempted only or perpetrated, what was the amount of harm done: the distinction between the creations of art and the products also, though we take count of common usage, it is yet necessary in the main to distinguish them by positive enactment.

183. Obviously we may set down any conception *M* as equivalent to any other conception *N* when we have by further specification so changed *N* that it is equal to *M*. Thus there arise a number of incidental aspects or variations of the expression for the same *M*, which we shall further on find to be of use in enabling *M* to be subsumed now under this law and now under that, such law leading to a new assertion about *M*. There is no limit to the extent to which this procedure may be legitimately carried so long as the transformed *M* really coincides with the original *M*, so long that is as *N* is equal to *M*. We may even bring a triangle *M* under the conception of a four-sided figure *N*, provided of course that we add that one of the four sides is reduced to nothing. This may seem mere trifling, but it is useful in practice: we can thus for instance easily picture to ourselves how every time that two sides of a polygon, which were before separated by an intervening side, are made to meet at their extremities by the vanishing of the intervening side, the sum of the angles of the polygon (in this case four-sided) is diminished by two right angles.

This use of transformation will engage our attention further on; what I wish here to emphasise is that the difference between the two conceptions thus brought together is of course not altered by it. The four-sided figure remains just as distinct from the triangle as it ever was, i. e. so distinct that it must be stripped of its very essence before it can be ranked with the other; and similarly the alterations, whatever they be, that must be made in order to turn *N* into *M*, give the measure of the *abiding* difference between the two conceptions. When we are dealing, not as in this case with abstract constructions of thought but with realities, which have an independent origin in the region of fact, such transformations have very little value; they are one chemical element we may in thought derive all the others by giving successively certain other values to the coefficients which the fundamental properties of matter take in the case of that one. But by such devices we cannot make the conceptions M and N approximate to one another, for their difference remains always as great as the number of steps that we must take to get from one to the other; neither can we thus establish between the actual things which exemplify these conceptions such a connexion that one might pass over into the other. For that it would be necessary to prove that the physical forces of the elements which build up an actual crystal of the form M are such as to make it possible for the same elements to be also in equilibrium when arranged in the form N; or that the concatenated system of forces which determines the structural type of the crocodile and maintains it in life may be so modified by other natural influences that the form of a bird may actually grow out of it,—that in short the order of nature actually contains impulses which realise the changes which we may choose arbitrarily to make in thought or upon paper. We cannot but remember, though happily as an error which we have outgrown, the wild caprice with which not long ago people would derive a word in one language from any casual word in another, and call it etymology; at the present day people need to be warned against proceeding in a similar way to satisfy the newly-awakened desire to conceive all the various kinds of organic beings as evolved from one other, all fixed specific differences being done away. But, whether Darwin has succeeded or not in his attempt, we must at any rate allow that he has taken the greatest pains to point out the real processes of nature by which the transformation of one organic form into another which we can conceive in thought may have been actually brought about.

CHAPTER III.

Schemes and Symbols.

1. In this chapter I shall continue to treat of the same subject
he foregoing, but from a somewhat altered point of view. The
and importance of the difference between several ideal con-
can, we ascertained, be precisely determined only when we find
lves able to compare several differences of the same kind, i. e.
the ideas to be compared themselves form series, whose mem-
proceed according to a law that can be more or less exactly
l, and when moreover from the nature of the feeling whose
ications, distinct both in quantity and quality, are represented
members of the series, such modification can only take place
e and the same direction. Compound conceptions whether of
or properties, situations or events, by reason of the number of
haracteristics or of the aspects which they include, may be
d in various directions; one or some or all of these characteristics
f these aspects may run through all the various phases of which
re capable; and again the bonds which connect them may pass
gh all the various degrees of laxity and strictness and all the
es of form to which they are by their nature liable.

w there is no reason why the value or the extent of the difference
en two such compound conceptions M and N should not fre-
ly be revealed to us by a direct impression with as much certainty
need require in the case in question: if however a more
ate determination were needed for scientific purposes, we should
first to determine the values of the various scales upon which
veral alterations take place, and thence to determine the value
total alteration which separates M from N or N from O. The
r may be inclined to object at once that in most cases at any

laid down so completely that the difference or the kinship between
y two conceptions M and N should be exactly indicated by their
osition in the universal scheme.

185. To illustrate this I will first go back to remote antiquity, to
ythagoras. To reconstruct a body of genuine Pythagorean philo-
phy out of the scanty and for the most part very questionable
aterials at our command is a task which I will not undertake, but I
ink I am able to state what *may* have been the fundamental idea
hich animated it, and which would enable us to understand why the
mpathy stirred by it has been so lasting though often so perversely
pressed. It is tolerably certain that the bent of the school was first
abstract mathematics, and secondly to their application to the pro-
sses of nature. The first line of study could not fail to lead them
picture the series of numbers and the world of shapes as two great
herent systems, and further to bring them to see how spatial figures
emselves depend upon the numerical magnitudes which they involve.
he second, besides other less known results, led to the discovery of
e relation between the pitch of a note and the length of the vibrating
ring, and thereby no doubt suggested the general idea that even
enomena whose differences are in the first instance felt by us as
fferences of quality are based upon mathematical differences that
lmit of comparison. The rash generalisation of results thus won is
hat the fancy of men is always prone to; the mathematically-trained
thagorean went so far as to make the reflexion that if it be once
tablished that a series of changes in phenomena corresponds to a
ries of changes in magnitude, then every other conceivable mathe-
atical relation along with all its modifications must have its counter-
rt in the phenomena,—or conversely, if a group of phenomena is
sed upon definite relations of magnitude, the coherence of all the
ocesses of nature necessitates the conclusion that all other pheno-

.d good for magnitudes in general: these data themselves, to which .thematics are only applied by modern 'mathematical physics,' were ,arded by Pythagoras as themselves forming a system whose inner iculation is based upon the same relations that determine the struc- e of the series of numbers and of all their possible combinations. ish to distinguish in this theory a general idea and the particular m given to it.

186. The so-called natural philosophy of the Ionians had devoted elf to describing the processes by which natural bodies were formed t of their primitive matter and returned to it again. As this philo- ohy very generally used for this purpose the ideas of condensation d rarefaction, it may appear, in virtue of its employment of quanti- ively determined means, to be closely akin to the Pythagorean ory. The two are nevertheless very far apart: for the Ionians ver betray any desire to show that the sum of that which is thus oduced at any moment of its existence or in the whole series of ps by which it comes into existence forms a coherent whole of utually dependent parts. Pythagoras on the other hand seems to ve troubled himself very little about this *origin* of the world, but the rld as it was after it had come into existence was to him a system, ch that not merely were its parts there, one beside the other, but ıt there would have been a gap in it if while one phenomenon were esent another had been absent. If *a* and *b* and *d* are present, then is there at all, it is not merely there along with the others, but it there because the law according to which the series *a b* advances to equires it as the third member of the series which is indispensable the presence of the fourth member *d*: or if *c* is absent, it is not erely absent as a matter of fact, but because the law which regulates series excludes the possibility of this third member before *d*. The ne consideration may be applied to other series in the actual rld, to $\alpha \beta \gamma \delta$ and to 𝔞 𝔟 𝔠 𝔡, and this application was made by the thagorean school.

How they conceived the relation between the different characters of these series, which I wished to indicate by the use of different alphabets, is a point upon which we are certainly in the dark, and upon which, as we may gather from Aristotle, the fullest information would probably throw but little light; but with respect to the law which in each of these series binds the homogeneous members together, it seems to be indubitable that it was regarded as precisely identical for all the series, i.e. that they maintained a complete parallelism between the relations prevailing in the various groups of connected phenomena. This is shown in the supposition that the earth has an invisible fellow, in order to bring the total of the then known planets up to ten, to which number the arithmetical mysticism of the system had once for all assigned a peculiar significance,—in the assumption of a fifth element, which together with water, earth, fire, and air, shall correspond to the five regular solids, tetrahedron, cube, octahedron, dodecahedron, eicosahedron,—in the attempt again to conceive the distances of the planets as arranged according to musical intervals,—and even in the meagre form of their tables of opposites. To us of course these tables do but illustrate the frequent occurrence of this relation of opposition between two conceptions even when these are arbitrarily chosen, but the fact that they always contain ten pairs seems to indicate that they were intended to represent this relation as essential for all the different stages in a series of ten members. Finally when they assigned life to the number six, intelligence and light to seven, and friendship to eight, we see that they regarded not merely the phenomena of nature, but also those of mind, and in a word every conceivable thing, as ordered according to the same serial law.

This philosophy then sought and fancied that it found precisely what we spoke of above, viz. a universal scheme which mounting from simple to complex was supposed to embrace the whole sum of possible forms, one of which was to serve as a pattern for the formation of every actual thing, while at the same time these forms or types were to be so arranged in the scheme that the position of its type directly determined the significance of every actual thing, and the amount of the difference or the kinship between it and other things formed upon the model of other members of the series. The general idea then that I would ascribe to the Pythagorean philosophy is this, viz. not merely a subsequent arrangement of things whose nature was originally settled without reference to the principle of this arrangement, but a harmony of the Cosmos—which name was first applied to the world by Pythagoras—based upon the notion that all things are from the beginning nothing but various realisations of a series of types, regulated by one law of development which is the same for all.

187. The general conception is undeniably grand, but grandeur is sadly lacking in the special form here given to it. Even in the present state of the mathematical sciences, various as are the magnitudes whose interesting mutual relations have been examined, it would be impossible to find adequate types or symbols or abstract expressions for the still more various relations that subsist between the elements of the actual world and the combinations that arise out of them; but the arithmetic of the ancients, which the Pythagorean school seems to have helped to develop, furnished in its then state but very few and very meagre numerical relations, whose significance must have been much exaggerated and from the beginning very arbitrarily interpreted before they could be regarded as the relations upon which the structure of the world is based. The grounds on which they justified their well-known veneration for the number ten,—viz. the fact that all numbers are generated by the repetition of unity; that in this series the even numbers alternate with the odd numbers, which cannot be divided by 'the principle of multiplicity,' i. e. by two, and which are therefore held to be of higher rank; that three is the first union of odd and even, four the first square of a multiple number, and ten the sum of these exalted four first numbers,—are grounds which could not be admitted except by a system of symbolism which was ready to accept any *interesting* motive without regard to its connexion with others: though the real grounds of that veneration undoubtedly lay in the habitual use of the decimal system. If these thinkers had been acquainted with all the algebraical and transcendent forms of functions which are the instruments of modern mathematicians, how much more various would have been the symbols employed, and how much more delicately would they have been adapted to the nature of the several phenomena! The same tendency still survives in us: even in cases where calculation in the strict sense is impossible we are inclined to use the term 'power'[1] when the meaning and importance of a conception is raised in some peculiar manner, as for instance when each of the centres of relation, whose determination by each other constitutes the meaning of the conception, is itself exalted into a small system, whose members determine each other in the same way.

We can imagine then how the Pythagoreans (if they had had our knowledge) might have illustrated many relations of dependence command, and as even these would still be insufficient, it would be quite useless to examine in detail the reasonableness of the Pythagorean symbols.

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[1] [In the mathematical sense.]

188. That it was the fate of the whole theory to be variously interpreted and misunderstood is easily explained by its nature. According to one statement of Aristotle it was the *principles* of numbers that Pythagoras identified with the *principles* of things. This seems quite intelligible. By these principles of numbers must be meant the relations between one and the other numbers, the way in which one can be repeated, the divisibility or indivisibility of the rest,—in a word the possibility of generating the whole series of numbers by the use of these constant relations and operations, or, as we should say, the possibility of exhibiting every number as a function of other numbers. Things then, ought also to have the same inner structure, their series ought also to be arranged according to the same principles, so that the nature of the one might be exhibited as a function of the nature of the other.

But it is also asserted by Aristotle along with others that the Pythagorean school declared that numbers were things, or at any rate that things were numbers. Even this is quite intelligible to any one who is acquainted with the history of philosophic ideas and the customary ways of expressing them. To a certain extent indeed the Pythagoreans would have been right in making this assertion, and this justifies us in supposing that they actually made it; for as already said what they intended was by no means merely to apply numbers to the quantitative determinations of things whose real nature is independent of these determinations,—e.g. you may have similar triangles of very various sizes: their numbers were meant to signify that which distinguishes the essential character of one thing from the essential character of another; a was a because its content was constructed according to α the function-form or the generating law of one symbolic number, and was thereby distinguished from b which was b because it followed β the generating law of another symbolic number. It was quite possible then to say, with a reservation to be presently noticed, that the essence of a thing, in the sense of that which distinguishes it from another thing, lies in the number immanent in it.

The other assertion that the essence of things, in the sense of that in virtue of which they all are things, or their reality, consists in these numbers, or that numbers are the real things, was perhaps not positively made by the Pythagoreans in this form : if they did make it, they certainly could not justify the latter expression, but they could assuredly justify the former: for if there is actually nothing whose nature is not determined by one of these symbolic numbers, the numbers are assuredly the *conditio sine qua non* of every reality; to treat them as more than this, and to speak of the numbers themselves as the real things, is an unwarrantable straining of language, though we shall presently see how prone to it the thinkers of all ages have been.

There remains one great imperfection which we have already mentioned. The same typical series of numbers has to repeat itself in a number of parallel series of actual things, in *a b c d*, *α β γ δ*, 𝔞 𝔟 𝔠 𝔡 ; how then are the members *b β* 𝔟 distinguished from one another if the whole nature of each of them is exhausted by the same symbolic number? To this there is no answer possible: at this point the theory, which aimed at embracing the nature of things completely, relapses again into a mere application of a general law of structure to various cases whose characteristic differences must be regarded as given. But this is what makes it serviceable for our present purpose as an illustration ; it thereby becomes an attempt to frame a universal scheme for the relations of kinship and difference between all the groups formed by kinds of content that can ever by any possibility come to be considered.

189. In order to justify the length of this discussion I would point to the extraordinary tenacity with which this desire to find a scheme for the whole contents of thought has maintained itself through the course of ages. It showed itself first in this form of mystical speculations about numbers ; over these we may pass very lightly; as such speculators were satisfied with anything however meaningless so long as it was interesting and startling, they were, to speak plainly, always in search of a secret truth which they never found, and it must always have needed a very sympathetic hearer to find in the symbols a better expression for the meaning put into them than could have been obtained without them.

Presently the speculators ceased to found their dreams on this purely arithmetical basis and wandered away in various directions. In the first place every discovery made by advancing science that has any important bearing upon the relations of things has almost without exception been extended into a scheme for the articulation of the whole world. For a long time people traced everywhere the behaviour of the four elements of the ancients; and in later days the mystic significance of this number four did not pass away, it was only transferred to the newly discovered constituents of organised bodies, carbon, hydrogen, oxygen, and nitrogen; it agreed admirably with the four quarters of heaven, for zenith and nadir of course fall outside our natural line of sight; it agreed equally well with the four seasons of the temperate zones, within which these speculations were carried on, and with the four indispensable cases of nouns; at a later date, as the theory of astronomy came to completion, the contrast between centrifugal and centripetal tendencies entered into men's notions of all things and was fused into one with the opposition of the sexes and the relation of acid to alkali; the discovery of magnetism and electricity caused the scheme of polarity to be carried even further if possible into the consideration of all conceivable things.

Other speculators proceeded in the opposite direction, starting from the just reflexion that even the relations of numbers are, in part at least, only instances of other still more abstract fundamental relations; these then (they hold) must be sought, and will be found if we simply reflect upon the operations by which our intellect does in fact arrive at its ideas of all things whatever. Now every idea, or at least every compound idea, is made by setting down an a, distinguishing from it or opposing to it a b, and finally bringing both into a relation c; thus thesis, antithesis, and synthesis come to be regarded as the scheme upon which all reality is constructed and as the rhythm which thought must maintain in the orderly consideration of that reality. But it is easy to see that the more abstractly these symbols are conceived the more they pass over into *notiones communes* which do indeed apply pretty well to everything but give us no adequate knowledge about anything. Logic then meets all this wild talk with the demand that things be considered, divided, and investigated simply and solely with reference to their several natures, for there is no universal scheme that can be applied, and the employment of merely fanciful models can only injure the impartial quest of truth.

190. Of this unfavourable verdict I can abate nothing, and in some remarks which I wish still to add I have no such intention. When the content M of a conception, an idea, or a perception is given to us in such a manner as to unite in the form μ a number of characteristics, or parts, or points of relation, it is a quite justifiable scientific curiosity that prompts us to enquire how the examples of

' simply changing the quantity of the characteristics, for these will, in most cases at least, resemble one other and only repeat ne thing on a different scale. But if one of these characteristics of such a nature that for it the opposition of negative and e has a plain and palpable meaning (such an opposition for :e as there is between right and left, attraction and repulsion, 'e and convex, and generally between ascent above a zero- ind descent below it) then it concerns us greatly to know what is to M when we substitute $-m$ for $+m$ in its generating law. sing $y = fx$ is the equation of a curve, we always take the : to set down in turn the positive and negative values of x, ot till we have united the results thus obtained do we think we rrived at the nature of the curve, which in this case presents o our perception not as a mere generality, but as the *whole* is got by combining every possible example of the general on. If we happen to see, in a piece of ornamentation, a volute bends downwards to the right, our imagination is stimulated imilar way; even if we have no mathematical knowledge of the ting law of this curve, we understand, by reason of the homo- isness of directions in space, that the volute might be repeated ecisely similar though opposite bend upwards to the right, and with another opposition upwards to the left and downwards to t. If now these continuations, suggested by the beginning we see, are not carried out, though the surroundings do not ny obvious reason for this incompleteness, our aesthetic feelings satisfied, but this demand for symmetry has also a logical foun-

It is of the very essence of a law that it shall apply to all ons of the points of relation which it comprehends; there is re a contradiction in a perception which suggests a law the tripartite division thus suggested becomes for us a *scheme*, which we take as the basis of our investigation of the whole extent of M. This is the point which I wish here to emphasize, in order to mark the difference between this proceeding and the wild dreams we have just condemned,—viz. that this scheme can be nothing but *an invitation to turn our enquiry in a particular direction*, and cannot give us by anticipation a picture of the result at which we shall arrive. It does not always happen, as in the case of the volute, that the counterparts we expect can be found: whether the change from $+m$ to $-m$ gives other possible kinds of M at all depends upon the nature of the form of union μ. Still less can we see beforehand whether the kinds thus obtained will be in any way proportional to the differences of the conditions, and if so in what way: it is quite possible that for a certain μ this absolute opposition of $+m$ and $-m$ is absolutely meaningless. Our method then will be to let μ likewise pass through all the possible forms given by the various alternatives; here also for mere additions of quantity we shall expect only a series of similar results, but for every cardinal point at which μ takes a qualitatively different significance or passes at a bound into its opposite we shall expect a quite new formation to appear in M which depends upon μ; and lastly for every remarkable feature which we find in a special case of M we shall expect to find as counterpart an equally remarkable feature in a similarly conditioned special case of a similarly constructed N (as for instance when we find that waves of light behave in a certain way we look for corresponding behaviour in the waves of sound): but all this remains only a *question* put to the object, to which we await the answer: the answer which enquiry yields may turn out quite contrary to what we expect, but must be accepted whatever it be. Where those dreamers deceived themselves was in supposing that whenever their scheme which they assumed to be universal was applied to any matter whatsoever, every place in it would always be filled by some remarkable form of that matter, none would ever remain empty, and further in supposing that as these various matters, passing through the same sequence of changes, filled up the several places of the scheme, the forms which filled the same places would by a striking resemblance or analogy in their whole character announce themselves as connected, as akin to or as counterparts of one another. When this was not the case, there was a strong temptation to try to fill up the gaps by groundless suppositions, and to restore the desired symmetry in the corresponding members by giving undue prominence to secondary features.

here with making a few remarks on its logical character. The Pythagoreans in conceiving development in countless parallel series with different contents took no count of the differences by which the correponding members of the various series are separated from one another in spite of their occupying the same place in the general scheme. The decimal system, with its ascending powers of the number ten, never led them, as it might well have done, to treat these parallel series as themselves successive periods of one and the same main series, esembling one another in their internal structure, but raised one above the other so to speak by the height of the level at which they exhibit this structure, like the octaves in the musical scale.

The imagination of the modern philosopher has supplied this deficiency; the many parallel series are contracted into a single eries, composed of cycles of similar structure, the last member of each cycle making a starting-point of a distinctively new character for the development of the next. If it is possible to find the first member of the whole series and the law which determines the form of the first cycle, the variety of the contents which form the members of the following periods may be explained by their distance from the tarting-point and the transformation which the initial member has undergone at each step of the way. Hegel then requires us to concede as a metaphysical presupposition, of whose correctness logic cannot judge, that the world is no sum of things that stand and events that go on one beside the other, the former standing quiet till hey are stirred to change by a stimulus from without, the latter determined in their inter-action and in their whole course by universal aws that hold good always,—but that instead of this all the variety of the world is only the development of a unity that never rests, all events only stages in this development or secondary effects of it, and hings themselves but appearances, either transitory or begotten anew at every moment, whose whole being lies in the active movements of hat unity, crossing each other and coming to a focus in them as long exposition and quite impossible in a short statement; but what has been said is enough to enable us to understand that within each dialectic cycle these different forms, whose significance somehow constantly increases, cannot simply occur one beside the other, but that each must issue out of the preceding one: development, in short, is the very essence of the system.

192. Now no development is imaginable without a definite direction which it takes in contrast to others which it does not take; but it is equally clear that in this case above all others it is impossible for the unity which develops itself to receive this direction from without; it must be determined by the nature of that unity itself. But here we find that no accurate and exhaustive expression can be obtained for the entire nature of that which under the name of the absolute is regarded as the one basis of the world, but that what we mean by it in a sort of presentiment is fully revealed to us, nay comes to be completely itself only in and through the development,—indeed, the very name indicates this, for as it is nothing but development, it cannot be itself before it has begun to develop.

The only point of departure then that is left for us is this fact itself, i.e. the knowledge that the absolute is not rest but development. Assuredly then its development must take that direction and form which follows from the conception of development itself, and which therefore must recur in every example of the conception. This opens up a very simple line of thought. If any *A* is to develop itself, it cannot already be that into which it has yet to expand itself; neither can it not be, or be void of content, for then it would not be the determining ground of that which is to be; as yet unexpanded and shapeless it must still be the determinate possibility of its future growth,—in a word it must be '*in itself*[1]' or potentially that which it is to become. But its nature would not consist in development if it were to abide in this potential state; it must actually become that which it is its nature to be able to become. But becoming or the process of development is only an intermediate step between possibility and fulfilment; as merely coming to be, hovering between starting-point and goal, that which is developing itself would be neither identical with itself as it was in its potentiality, nor yet already that which it has to become. This at once enables us to see why the second stage of the development, in which that from which we started is as it were divided against itself, was called by Hegel 'other being' or 'being otherwise[2]'; we see it still more clearly when we remember that it is into a unity in his thought. But that which is developing itself must be this unity not only for others but for itself, if it is actually to become that which it was its nature to become; and thus the name of 'being for self[1]' is given to this third stage of the cycle, signifying the completion of becoming, the attainment of the end of development, the return of the potentiality into itself. This return of course is not a simple return; i.e. we do not mean that the intermediate stage of the process is set aside[2] without leaving any result behind or wiped clean out; it must be set aside in the sense of being stored up and preserved; the last stage, being for self, is richer than the first, the potentiality, by the history of the process through which it has come into being.

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[1] ['An sich.']

[2] ['Anderssein.']

It is easy to find images for this; thus the octave of the initial note is a return of the latter into itself, and yet preserves in its heightened pitch the result of the intervals through which it has passed; thus when a mind, in which universal truths were innate in the form of methods which its thought instinctively followed, had, by passing through various experiences and enquiries, involving doubt and the removal of doubt, arrived at a full consciousness of these truths, it would merely have returned to itself and yet would be enriched. I will forbear however to explain in detail the peculiar meaning of these phrases; for us it is enough that in the third stage of the development something is given which is indeed a consequence of the first stage, yet is not identical with it but opposed to it as actuality to possibility.

Thus understood the three moments or stages of 'being in itself,' 'other-being,' and 'being for itself,' are but the component parts of the conception of development, and we shall be able to recognise rhythmic order, each founded upon and issuing out of that which goes before, and that accordingly the sum of all that is intelligible and all that is real would present itself to us if we knew it completely as a great series, whose several periods are similarly constructed but have each a peculiar significance in its content which is ever rising higher and higher. Upon this conviction we do not here intend to pronounce any judgment; but it remains for us to ask what is the logical value of the dialectic method just described.

193. It is easy to see that it is not strictly speaking a method in the sense of a direction how to find something that we are in search of; it is rather a *scheme*, in the sense in which we have used the word above, which only invites us to enquire if anything is to be found in a given direction or in a spot already marked out, and if so what it is, though of course it implies a confident expectation that the search can never be in vain. If we try to apply this scheme to the independent treatment of a generic conception M, in order to arrange its various species in a series corresponding to their essential resemblances and differences, or if we try by means of it to exhibit in their true relations to one another a series of conceptions which are connected by a variety of other circumstances (as e. g. right, wrong, crime, and punishment are connected), we at once find how uncertain it leaves us as to the direction in which our thoughts are to be turned. It is possible that this uncertainty might vanish if we could appeal to a complete philosophy which had already set down in a universal series the history of the development of all that is thinkable, and had therefore arrived at a conception of right so perfect as to reveal at once the direction of its further dialectical development. But to say this would be to deny from the beginning the applicability of the method as a universal direction for the discovery of truth; it can prove itself such only by this independent service which we require; i. e. it must be able merely by means of its form of procedure to teach us how to develop any given conception in all its proper consequences.

Suppose then that we have given us the general conception of right, for evidently the other three that we named refer to this as a primary conception already fixed: what now is it 'in itself' or potentially? into what 'other-being' does it pass over? into what 'being for self' does it return? It is at any rate evident that a right involves an estimate of relations which prevail between the claims of various persons to exercise their wills upon some object which brings them into collision. It follows that there can be no right if there be no e same world. Right then is only potentially right and not yet that hich according to its conception it is to be, so long as it only enotes by anticipation the approval or disapproval of relations which o not yet exist.

Its 'other-being' is also quite intelligible; it all comes to the imple truth that general conceptions mean nothing when there are o particulars for them to connect; the 'other-being' of right consists in the various rights whose conditions lie in the existence of this ature, of these human personalities with these definite wants and laims; after the general doctrine which sets forth the conception of ight will come the special doctrine which contains its applications. his direction is so simple that we do not need to wait for the ialectic method to teach it to us; but that method does not help s in the least to carry it out; for after all experience alone can teach s what conditions do in fact exist which give occasion for the evelopment of the general idea of right into special forms of right.

194. There is, however, yet another kind of advance that we can onceive. 'Other-being' certainly does often mean the passing of e universal into its various particular forms; but I have already emarked that the Hegelian doctrine lays stress upon the relation of pposition which prevails between the two members, including the pposition of the universal to the particular: this idea of opposition, niversalized and carried to its extreme pitch in the conception of ontradiction, gives a further meaning to 'other-being,'—it may stand r the simple contrary of that which the first (the being in itself) tands for. In pursuance of this train of thought, right was made to ass into wrong; and wrong was made to issue in punishment, not ndeed as the 'being-for itself,' but as the means of reasserting the iolated right by the negation of its 'other-being,' i. e. of the crime.

Now here again we have nothing that would not be just as clear y itself without all this apparatus of the dialectic method; and rther, the method is actually confusing. Any unprejudiced person ould say to himself on reflexion that all right has living reality only the dialectic method gives there is none of this cautious bridging of the gap between the two conceptions; it is represented as part of the very conception of right that it shall pass over into wrong, and the paradox is not to be justified by a plea which will be presently considered.

The transition to punishment as the third stage offends us less merely because we supply the motives which are in truth not given at all by the method itself. The method does indeed demand restoration of the right, and that by negating its negation the wrong; but it does not tell us by what procedure this task, stated abstractly as the negation of the wrong, is to be carried out. Why should it take the shape of punishment? The evil disposition out of which the wrong sprang is equally negated by disapproval and by improvement, the harm done by payment of damages, the violation of the dignity of the law by repentance, and by a fresh recognition of its bindingness. All these considerations show that the dialectic method was of no use here except as a scheme, with places marked out which we might seek to fill, but that, though we were tolerably successful in filling them, the content with which they were to be filled was only to be got from a quite independent examination of the peculiar nature of the object in question.

195. We said that it seemed to us absurd to maintain that it is part of the very conception of right to pass over into wrong; but this *swinging round of a conception into its opposite* has been so often and so emphatically claimed as a higher truth discovered by dialectic, that it is worth while to return to the point. Hegel remarks[1] that at first of course the understanding fancies it can apprehend the nature and truth of the real world by a number of fixed conceptions complete in themselves and exclusive of each other; but that the truth is that different conceptions do not simply stand one beside the other with equal claims to represent the finite, but that the finite of its own nature does away with itself, and passes over of itself into its contrary. Thus we say that man is mortal, regarding death as something whose ground lies merely in external circumstances; and according to this view man would have two distinct properties, that of living and that of being mortal also. But, according to Hegel, the true way of regarding the matter is that life as such contains the germ of death, and that in a word the finite in itself contradicts, and thereby does away with itself.

Here we can detect, more readily than we can in some of the other passages in which Hegel treats of dialectic, a confusion between two different statements. It is to the conceptions by which we try to apprehend reality that fixity and completeness are attributed in the first sentence: it is not the conceptions but the finite thing to which we apply them that is said to pass over into its contrary,—and in this latter statement lies all the truth that the passage contains, which truth is shown by what follows to have been uttered unintentionally or even contrary to the intention of the author. For when the finite as such does away with itself, it does so not because the general conceptions which apply to it have lost their definiteness and swung round into their contraries, but because it, the thing to which those conceptions are applied, as finite or as actual, is unable permanently to fulfil what is required of it by these conceptions, though each of them is true of it at one moment; through a defect in its nature it passes out of the province of one unchanged conception into the province of another which is equally unchanged. But the conceptions themselves do not alter their eternal meaning because it is only for one moment perhaps that they are a correct measure of the changeable objects to which they are applied.

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[1] [Vol. VI. of his collected works, p. 152 f.]

The true view of the matter then cannot be that life as such bears in it the germ of death, and that the finite in general contradicts itself: it is rather the two parts of this statement that contradict each other. Life as such does not die, and the general conception of life obliges the living thing to live, not to die; it is only the finite, mentioned in the second part of the statement, i.e. only particular living bodies that carry in them the germ of death. And even they do so not in virtue of the idea of life which is realised in them, but assuredly only by force of external circumstances, i.e. only because that combination of material elements through which alone life is manifested on the surface of this earth is unable to exhibit an undying example of life, though that would in no way contradict the idea of life,—whether this inability be regarded merely as a result of the laws of nature which are here in operation, or as part of a universal plan.

Similarly right never itself passes over into wrong, but sometimes the will of a living person which ought to embody it may, through want of judgment or through the impulse of passion, be led into wrong while striving to do right, and sometimes the law, which, men being what they are, could not be administered at all if it allowed exceptions, may do a wrong in a particular case involving complications for which no provision has been made.

Logic then can in no way accept this doctrine that conceptions with itself, yet does of its own nature pass from the province o conception into that of another; and the fact that we find it worth notice, as a fact about things that is to say, not as a pecul of the intellectual tools by which we come to know them.

196. In any case, even apart from all the objections here r the dialectic method would in the end give us only an arrangeme our conceptions, — an arrangement which might no doubt pr various points of interest to persons fond of reflecting and compa and might move their feelings by the discovery of analogies, para and contrasts, but which would scarcely open up a new way of k ledge that could lead to definite new judgments or propositions, a better and more precise settlement of questions hitherto dou To supply this want which the dialectic method fails to supply is cisely the aim of other vast attempts, viz. the attempts to fou logical language, a *universal mode of characterising conception* a philosophical calculus, at which Leibnitz laboured so long. mere addition of a series of large numbers would be an endless if we were obliged to have a distinct image of each one of thousands or hundreds of units composing them, and to build up of these numbers separately and at last their sum by repea adding unit to unit. But our system of ciphering enables us, wit the need of distinctly forming even any collective idea of the num to set units under units, tens under tens, hundreds under hund and then, by adding up each of these simple columns, unerring bring out a result which itself in turn we are quite unable to repr adequately in a single picture by any effort of our imagination.

Now our conceptions so far resemble numbers that they also tain for the most part a variety of individual images, whose u with each other is not distinctly before us at every moment, but thought of in one collective impression; but they are denote words far less perfectly than numbers are by figures. By the u words that are akin (though we are often no longer conscious o fact) speech does indicate the kinship between contents, but imperfectly, for kindred ideas are also denoted by independent r the kind of kinship between them is no less imperfectly expre for the small variety of ways in which derivatives may be form quite inadequate to the manifold relations that have to be indic moreover, instances of each relation occur which, as the first to the fancy of the framers of language, are denoted by simple wor o the meaning of the whole and apply ourselves to combine some of the component ideas with the same certainty of arriving at new and correct results that our system of ciphering gives us in numerical calculations.

These defects of language then we are called upon to try to amend; we are to dissect all our conceptions till we have found the simple primitive ideas of various kinds which admit no further analysis and the simplest ways in which they can be combined, and we are to characterise these by fixed signs, in order to obtain by their combination a symbol for each conception which shall adequately express its content. We need not think that the object of this underaking is the formation of a new speakable language, which could never supplant the national and historical forms of speech: its result would be a collection of formulae for the purposes of scientific thinkng only, to which recourse might always be had for the settlement of the doubts which arise from the employment of ambiguous expressions: for Leibnitz flatters himself that if we once got such an nstrument disputants would always cut their quarrel short by an amicable agreement—'Let us reckon it out.'

197. This is no doubt one of those enterprises whose execution alone can finally decide whether they are practicable; it would be over-hasty to deny the possibility of that which might after all perhaps be realised by a happy invention. However, the utter want of success hitherto makes the inherent difficulties of the task more evident for the present than the possibility of overcoming them. If all we had to do were to make a system of signs for marking the contents of our conceptions, the problem might appear difficult but not insoluble. For then we should probably begin by passing over all the generic conceptions of natural history and limit ourselves to those conceptions whose union in thought leads to difficulties which impede science or the practical deliberations of life. Nevertheless but as the functions and equations thus obtained grow more and more complex, we see more and more plainly even here a sort of deterioration in their employment. In the place of denominations which really exhibit the inner structure of the magnitude in question so as to indicate quite plainly how they are to be treated in the calculation, we find introduced in order to secure the necessary conciseness arbitrary symbols which no longer have this property, but resemble the words of ordinary language whose meaning must be known quite independently of their sound. The expression $\sqrt{-1}$ still expresses the origin of the function for which it stands, and from this we can determine by general rules what results when we multiply it once or several times by itself: but this expression has already been discarded as too lengthy and replaced by the other expression i which as it stands gives no clue to its signification, and whose meaning must be otherwise already known if it is to be used correctly. When we go on to speak of B-functions and Γ-functions, these expressions are certainly concise, but we can only understand them by representing them as equivalent to other lengthy formulae, which in turn are only made intelligible by a previous explanation of the meaning to be attached to the general signs of magnitude and symbols of combination employed in them. All this is no reproach to mathematics, nor is it any proof of the impossibility of a universal system for characterising conceptions; it only shows that any formulae that the latter could give us would not by themselves tell us all we need know, but would presuppose a great deal which we should have to learn before we could even understand them.

The symbols of chemistry make this still plainer: as yet they refer only to the quantitative relations of the combining elements, and to some extent to the supposed form of their union; what letters are to stand for the several elements, and how their sequence is to denote the arrangement of those elements, we must of course learn or know by heart, as both can only be determined by convention: but no one can tell merely by looking at the formula thus constructed whether it stands for a gas or a fluid or a solid body, nor what its density is or its specific gravity, nor what its colour may be, whether it is fixed or volatile, soluble in water or insoluble. If a man after looking at the formula answers these questions correctly he does so upon the basis of analogies with which his experience supplies him, and which he could not draw from the formulae themselves with any certainty that they would be correct. And yet all that is wanted here would be the determination of properties or modes of relation, which though not to the results thus obtained issue in these elegant and certain methods of solving problems. It is these rules that we must feel the want of when we try to combine conceptions which denote something more than magnitudes so as to produce a certain result; and I believe that we have absolutely no reason to flatter ourselves with the hope that these rules would of themselves suddenly become perfectly clear so soon as we had analysed into their ultimate constituents the essences, contents, and matter to which they were to be applied. Assuredly there is no need to insist on the fact that increased clearness in the objects cannot but have a favourable effect on the certainty of our conclusions regarding them; but in the main it is not by analysing our conceptions and tracing them back to primary conceptions, but by dissecting our *judgments* and tracing them back to simple *principles* that we must hope gradually to fix our convictions which on so many points are still in flux.

But there are two things which we shall require to know: first what are the necessary consequences which follow from certain definite relations which, as we either arbitrarily assume or are forced to believe, hold between the contents of various conceptions; and secondly what general laws, not proved to be necessary but found to hold good in fact, connect various ideas in such a way that our reason, founding upon these laws, can deduce the consequences that will then necessarily follow from given conditions. These problems, which concern the application of the form of judgment, we must for the present attempt to solve without the valuable assistance which that universal system of signs would no doubt afford if it were once completed.

Note on the Logical Calculus.

THE idea of a logical calculus has been often taken up and often abandoned: but the Englishman Boole has recently made an elaborate and careful attempt to carry it out, which is beginning to attract attention in Germany as well as in his own country. Though I freely admit that the author's ingenuity makes his able work[1] very charming, I am unable to convince myself that this calculus will help us to solve problems which defy the ordinary methods of logic.

Boole does indeed insist that the result of a calculation when completed must be expressible in logical terms; but he holds that between the statement of the problem and its solution a course of operations may be introduced whose several steps allow of no logical interpretation; and he appeals to the extension of mathematics by the introduction of imaginary quantities. This appeal is hardly relevant. The mathematician could not avoid imaginary formulae: he lit upon them in the course of well-founded calculations: he has always sought for the interpretation of the enigmatic expression and has actually found it in the province of geometry. In the logical calculus on the contrary this working in the dark to which recourse is had from time to time would have to take place by means of symbols which have been arbitrarily chosen to denote logical elements and the relations of these elements. If therefore a calculation is really of use only when it allows us to solve single problems mechanically, without requiring us to be conscious at every moment of the logical meaning of what had taken place, it becomes all the more necessary that the rules which make such labour-saving processes possible should be determined upon purely logical principles without any rash and misty analogy from the province of mathematics. Though on this point I entirely agree with the admirable exposition of Schröder[2], yet I cannot entirely follow him: his demonstrations, which after the manner employed by mathematicians follow upon the statement of the theorems to be proved, have in my opinion no significance beyond that of establishing that the whole calculus is consistent with itself and that all the transformations and combinations of its elements which it allows lead to the same results when applied to the same arks which combine in various ways to form the different species at fall under a concept. Supposing that the three marks $A B C$ elonged to the general notion M, the principle of disjunction would irect us to reduce each of them to its subdivisions $a_1 a_2 a_3 \ldots$, $b_1 b_2 b_3 \ldots$; e complete set of triplets of the form $a b c$, of course not counting epetitions or permutations, would represent all the kinds of M, which, iling any closer determinations, may be regarded as equally possible. hese groups obtained by combination express *per se* merely the imultaneous presence of their elements; they leave the nature of he connexion between the latter undetermined in two respects.

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[1] ['An Investigation of the Laws of Thought,' London, 1854.]

[2] ['Der Operationskreis des Logikcalcüls,' Leipzig, 1877.]

First of all they do not assign the final form which is to be the esult of the completed combination. Where logical classification s aimed at this want is supplied by the image which is retained in hought of the abstract M, of which the kinds are in question; this M is to be added in imagination to each combination $a b c$, as the eneral outline which the union of the elements is to fill in; apart rom such an occasion for the procedure by combinations, $a b c$ aken by itself only designates any object of thought, no matter how onstituted, in which the marks a, b, and c are found together, or what s more important, any case, which it has not yet been possible to haracterise more closely, in which the *conditions* a, b, and c are found ogether. This uncertainty does not exist in mathematics, for the orm which the result of the calculation is finally to take, is here ompletely and solely determined by the definitely assignable nature f the connexion which this science requires to be introduced between ts elements.

Now with regard to this second point also, the reciprocal determina-ion of their component parts, the formulae employed in the commultiplier, which must be thought of to begin with as a whole number, leaves the value of the multiplicand as a separate number unaffected and only repeats it several times over; while every mark *c*, which is annexed to a combination *a b*, not only modifies the reciprocal determination of these original elements, but at the same time by adding to the matter of the thought limits the extent of its application. Anyone who cared to dispute the question might perhaps find it easy even on this point to make more of the analogies between the two sets of relations than of their differences. But it is an essential fact for our purpose that while multiplication is forced to retain both the recurrences *a a*, *b b*, and the permutations *a b*, *b a*, as indispensable components of its product, logic can admit no meaning in the former and no distinction between the latter. Thus the nature of the case presented no occasion for departing from the neutral significance of combination-formulae which can have many kinds of meanings, and applying to them the mechanism of calculation, which has strictly speaking no suitability to them except as symbols of quantities that can be multiplied. It could only be ventured on in the hope that the more extended application of the calculus would compensate, by results which no other means could attain, for a cumbrousness inevitable at the outset, seeing that exceptional rules were necessary to bring such an inappropriate mode of calculation into harmony with its logical object-matter.

Every A, according to the law of Identity, must $= A$. Natural thought has no motive to determine such an A over again by a characteristic A, in the same way in which A would be determined by a second mark b. No doubt we speak of a human being as truly human, or emphatically of a man who is indeed a man; but we only employ such expressions where it is permissible to distinguish the conception M of an ideal from the conception μ of the particular facts from which the realisation of the ideal is expected. At bottom, therefore, we are not determining a single M by itself. The human being $M\mu$ that is thus pronounced to be truly human, corresponds to its determination M once only and then completely, and just so in another aspect corresponds to its zoological conception μ once

troduced to restore logical truth, should at least abstain from pro-
ssing to be a newly discovered fundamental law of thought, or
deed anything but a make-shift contrivance to correct an improper
ocedure. The determination of a by a is logically speaking an
eration that cannot be performed; it is only because and in as far
, in the context of our thoughts, such a fruitless attempt does not
sult in cancelling the a on which it is made, that it is permissible
substitute a by itself for the a^2 to which the calculus would bring
; but by no means to treat this a^2 as existent, and pronounce it
ual to a. The left side of this equation contains an insoluble
oblem; the right contains, not the solution, but what has to be
quiesced in because there is no solution.

This is no mere verbal dispute, as may be seen from some con-
lerations which Boole subjoins. If we accept $a^2 = a$ for an equation,
is an easy step to the inferences $a^2 - a = 0$ or $a - a^2 = 0$; Boole
solves this last formula into $a(1-a) = 0$. Now the law of excluded
iddle teaches us that everything that is thinkable is either a or not
; this truth is expressed by Boole, who indicates the totality of the
inkable by the symbol 1, by saying that not-a is what remains
this totality when we subtract a from it; so that $(1-a)$ is the
ntradictory opposite of a. Now the meaning of giving the equa-
n the form in which one side is zero can only be that the
mbination on its left side has no extension that falls under it,
d cannot therefore occur at all. Thus the formula $a(1-a) = 0$
comes the expression of the law that nothing thinkable can be at
ce a and not-a. We may be delighted with the plasticity of the
lculus which furnishes such a graphic expression of a familiar
uth; but we shall be the less prepared to admit the interpretation

hair-splitting if I object to this extraordinary piece of argument. Boole himself mentions that from $a^2 = a$ we can further deduce $a^3 = a$, but he disposes of this cubic equation with the remark that two of the factors which it presupposes, $\pm(1 + x)$, are incapable of logical significance; and it was clearly the same reason that decided him at an earlier stage to attach his inferences not to $a^2 - a = 0$ but to $a - a^2 = 0$. This procedure implies an idea which is quite correct; among the numerous formulae which can be mathematically derived from the supposed logical principle $a^2 = a$ none have any meaning but those which express something that is of use in logic; the validity of the logical law does not depend on the shape of the formula; it is the value of the formula as a symbol that depends on its agreement with the import of the law. But the quadratic form itself and its interpretation are altogether a mere caprice. I shall not insist on the point that according to $a^2 = a$, a should have been at once substituted for a^2, which would have brought us back quite intelligibly to $a - a = 0$; for even if we believed it possible to retain a^2 as a real result of a practicable determination of a by a, and *as such* to equate it with a, still there was no sort of logical justification for resolving $a - a^2$ into $a(1 - a)$. In mathematics, where we are speaking of magnitudes, the transformation is correct and in it 1 really means unity; but in logic the difference $a - a^2$ does not present the least motive for regarding it as the product of two factors. The 1, which is introduced in doing so, is not unity, which it would have to be if the resolution were to be mathematically correct, but is Boole's arbitrary though not inappropriate symbol for the totality of the thinkable; the truth that a and $1 - a$ taken together exhaust this totality must therefore be established to begin with, in order to so much as make the interpretation possible by help of which the formula is intended to yield it.

These chimeras have not found their way to Germany; but I have mentioned them at length because of their connexion with a general conception which does meet with some assent among us. We do not overlook the differences between arithmetical and logical computation; but there is an inclination to the idea of a more general mathematical calculus[1], for which this distinction of object-matter would be indifferent. And it is true that every single act of thought, apart from the logical import of its result, admits of many uniform repetitions, and the result admits of many connections and rearrangements; further, the notions of equality, inequality, and opposition have raced under the same rules of union, severance, and arrangement which hold good of all that is recurrent and that has number. Only he laws which are specifically logical and, like the law of excluded middle, govern the formation of the actual elements which are to enter nto this new connexion, must stand on their own feet; and it is an dea as incorrect as it is confused to expect that they can be established by any mathematics however abstract which should still merit hat name in contradistinction to Logic. On the contrary, all that uch a science would have to teach would be the development of the simplest logical truths, which are uniformly true of the manifold and ts combinations, whether those of what has number and is homogeneous, or those of what has mere relations and is heterogeneous. Many things may be proved by mere verbal deductions; and so it may be held an important task to reckon up these truths, in their abstract form apart from their applications; I think it rather tedious than indispensable.

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[1] ['Eines noch allgemeineren mathematischen Algorithmus.']

As direct expressions of such extremely simple truth we at once hink of the axioms, the separate introduction of which is hardly more than a matter of form. Obviously the logical calculus must agree that $a = a$, and that every a and b which are equal to a third hing c are equal to each other; only the definition of equality demands a few words. Logic uses a to indicate a general mark, a general class, or a general case; and is therefore able to accept the anguage of the calculus, according to which a is the symbol of a class, whose extent comprehends all individual things or cases of whatever nature which share the character a. These relations of extent are all hat the calculus notices; it therefore sets down two class-symbols, a

contents which they directly declare as their own meaning. It follows just as simply from those simplest truths that it is always possible to comprehend two acts of thought and their results in a sum $a+b$; that $a-b$ is also possible in logic if the necessary homogeneity is obtained by b being included in a; that the other combination ab, which collects the two characters into one idea, represents a new class-symbol with a defined extension; and finally, that where the problem put before us is only that of carrying out some uniform mode of connexion, no difference can be made by the order of the *summanda* or factors which we combine to make a sum or product.

These easy analogies between mathematical and logical reckoning are less deserving of mention than the differences which are derived from the specific nature of logical thought. I have already mentioned the equation $a^2 = a$; and not less paradoxical is the form in which the law $a+a = a$ veils the logical truth that each universal conception exists once only, that therefore every logical assertion about what comes under such a notion is completely exhausted when it is once thoroughly admitted of the conception itself, and that no new truth can be obtained by repeating the same process on the same object. Just so the theorems $a+ab = a$ and $a(a+b) = a$ remind us that every assertion which is once granted to be universally true of a is also true of every species of a that is still further determined by any mark b, and that therefore the mention of ab beside a remains ineffectual, in other words, the former is 'absorbed' by the latter. It is only the improper employment of the sign of equation that gives these theorems their appearance of peculiarity; all that they really say comes to this; wherever the mechanism of the calculus would naturally lead to the forms a^2, $a+a$, $a+ab$, these useless incidents of its method are to be replaced for logical purposes by a simple a.

More important is the extended use which the calculus makes of the law of excluded middle; for the principle of Duality, which appears at this point as a new law of thought, conceals nothing more than this familiar law. If we use a' to designate the contradictory opposite of a, and 1 for the totality of the thinkable, then we have, really as equations, the formulae $a+a' = 1$, according to which all possible matter of thought is exhausted by a and not-a, and $aa' = 0$ which declares the impossibility of a union of a and not-a. No further proof is either possible or necessary, whether for these laws or for the remaining one that the negation of not-a brings us back simply to a and not to any third thing; they are logical truths which have no doubt received in those formulae a very clear and convenient expression.

The old Logic had its chapters about immediate inference, conversion, and contraposition of judgments, and endeavoured by help of this same law to pursue the content of an enunciated judgment into its relations to judgments not yet uttered. Boole in a more comprehensive spirit sets before himself the problem of developing the different and mutually exclusive divisions of the thinkable that may be formed by the affirmation and denial of the concepts, class-symbols, or elements of whatever kind united in a judgment. If x and y are the given elements, and x' and y' their contradictory opposites, then xy, xy', $x'y$ and $x'y'$ are evidently the four classes into which all that is thinkable must be divided; that is, the constituent parts of the complete division which Boole calls the expansion or development of the given relation between x and y. It is somewhat inconvenient that following mathematical tradition he designates that relation between x and y as a 'Function' of the two, $f(x, y)$; logically such an expression can mean nothing, unless it is understood as the definition or predicate of some M, for then all the constituents xy, xy', etc., might be deduced from the given connexion between x and y, and with them the coefficients which would indicate them as possible or impossible within the extension of M. Boole however employs for the moment the independent function $f(x, y)$ in order to develop out of it in an equally general way the law of the formation of those coefficients. His original equation $x^2 = x$, as he can find for it only the two arithmetical analogies $0^2 = 0$ and $1^2 = 1$, induces him to make the assumption that the logical and the mathematical calculus would completely coincide if these two values were the only ones which any magnitude could assume; and conversely, he takes all mathematical operations to be permissible in logic, on condition that the class-symbols to which they are applied are treated as magnitudes which admit of these two values only. So taking $ax + bx'$ as the given function $f(x)$, and $f(1)$ and $f(0)$ as the two values which it assumes if we take $x = 1$ and $x = 0$ (x' always assuming the opposite values), it is shown that $f(x)$ may be obtained by the combination of the two values: $f(x) = f(1)\,x + f(0)\,x$. The same consideration leads, in the case in which the given function contains the two elements x and y, to the formula:

$$f(x, y) = f(1, 1)\,xy + f(1, 0)\,xy' + f(0, 1)\,x'y + f(0, 0)\,x'y',$$

in which the two bracketed values refer in their order to x and y respectively.

If any stress is to be laid on this scheme of the logical development of a function, it would have been easy to establish it in a less metic. Granted that they rank as magnitudes when considered by themselves, still in combination or multiplication with other magnitudes they have the general logical import of affirmation and negation. What was required in the above theorem was only this logical meaning, valid indeed for arithmetic but not derived from it; it was therefore improper to give currency to the illusion that logic is indebted to the peculiar laws of arithmetic for the instruments with which it operates. I will take two examples to show what I mean.

First, if $M = ax + bx'$, the value of the right side will obviously be reproduced if we first suppress the first term and leave the second, then suppress the second term and leave the first, and finally add together the two that are left:

$$ax + bx' = ax + 0 \, . \, bx' + bx' + 0 \, . \, ax;$$

then the coefficients can of course be expressed by $f(1)$ and $f(0)$, and

$$ax + bx' = f(1)\,x + f(0)\,x'.$$

Again, let the function $f(x, y) = ax + by$ be given and its development with reference to the terms xy, xy', $x'y$ and $x'y'$ required; and further, to make sure of what we are speaking, let us regard $f(x, y)$ at the same time as a definite M, whose definition, or specification of extent, is contained in the right side of the equation.

Within this M, the combination xy is possible in three cases, being the ax's which are also y, the by's which are also x, and the ax's which are also by in full, or the by's which are also ax in full; for none of these combinations are expressly excluded by the right side of the equation. We should therefore get axy, bxy, $abxy$; but as logically speaking the ab are included besides both under a and under b, it is sufficient to exhibit $a+b$ as coefficients of xy; and of course $a+b = f(1, 1)$ is equal, that is, to the value of the right side for $x = 1$, $y = 1$. The second term of the development would contain xy'; the equation tells us that if we suppress by which can never be combined with y' there can occur within the compass of M no y' or not-y besides ax; consequently a is the coefficient of xy', and a of course $= f(1, 0)$. Just in the same way it follows that within M there can be no other x' or not-x but by; consequently $bx'y$ is the third term, and b of course $= f(0, 1)$. Finally the equation tells us that the more clearly what is the purpose of the whole proceeding. The first examples which Boole gives can only be regarded as exercises. If clean beasts x are according to the Jewish law those which divide the hoof y and chew the cud z, and then the development tells us there are no clean beasts which divide the hoof but do not chew the cud, and none which chew the cud but do not divide the hoof; that again there are no clean beasts which do neither the one nor the other, and lastly there can be no beasts which do both and yet are not clean; I have my doubts of the frequency of the logical desire to go through these deductions of given fact; but if any one feels the want, it is beyond a doubt more easily satisfied without a calculus than with one. But there are two other problems which Boole hopes to solve by help of such use of formulae; first, if a number of elements are given in any combination, the equation which expresses this combination is to be solved with reference to any of its elements at pleasure; and then it is to be possible to eliminate any one from the equation, in order to display the relations of the rest to one another.

As regards the first problem, I can only regret that Boole abandons himself recklessly to his principle of permitting himself all operations of reckoning if only their result can be logically interpreted. From the given proposition 'All men y are mortal x,' he obtains by contraposition 'No man is not-mortal' $yx' = 0$. Now as $x' + x = 1$ and therefore $x' = 1 - x$, we get $y(1 - x) = 0$ or $y - xy = 0$, $xy = y$; then further $x = \frac{y}{y}$, and by development of $\frac{y}{y}$ we obtain $x = y + \frac{0}{0}(1 - y)$ or $= y + \frac{0}{0}y'$; this he takes to mean, introducing the mathematical significance of the symbol $\frac{0}{0}$: 'mortal includes all men and an in-

$y = vx$. The proposition 'All men are mortal' means simply this and nothing in the world besides; it merely regards y as subordinate to an x within the compass of which there is something else as well. There is no possible meaning in finding over again by calculation precisely what was presupposed, and what is self-evident, that is, that x comprehends beside the vx which are y a further indefinite number of kinds which are not y; that therefore $x = y + wy'$.

With respect to the process of elimination, I shall content myself with giving an example. Every logical equation can, by applying contraposition to the affirmative judgment which it expresses, be reduced to the form in which one side is zero; for the equation $xz = 0$ simply means that no x is z. I pass over Boole's doctrine about the procedure of collecting all given single judgments or equations into one solitary resultant equation, and suppress the scruples which I feel as to the necessity or productiveness of such an operation.

It is granted then that the equation is to be presented in the following arrangement; $pab + qab' + ra'b + sa'b' = 0$; then the product of the coefficients $pqrs$ equated to zero, is assigned as the result of the simultaneous elimination of a and b. This is easily seen with the ordinary appliances of Logic. For logically this equation cannot have the value 0 unless each of its terms taken by itself $= 0$. Further, $pab = 0$ says that No pa is b; but $qab' = 0$ gives by contraposition All qa are b, and so in Cesare, No qa is pa, or, $pqa = 0$, and this again gives No pq is a, or by contraposition, All pq are a'. Again $ra'b = 0$ gives, No ra' is b; but $sa'b' = 0$ gives by contraposition All sa' are b; so we get in Cesare, No sa' is ra', or, $sra' = 0$, or, No rs is a'. If we subsume the second conclusion No rs is a' under the first All pq are a', there follows in the same figure, No rs is pq or $pqrs = 0$. It is easy to see that if a similarly arranged equation with one side zero contains besides a, b, and a', b', more such pairs of opposites c, c', the elimination may be continued in the same way. But no doubt for such cases there is value in the abbreviated rule that the result of the elimination consists in the equation of the product of the coefficients to zero. If the equation had contained besides a term $z = 0$ independent of the pairs to be eliminated, it would persist without change, and might be added to the preceding term, so that in the result $pqrs + z = 0$ each of the terms by itself is $= 0$. Schröder remarks on this question at p. 23 of his work that the results of the elimination of a symbol a from several isolated equations are less comprehensive than those of an elimination from the combined final equation; $xa + ya' = 0$ and $pa + qa' = 0$ when taken apart, only give $xy = 0$ and $pq = 0$; while on the other hand the combined equation gives $xy + qx + py + pq = 0$; and for this reason he thinks the latter order of procedure preferable. Is he not in this artificially making little difficulties, simply out of the order of procedure, which must ultimately depend on the development of the functions? Why are we forced to unite the four terms $xa = 0$, $ya' = 0$, $pa = 0$, and $qa' = 0$, although they must be true by themselves, in two equations, instead of regarding them as four terms to be employed at pleasure? Then we might find without difficulty all results of elimination which we had any interest in ascertaining.

I do not maintain that the same syllogistic process will easily bring us to our goal in every case, especially in more complicated cases. But Boole himself insists that we must carefully analyse what we mean in every case, before translating our notions into the language of the symbols; and I certainly believe that the fulfilment of this postulate would enable us to dispense altogether with the calculus, and that Logic would prove rich enough to allow of the invention of adequate means of solution corresponding to particular problems, even if these means were not stereotyped beforehand. With reference to this point I mention a problem which Boole[1] puts and which Schröder repeats.

It is assumed to be known from an analysis of experience that in a certain class of natural or artificial products the combinations of the marks *a b c d e* are subject to the following rules; and in such a way, that not only the occurrence but also the non-occurrence of each particular mark belongs to the conditions from which the presence or absence of the others has to be inferred.

1. Wherever *a* and *c* are absent at the same time, *e* is present, together with either *b* or *d*, but not with both;

2. Where *a* and *d* occur, but not *e*, *b* and *c* will either both be found or both be missing.

3. Wherever *a* is found in conjunction with either *b* or *e* or with both at once, either *c* or *d* will be found, but not both together.

4. Conversely, where, of the pair *c* and *d*, the one occurs without the other, *a* will be found in conjunction with either *e* or *b* or with both at once.

It is required to ascertain :—

1. What can be inferred from the presence of *a* with reference to *b*, *c*, and *d*;

2. Whether any relations, and if any, what, exist between *b*, *c*, and *d*, independently of the other marks;

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[1] ['Investigation of Laws of Thought,' p. 146 ff.]

3. What follows from the presence of b with respect to a, c, and d, and

4. What follows for a, c, and d independently of the other marks.

Boole anticipates that no logician would find the right answers to these questions by syllogistic process, unless he knew them beforehand; I fully admit this, but who would be tempted to select that process for attacking this problem while the more suitable one offers itself spontaneously? We have only to make a list (it is a purely mechanical process) of all the combinations of five which can be formed out of $a\,b\,c\,d\,e$ and $a'\,b'\,c'\,d'\,e'$, avoiding repetitions and the inclusion of contradictory elements, and then, or while making the list, to suppress those which are excluded by the totality of the given conditions. This leaves only 11 combinations;

$a\,b\,c\,d'\,e$
$a\,b\,c\,d'\,e'$
$a\,b\,c'\,d\,e$
$a\,b'\,c\,d'\,e$
$a\,b'\,c'\,d\,e$
$a\,b'\,c'\,d'\,e'$
$a'\,b\,c\,d\,e$
$a'\,b\,c\,d\,e'$
$a'\,b\,c'\,d'\,e$
$a'\,b'\,c\,d\,e$
$a'\,b'\,c\,d\,e'$

From these we can read off the answers to the questions proposed:

1[1]. We infer from the presence of a that either c or d is present, but not both, or else that b, c, and d are all wanting.

2. There is no independent relation between b, c, and d, for all conceivable combinations of them with b', c', d' are equally realised.

3. From the presence of b it follows that either a, c, and d are all absent, or some one alone of them is absent.

4. If a and c are both present or both absent, d is impossible.

Similar questions about e which are not proposed could be answered out of the same conspectus without any distinct operation.

I borrow from Schröder's treatise for purposes of comparison no more than the beginning of the solution by calculation; not so much to show that if all the intermediate terms are actually supplied it is by no means distinguished by brevity, but chiefly with the general object of elucidating the use of the calculus by help of an instance that involves a real problem, and is not merely going back upon what we know to clothe it in awkward formulae.

By contraposition of the positive judgments which constitute the given conditions of the possible combinations, and so reducing them, as equations, to the form in which one side is zero, we obtain

from 1. $a'\,c'\,[e' + b\,d + b'\,d'] = 0$;
from 2. $a\,d\,[b\,c' + b'\,c]\,e' = 0$;
from 3. $a\,[b + e]\,[c\,d + c'\,d'] + [a\,d' + c'\,d]\,[a' + b'\,e'] = 0$.

As the questions ask nothing about e and e' the first operation to perform is the elimination, which we dispensed with, of this pair of

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[1] [Cp. Boole, pp. 148-9.]

$$a\,[c\,d + b\,c'\,d'] + a'\,[c\,d' + c'd + b'\,c'\,d'] = 0.$$

Now to answer by this result in the first place the second question, about the relations between b, c, and d, we should have to eliminate a and a'; but the requisite product of their coefficient is $= 0$ because each individual product as it arises takes independently the value 0 owing to the combination of contradictory elements; the result is therefore $0 = 0$ and we must accept this as a sign that there is no independent relation between these three marks. However, we see at once that if we give the symbol p to the coefficient of a that of a' will become p' or not-p; we therefore obtain from $a\,p + a'\,p' = 0$ the two equations $a\,p = 0$, or No a is p, and $a'\,p' = 0$, or No not-a is not-p. The first of these gives at once; all a are not-p, or p'; hence $a = c\,d' + c'd + b'\,c'\,d'$, which formula answers the first question.

I omit the continuation which would be needed to answer the third and fourth questions, and confine myself to remarking that in the whole of this problem no use has been made of the development of functions, of the importance of which I expressed my doubts above; the required equations were obtained directly from the given propositions, and the eliminations out of them were conducted on a method, the origin of which we explained to ourselves by help of syllogisms in the second figure. Thus there is nothing to be said against the appropriateness of the present method; but just as little against the superior simplicity and plainness of that which we adopted. This, by the way, had not to wait to be discovered by Jevons, for it was already forthcoming in the doctrine of classification, which long since required in the first place the tabulation of all the marks in their combinations, and then the cancelling of all combinations that become inadmissible adapted to the proposed problem. It is inevitable that a symbolic method intended to make uniform provision for every case should purchase its suitability for the solution of one problem at the cost of a useless prolixity in its treatment of others and of manifold discords with the custom of language.

Even the quantification of the predicate, which was the starting-point of recent English logic, was no new discovery, but the superfluous inflation of a familiar idea to an excessive importance. That the predicate of a judgment, except in case of simply convertible judgments, has a larger extent than the subject which in part takes its place within this extent; that therefore it is not merely the predicate that determines the subject, but also the subject that restricts the predicate to such a modification as is true of the subject's self; these were old doctrines of logic, and in its rules of conversion it went so far as to provide for their application. It is true that the scheme of judgments gave no special expression to this truth, just as the ordinary linguistic form of the sentence did not. But what harm was there in that, when the fact was known? Did the want of such an expression ever deceive a considerate thinker? And was it worth while, for the sake of amending such trifles, to have recourse to such dangerous contrivances, as to connect the natural expression of thought with a new symbolism and a new calculus? There could be no real gain in expressing the proposition 'All men are mortal' by $y = v x$ unless a means could be discovered of defining this v; as long as it remains an undefined coefficient, it is an ineffectual indication of what we knew before. In the converse of this judgment 'Some mortal is man,' the old logic would bring to light this indefinite Particularity[1] neither better nor worse than that v would; if we object to the expression 'some,' our objection might be easily removed by the consideration that these indefinite particular judgments are at the same time forms of modality, and express the possibility of a conjunction of their predicate with the general notion which forms their subject, by affirming such a connexion for some but not for all cases of the notion.

There is a passage of Jevons ('Principles of Science,' London, 1877, p. 59) which among others has occasioned these remarks. He forms two premisses; sodium[2] = sodium metal, and sodium = sodium capable of floating on water. He draws the conclusion sodium metal = sodium capable of floating on water. To this he subjoins these remarks. "This is really a syllogism of the mood Darapti in the of being tediously minute and accurate." Oh no! we might admit the 'tediously,' but otherwise Aristotle is in the right. Jevons' whole procedure is simply a repetition or at the outside an addition of his two premisses; thus it merely adheres to the given facts, and such a process has never been taken for a *Syllogism,* which always means a movement of thought that uses what is given for the purpose of advancing beyond it. So the combination of words which he proposes is not a syllogism at all, and consequently not one in Darapti. The meaning of the syllogism, as Aristotle framed it, would in this case be that the occurrence of a floating metal Sodium proves that the property of being so light is not incompatible with the character of metal in general. If he expressed this by saying 'Some metal is capable of floating,' he intended of course not to repeat the premisses which were known before; but to enunciate the possibility of a general distribution of this property among metals, as a supposition whose correctness in fact there is ground for testing further, since it is logically not inconceivable. Even the expression 'Some metal' is at bottom quite correct, for Sodium certainly is some metal; the expression does not enjoin us to think of other metals at the same time with it; it is true that it does not prohibit our doing so, but this need not give rise to any error.

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[1] ['Diese unbestimmte Particularität.']

[2] [See Professor Lotze's Preface to the Logic.]

How often have modern enterprises like these proclaimed the dawn of a wholly new epoch in logic, and the fall of the contemptible system of antiquity! I am convinced that if the ancient logic were to be really forgotten for some generations and then rediscovered by some fortunate thinker, it would be welcomed as a late discovery, after long search, of the natural march of thought, in the light of which we

CHAPTER IV.

The forms of Proof.

199. It was our business in writing of systematic logic to enumerate the various forms of judgments and to point out the precise mode of union which in each of these forms is conceived as subsisting between S and P or as to be effected between them: it is the business of applied logic to consider what contents S and P can properly be joined in one of these forms of union. Various problems which we shall not always hold apart fall within this scope. In the first place the communication of the thoughts of others gives us numerous propositions of the form S is P, whose meaning and purport is perfectly plain, but whose validity is questionable: then there arises for us the problem of a *proof* for the *given* proposition T. In the second place our own observations may lead us to suppose that between two ideas S and P there subsists a relation which if it were known could be expressed in a judgment of the form S is P: then we are called upon to *discover* the yet unknown proposition T which would be the precise expression for this supposed relation.

These two, discovery and proof, differ only in their different use of the same materials. The same combinations of thought by which the truth or probability of a proposition T was first discovered may always be applied, when put somewhat differently, and sometimes even without any such transformation, to prove the truth or probability of a given proposition T. Moreover it is easy to see that the reflexion of the discoverer, if it is not to miss its aim, needs at every step slight connecting links, resembling a proof in form: and conversely that a proof will never reach its goal without some inventive play of thought. On the whole however discovery reaches farther than proof: and so I will separate the two problems, though I shall not always avoid the natural mixture of the two. Scientific investigations lead to both in about equal measure; the needs of life more frequently lead to discovery.

can seldom be established between S and P without the employment of knowledge supplied by experience; but as such knowledge, if it is to lead to universal conclusions, must itself have universal validity, we may regard it as knowledge which, though originally derived from our experiences, is yet, now that we have full confidence in its universality, to be counted among the proper instruments of thought. The proof of particular facts on the other hand, of historical events or of the ordinary transactions of life, can never follow from universal propositions alone, not even from such universal propositions as are themselves derived from experience: it presupposes the knowledge of a number of particular circumstances, occurring only here and only here united in this precise manner. The preliminary process of getting at all these conditions, from which the conclusion is to be drawn, requires peculiar instruments which we shall consider presently. The solution of a proposed problem on the other hand, even when the result is to be not a universal proposition, but the establishment of a single fact, may be connected with the proof of universal propositions: under the conditions which here do not need to be sought but are given, and so far as they are given, the definite proposition T which satisfies them all is always to be found by employing instruments of thought which are of universal application, and these theoretical results are inaccurate and need correction in practice only so far as we have failed to state *all* the conditions which T had to satisfy.

200. Every proof is a syllogism, or a chain of syllogisms, which completes the premises required for the given proposition T, so that they fit into one another in such a way that T follows as their necessary consequence. But the validity of every conclusion depends upon the validity of its premises: these again might be established by fresh proofs, but this procedure would go on *ad infinitum* without any result were there not a number of universal propositions which we accept as immediate truths, which therefore neither need nor are capable of proof, but are themselves the ultimate grounds by appeal possible proof of an axiom, this mark may in the last resort be nothing but the *self-evidence*, the immediate clearness and certainty with which the content of a universal proposition thrusts itself upon us as a necessity of thought; and in fact this is what we always come back to in the end.

Experience however abundantly shows that propositions which later generations have proved to be false, were as self-evident to earlier generations and produced in them as strong a conviction as any propositions whatsoever: relations which in the limited sphere to which our observation is confined are seen to be permanently present or constantly recurring, without any contrary experience to disturb us, very commonly assume the appearance of necessities of thought. There is only one way of distinguishing the spurious self-evidence of a prejudice from the genuine self-evidence of a true axiom: we must try whether the contradictory of T the proposition in question is as impossible in thought as T itself seems to be necessary. This test will often be quite decisive; we shall often find to our astonishment that the attempt to join S and P in the opposite way to that asserted by the given proposition T leads to no inner contradiction in our thought at all. In that case T is no axiom, but either altogether an error, or a truth that holds true in some cases only, or a truth which though universally true requires to be proved. In the other case, when the contradictory proposition non-T appears as impossible in thought as T appears necessary, we may with greater confidence regard T as an immediate axiom; but the test does not even now give perfect security, for it is quite possible that the inconceivability of non-T and the apparent necessity of T may both alike rest upon a spurious self-evidence. Should these two simultaneous errors be made, logic furnishes no short way of detecting them: our mistake could only be gradually amended by our becoming aware of the contradictions which experience offers to the assumed validity of T, and by a slow and far-reaching modification of our system of thought suggested by those contradictions.

Such a double error will seldom be found in the case of purely theoretical principles, more often in the case of the principles upon which our moral judgments are based, and which may be classed as genuine or spurious axioms, although strictly speaking they do not seem to be necessities of thought but only unquestionable truths, and their opposites do not seem to be unthinkable but only absurd.

201. Supposing now that T is a universal proposition whose validity is not axiomatic, i. e. that it is such as to need proof, we yet shall not set about proving it till we know that T is worth proving. In three cases it will not be worth proving. In the first place it will not be worth proving if its content is an incomplete, and therefore an indefinite thought. A man of untrained intellect, so long as he confines himself to the objects which naturally come within his scope, is usually conscientious in enumerating and examining all the points which are important for the understanding of a fact: he follows the old rule which tells us to ask 'quis? quid? ubi? quibus auxiliis? cur? quomodo? quando?' and to omit none of all these questions. But he is quite helpless when he wanders off into general considerations which belong to the province of speculation: he then usually does not get beyond a clumsy expression for something which he perhaps rightly believes, demands, or assumes, but is unable to connect with any determinate or determinable points. The philosopher on the other hand, revelling in his abstractions, does not always come to meet him half-way, but often contents himself with employing conceptions which when severed from their proper application become utterly meaningless: the result is that vague theses are nowhere so common as in the attempts of a man who has had no logical training to philosophise by the light of nature. That God and the world are one is a proposition that no one can prove except him who propounds it; so far as his proof is correct at all it is the proof itself that tells us what he meant by the proposition: any other person than he who propounded it will, if he be wise, attempt neither to prove it nor to refute it; for that God and the world are in some sense two is asserted by the proposition itself, for otherwise it could not have distinguished them; but that they are also one in some one of the many senses of unity, may be supposed without more ado.

That things are appearances is an equally ambiguous proposition:

simply to be returned as they are to him who brought them, just as in a court of law we refuse to listen to a man who complains that he has suffered wrong without saying what has been done to him and who has done it.

202. The second case is when though a perfectly clear nominal definition may be given of S the subject, or P the predicate of the proposition T, the definition contains a combination of ideas which can be shown to be impossible, or cannot be shown to be real. No one would trouble himself to prove or to refute a proposition the subject of which is a wooden iron: no one would seriously enquire whether this wooden iron will burn in the fire like wood or melt in it like iron. There is no such logical contradiction in the ideas of ghosts and will-o'-the-wisps, but we defer asking whether the former need sleep, and whether the latter are attracted by buried metal, till their existence is proved. What we here require may be called in general the *justification* of a conception, which must without fail be added to its nominal definition when use is to be made of it.

This may be effected in various ways. If M stands for something which is supposed to have external existence, the shortest way to justify M is to *point* at once to an instance of it or to a fact in which its existence is given and accessible to observation. If M denotes a combination of ideas the validity of which means that it can be carried out and that its result can be imagined or realised in a mental picture, this very realisation of the content which M demands, or in other words its *construction* will justify M itself: thus geometry establishes the admissibility of the conceptions it has formed by presenting in a visible form what they till then only contained as a problem, thereby proving most conclusively that the problem was soluble. If we can neither point out any instance of M nor carry out its construction, we must at least show cause or give a 'deduction' which explains how in connexion with some demonstrable reality or in pursuit of some problem we have been properly and justly led to form this conception. Such a 'deduction' cannot always directly prove the validity of M in the shape in which the conception is presented, but it may always show that M is a preliminary designation for some content which we are reasonably and rightly looking for; it remains for the further enquiry whose beginning is hereby justified to determine whether M itself can be justified as a valid conception, or else how its content must be modified in order to make it valid.

The ancients regarded the doubling of the cube as a serious problem: but though they could not geometrically construct the required line, whose cube should be double of a given cube, yet it was all along certain that the problem was soluble and that the required line was a magnitude which could in some way be found. For it could be shown that as the side of a cube increases its volume must also continuously increase without any alteration in its shape: among this infinite series of larger and larger cubes then must be found that particular one which is double of a given cube, and this implies that its side actually occurs in the series of existing lines. We here show cause for the necessary validity of that which is sought instead of actually realising it in a construction.

Again it may be doubted whether one and the same conception of length fits both curved and straight lines; but setting this doubt aside it was not unreasonable as things then were to hope to find by a simple geometrical construction the straight line which is equal to the circumference of a circle of given radius; for it was certain that the length in question depends upon the length of this radius and upon nothing else. This hope was only banished by the completion of the enquiry, which showed that the circumference cannot be expressed as a determinate real and algebraical function of the radius. In the natural sciences a hypothesis often assumes facts which we can never hope to establish by direct observation: often indeed we must leave it to God and the future to show even the possibility and constructibility of that which we are for the present absolutely obliged to assume. The only way of justifying ourselves in such a case is to show from the given facts the pressing need of the idea which we employ, with the reservation of course that we may at a future time so alter it as to enable us to construct it without impairing its usefulness. We shall return to this point on another occasion; for the present it is enough to refer to the instances above employed as showing what kind of justification is needed for conceptions if their union in a proposition is to deserve proof or refutation.

203. We will now suppose that the conceptions which are joined in the universal proposition T have the requisite definiteness and validity: but even so we do not start in search of a proof that shall exhibit T as the necessary consequences of premises that must be discovered, until we have got some preliminary warrant that the proposition is true as a matter of fact; for it would be lost labour to try to prove what is not even true. If T is a universal proposition of whose field it is not easy to take a comprehensive survey, we first try whether T holds good in some examples that lie near at hand: a single case in which it did not hold would do away with the universal validity of *T*, and the problem would then be changed into finding the conditions under which *T* has at least a partial validity; if on the other hand that which *T* asserts is found to hold good in all the cases of its application which we compare, this trial, though being necessarily incomplete it cannot prove that *T* is universally valid, may yet corroborate what it alleges so strongly that it will be worth while to search for a proof. This very needful preliminary procedure, which will further on take its place among the forms of proof, is in fact neglected but seldom, and that mostly in cases where the validity of *T* cannot be tested by mere reflexion upon instances supplied by the memory, but only by observation or experiment. The courtiers of Louis XIII exhausted themselves in ingenious proofs of the proposition that a living fish thrown into a bowl full of water makes it overflow while a dead one does not, until the gardener who was called in made the experiment and showed the assertion to be entirely false; but others make the same mistake, and in the less exact departments of natural science we frequently find subtle demonstrations and explanations of phenomena whose actual occurrence is entirely problematical.

204. Supposing now that this preliminary question is settled, and that *T* is recognised as a universal proposition that deserves proof, its truth or falsehood may be established either in a direct or in a roundabout way, and this makes the first division of proofs.

A proof is *direct* when it shows immediately that the given proposition *T* is necessary or impossible; it is *indirect* (or *apagogic*) when it establishes the truth or the falsity of *T* mediately by showing the falsity or the truth of its contradictory non-*T*. In each case there are two directions which the train of thought may take. We may call a proof *straightforward* or *progressive* when it starts with that which in the nature of the thing conditions something else and makes that which is conditioned issue from it as its consequence; it is a *backward* or a *retrogressive* proof when it starts from that which in the nature of the thing is conditioned in order to arrive at knowledge of that which conditions it. The first form of proof, since it proceeds *a principio ad principiatum*, may equally well be called *deductive*, though the opposite name *inductive* will not be found so generally suitable for proofs of the second form which proceed *a principiato ad principium*. And finally there is yet another distinction applicable to both these lines of proof: you may go forward (progressively) either from general truths to *T* or from *T* to its proper consequences, and similarly you may go backward (retrogressively) either from *T*'s consequences to *T*, or from *T* itself to the truths upon which it is series of epi-syllogisms, if the proof can only be completed in chain of reasoning) it is then shown in what relation S and P wh are joined in the given proposition T stand to that major premi and lastly the conclusion infers that by reason of these relations S and P the proposition T which was to be proved must hold go If the problem be stated in this general way it seems as if all three figures of Aristotle might be employed in this form of proc the fact is however that the first figure alone answers to the spirit it. I do not reject the third figure on the ground that as usua described it only gives particular conclusions, while we here wish prove universal propositions; if we put the particular conclusi 'some S are P' into a modal form, 'that which is S may be P,' get a universal proposition which it may be worth while to pro For instance if we want to produce an effect P, and have nothing get it out of except an unpromising material S, we shall be glad see it shown by a syllogism in *Bramantip* that S and P are co patible with one another in the case of a subject M, and that the fore S does not always make the desired effect P impossible. I the third figure does not exhibit this proof in the progressive for It only states in the premises an instance of the coexistence S and P, from which we may argue regressively, *ab esse ad posse*, their compatibility. The second figure admits universal conclusic indeed, but only negative ones; these too may be valuable, but tl cannot be obtained by this figure without premises of oppo quality, and therefore fail to satisfy us. For a universal negat proposition T, which simply denies a predicate P of a subject because S and P stand in opposite relations to a third M, appe to a *mark* which shows that S and P cannot be combined, but not a *reason* which explains why they cannot: it merely expresses a us to understand not only *that* T holds good, but *why* it holds good.

206. This opinion is as old as Aristotle: it is worth while to observe however that this form of proof is to be regarded as an ideal in another sense than this: it cannot fairly claim the praise bestowed upon it unless we succeed in filling it with the content which its articulation requires, i. e. unless we set down for major premise a general proposition under which the special case of the minor premise demands to be placed in virtue of its very nature, and which therefore would actually be the reason upon which the validity of the proposition to be proved depends not merely for our reflexion but in the nature of things. But it is clear that we may use the form of this proof without in the least satisfying this last condition. Many instances occur, and that precisely in the field of mathematics where exact treatment is required, of propositions that admit of various equally convincing proofs all couched in this form of subsumption, none of which therefore can claim exclusively to express the proper connexion and development of the thing itself. The possibility of presenting the same idea in very various forms without altering its value enables us here to subsume it under a great variety of universal major premises, and to proceed from any one of these arbitrarily chosen starting-points to the same assertion T. I am anxious not to be misunderstood here and will therefore go into detail.

I will in the first place allow that we often find in mathematics a proposition T which is so evidently only an application of a definite major premise M that its deduction from this major alone seems natural, from any other artificial. I will remark in the second place that when T may be deduced with equal ease from a variety of majors $M N O$, I do not find in this alone any reason for saying that these various proofs are foreign to the natural sequence; for it may be (though I do not propound this as the true theory but only suggest it as a possible view) that the whole of our knowledge (e. g. of geometry) rests in fact upon a number of original and equally self-evident perceptions, none of which can be deduced from any other, but which, like the several components of one *complete* thought, are each and all valid at once and connected in definite ways with one tion of the content itself, which makes a harmoniously articulated whole not on one line only but on several lines at once. But I must nevertheless add in the third place that there remain many propositions *T*, whose proof (I mean in this form of subsumption) can only be effected by devices, which can be justified after they have been applied, but to the application of which we cannot find any invitation in the thing in question. It is to these proofs, of which many occur in pure mathematics, and a far greater number in applied mathematics, that the remark above made is intended to apply; though these proofs are as conclusive as can be wished, it is yet quite beyond our power to take them all in at one view, especially when they form chains of many links; and as they scarcely allow us to do more than see the necessary consequence of coupling each link to the one which follows, while the inventive ingenuity which forges the chain seems to be guided by pure caprice, we cannot honestly say that they *show* why the conclusion *T* is true; they only constrain us to admit that it is true. I have introduced this point because of its practical importance. Our ideal of knowledge and demonstration no doubt is that we should deduce each given proposition *T* from the determining grounds by which it is in fact determined *in such a way as to explain it*, and not simply assure ourselves of its *certainty* by a logical device; and if this problem is to be solved, it can only be by a direct progressive proof of this form. But it is soluble only within narrow limits, and where it is not soluble, where therefore we must content ourselves with the mere certainty of *T*, this form of proof by subsumption has not the least advantage over other forms. It is mere pedantry on the part of the logician to wish in spite of this to enforce it and when a proposition can be conclusively proved in two words by an indirect method to look about for a direct deduction, reveal the existence of a contradiction hitherto concealed, but at any rate this procedure suffices in the field of science to *recommend* a hypothesis, which is then reserved for further examination. But the true province of this method lies in practical life: it is the method we employ to recommend proposals, arrangements that are to be adjusted, resolutions that are to be adopted. And here the incompleteness of the development of the consequences is no obstacle; in all human affairs it is enough to ascertain what effects will follow from the application of a proposed measure within such a limited time and in such a limited field as we can readily survey: he who wishes to take count of all the subsidiary effects which a microscopical examination might disclose, all the consequences centuries hence of what we do to-day, is a supercilious pedant; fresh measures will be taken to avoid minor disadvantages, and the remote future must take care of itself.

208. A *third* form, the first *directly regressive* form of proof, proceeds from the assumed validity of T and works back to the conditions under which this validity is possible. The difference between this form and that just discussed is not considerable, but there is a difference: it is not considerable because the conditions requisite for the validity of T can only be found by taking T as their basis and deducing them as consequences from it, a procedure which coincides with our previous direct progressive method: but we see that there is a difference when we consider the nature of that which is thus deduced. We may take as an instance of both forms at once the ordinary way of solving a problem in mathematics; for every such solution is at the same time a proof of the solubility of the problem, i.e. of the validity of the combination of ideas contained consistent with truths otherwise established. If we only come upon particular consequences which disagree with given facts or secondary conditions, we may with certainty infer that T does not hold, though we do not see the reason why it does not; if T is a practical proposal, it may be that it is quite acceptable in itself and that it is only ts execution that encounters some obstacle; and then we should have a case of the second form of proof: if on the other hand we come upon absurd general propositions which must be true if T is to be true, then besides the certainty that T is impossible we get also a strong hint as to the reason why it is impossible; that reason must lie in the general truths which conflict with the absurd conditions we deduced, and herein we find what this third form of proof does for us. It not only clears the ground for the subsequent discovery of a direct and progressive proof of the contrary proposition, but gives us a remarkably conclusive and palpable negation of a given proposition T in the disclosure of all the absurd assumptions that would be necessary if it were true; and on this account this regressive proof is often preferable to a progressive one.

It cannot establish anything but the falsity of T, and so remains a form of *refutation.* If in working backwards from T we come upon none but admissible conditions, we cannot infer that T is true except in mathematics; for only in mathematics is it possible to develop from a proposed problem *all* the conditions necessary to its solution; in other cases we can never be certain that we have really deduced from T everything without exception that is implied as a condition necessary to its truth; the next step we took might bring to light an absurdity that we should have to assume. Affirmatively then this method is in matters of theory only able to establish the probability of T; in practice however we use it to recommend a proposal just as much as the foregoing progressive method. For when we want to secure the acceptance of a proposal we not only point out the consequences to be expected, but also show that the conditions of its execution are not incompatible either with the general requirements of justice and morality, or with the means which are actually at our the price we must pay for it.

209. A *fourth* method, the second *direct regressive* method, starts from given propositions and proceeds to prove from them the validity of T as the condition of which they are the result. This is a line of thought which we are very constantly impelled to follow: for the greater part of our knowledge of general laws is won in this way by reasoning back from given facts to that which must be assumed as the condition of their possibility. It is easy to see however that its most important applications belong to the method of discovery which tries to elicit from that which is given a T which is as yet unknown. When the general proposition T is given and we are looking about for the several propositions which may serve to confirm it, the proper method is always to begin with the progressive development of that which as consequence of T must be true if T be true: only when we have made a comprehensive survey of these consequences do we proceed to compare the result obtained with experience or with other truths, in order to reason regressively from the truth of this result to the truth of T.

I will therefore postpone the consideration of much that might be introduced here, and will only mention one species of this method, viz. that which infers the universal truth of T from its truth in particular instances,—*complete induction* or the *collective* proof. We are often compelled to employ it: it is not always possible to prove at one stroke that a proposition T holds good for all quantities, integral and fractional, positive and negative, rational and irrational, real and imaginary magnitudes; but each of these several kinds of quantities may offer some special point of attachment for a proof that T is true of it; if then we are sure that we have included all possible cases of T, that is in this case if we are sure that there is no conceivable kind of quantity besides those named, then we know that T is true of all quantities whatsoever. The general conception of quantity will then no doubt contain some reason for this universal validity; nevertheless we cannot always point out this reason, or at least we cannot always make it quite clear and self-evident; and then we must have recourse to the collective proof.

210. The necessity of including without any omission all the kinds of cases to which T can apply if T is to be proved universally true leads here to an interesting special form of this proof. Mere completeness of course can always be secured by dividing all the cases into say Q and non-Q, the non-Q again into R and non-R, and so kinds of cases Q R U, it is very difficult to find one for the negative remainder non-U which embraces a miscellaneous crowd of different cases. We are constrained therefore to take a case Q, for which we happen to be already able to prove that T is true, and try to derive the other cases R U.., etc. from Q in such a way that it may be evident that the changes by which Q passes into R, and R into U, either do not affect the conditions which made T true in the case of Q, or else constantly reproduce them. This is the method, familiar to mathematicians, first formulated by *Jacob Bernoulli*, of proceeding from n to $n + 1$,—chiefly applicable when the several cases in all of which T is to be true form of themselves a series in which each successive $(n + 1)^{\text{th}}$ member is formed in the same precisely definable way out of the preceding n^{th} member. If then it follows from the way in which the member $n + 1$ is formed from the member n that T when true of the latter must be true of the former also, it follows for the same reason that it must be true of the member $n + 2$, and so on for every member of the series. For instance in teaching the elements of algebra this method is usually employed to prove the binomial theorem for integral exponents in a palpable way by repeatedly multiplying the binomial into itself.

The general idea of this proof however is by no means confined to mathematics, but is very often applied in common life, sometimes under the not quite appropriate name of a proof by *analogy*. In support of a plan or a statement we first mention an instance in which the plan is evidently advantageous, the statement obviously true; then we show that the other conceivable cases are in reality distinguished from this case by no feature that could possibly make a change in this respect; and thence we conclude that T holds good universally. It is easy to see how a careless or sophistical use of this method may lead to error. Between two very different cases A and Z we insert a great number of intermediate cases, each separated from the next by an inconsiderable difference d. Then instead of showing that if T is true of A it must also be true of but at T: they are therefore not affirmative but negative proofs in respect of non-T. The *fifth* method of proof, the first *indirect progressive* method, would have to show that non-T is false on general grounds, and this may be done by syllogisms in the first and second figures with a universal negative premise. But we shall seldom find an opportunity of applying this form of proof: if there be a direct proof for T we shall prefer it; if there be none, a universal refutation of non-T is usually no easier.

The only form of this method therefore which is practically important is the secondary form, which in the place of non-T, the contradictory of T, substitutes the complete sum of all its contraries. As these contraries are all quite definite positive statements, there is more hope of being able to disprove each upon general grounds, and therefore by a progressive method. The proof that non-T is universally false which is formed by the union of these several negative proofs is then evidently a regressive argument corresponding to the positive collective proof. When T and all the contraries of T are conceived as together forming the sum of all possible relations which can subsist between S and P, the subject and predicate of T, the form of proof of which we are speaking becomes that which is known under the name of *proof by exclusion:* the truth of T then follows from the falsity of all the other members of this complete disjunction. One of the most important applications of this form is the special case of a tripartite disjunction, in which T has two contraries, i.e. in which non-T divides into two contradictories: then we get the *proof by the method of limits.* We are familiar with this proof and its very great importance in mathematics, where it belongs equally to inventive and demonstrative reasoning: every magnitude a is either equal to or greater or less than another magnitude d with which it may be compared: if it can be shown that it is neither greater nor less than d, the proposition $a = d$ is proved. In practice this train of reason generally takes another line: for the above statement presupposes that our attention has already been directed to the definite magnitude d which is proved in the end to be equal to a. As a rule this is not the case, but we only know that a is less than a second proximation to the truth which may be carried as far as we please. he best known and most elementary example is the determination the length of the circumference of a circle by enclosing it between larger circumscribed and a smaller inscribed polygon, and diminishg the former and increasing the latter without limit by continually iding to the number of their sides. Such forms of proof deserve r attention; they are the potent instruments by which we actually large our knowledge; the development and application of this ethod by *Archimedes* is a greater advance in applied logic than any at ever proceeded from the merely syllogistic art of Aristotle.

212. A *sixth* method, the second *indirect progressive* method, would egin by assuming non-T, and proceed to develop its necessary conequences, and then from their falsity infer the falsity of non-T, the st step of course being regressive. I will here refer the reader ack to the second direct progressive proof, and only add with eference to this indirect method that it does not matter how many ue propositions may be deduced from non-T: for it is quite possile for a number of true inferences to flow even from a false proposion with respect to points whose mutual relations are not affected by e error: but a single false proposition which follows as a necessary onsequence from non-T does away with its universal validity. If is consequence merely conflicts with given facts there is properly no eason for calling this proof a *deductio ad absurdum*, though the name sometimes given to all applications of this method: all that has een done is to prove that an idea which in itself is not unthinkable or absurd is as a matter of fact untrue. But again absurd or nonensical is strictly speaking not that which is known to be impossible thought, but that which conflicts with all probable suppositions, ith our general feeling as to what is true, and a number of truths volved in that feeling, provable perhaps but not yet actually proved. hat 2 = 3 is more than absurd, it is impossible; but that the whole orld is a thoughtless jest, that parents should obey their children, in daily life, especially whenever non-T states a thought, which is perhaps in itself correct, in too general language, i.e. when it proceeds from too wide a definition of the subject S to which a predicate P is to be attached, or from too wide a definition of this P. It is in this way that we prove the unreasonableness and foolishness of a proposed law, whether it gives or takes away rights and duties, by showing what further intolerable and monstrous consequences would follow if the proposal were carried out universally. Usually however the *deductio ad absurdum* is made to include also that form of indirect proof which deduces impossible consequences from an assumed proposition and thereby refutes it.

A particular case of this is when the development leads to a consequence which at once does away with the proposition from which we started, so that the inner contradiction which lay in the assumption of its truth of itself forces us to infer that it is false. As a simple instance we may take the indirect proof of the proposition T that on a straight line ab in the same plane and at the same point c only one perpendicular cd can be made to fall. Non-T then would assert that several perpendiculars were possible at the point c under the same conditions. Now assuming that this is correct, assuming further that cd is the first perpendicular, i.e. that it makes with ab two adjacent equal angles α, any second perpendicular ce must, in order to be distinguished from cd, make with it at the point c some angle δ, while at the same time in order to be perpendicular to ab it must make with it equal adjacent angles. A look at the figure then is enough to show that the two angles $\alpha + \delta$ and $\alpha - \delta$ must be equal, and each equal to a right angle: but if $\alpha + \delta$ be a right angle, α, being a part of this right angle, is not itself a right angle, which contradicts the original supposition that α is a right angle. The equation $\alpha + \delta = \alpha - \delta$ can only hold good when $\delta = 0$, i.e. when ce and cd coincide. The proposition T therefore holds good: at the same point in a straight line there can be only one perpendicular in the same plane.

We are constantly led to proofs of this kind when we have to do with the simplest fundamental perceptions or propositions concerning a coherent field of thought: the impossibility of apprehending the relation of S to P otherwise than as it is expressed in T, i.e. the fruitlessness of the attempt to affirm non-T, will always betray itself by the fact that the consequences which follow from it destroy or alter the subject S or the predicate P, which were both assumed to be valid for non-T in the same sense in which they were valid for T.

o distinguish them, they bear just the same relation to the falsity of on-T that the two direct regressive proofs bear to the truth of T.

The former (the seventh) method would work back from non-T o the conditions necessary to its truth, and then reason back again rom the falsity or inconceivability of these principles to that of on-T. In its application this method differs but little from the corresponding progressive method; for the principles which are neessary to the truth of non-T can only be found by taking non-T s their basis and developing them from it as its consequences, i.e. rogressively. The latter (the eighth) method would start from given acts or principles and proceed to show that they cannot be founded pon non-T as their basis, but rather expressly require the falsity of on-T. This also we shall find can only be carried out by either leveloping non-T progressively into its consequences, and ascertaining that if they held good they would make the given facts imossible, or by taking these given facts for basis and deducing from hem, progressively as before, their necessary presuppositions: but his will very seldom be of much use, for in that case it will sually be easier to establish directly that T as such a presupposition nust be true, than indirectly to establish that non-T cannot be true.

I will conclude this survey with the general remark that I believe hat I have correctly distinguished in my classification the various ims of demonstrative reasoning, but that not every one of these ims has corresponding to it an equally important and equally peuliar form of proof, clearly distinguishable from all the other forms; t was enough therefore to examine in detail only those which have n practice shown themselves to be methods that are frequently pplicable.

214. The reader will be surprised at the absence from my list of he proof by *analogy*; I do not believe in its existence. In all cases vhere we believe we can prove *by* analogy, the analogy in fact is listinctly not the ground of the conclusiveness of the proof; it is only he inventive play of thought by which we arrive at the discovery of a ufficient ground of proof: it is upon this ground, by means always of subsumption of the individual under a universal, that we establish he necessity of the proposition to be proved. Although it will take a under a condition X, S and X may be comprehended in a general conception M, of which as such P is true; under the same M we may subsume any other S which is like the first S and subject to a like condition X; therefore the same predicate belongs to this S as to the first. This transformation, which may here seem arbitrary and superfluous, cannot be dispensed with in the case of the second principle,—of unlike things under like conditions unlike assertions are true. We may be inclined to regard this also as unconditionally true, but difficulties thicken upon us when we try to apply it. Suppose that unequal magnitudes a and b are divided by the same third magnitude c; in this case the principle will hold good; the quotients will be unequal. But take a second case: divide each of the unlike magnitudes by itself, and the principle seems to fail; the quotient in both cases is 1. Of course it will at once be urged that the condition X, to which the unlike elements a and b are subjected, is just not alike for both; for when we divide each magnitude *by itself*, we introduce the unlikeness again into the meaning of the condition which was to have been alike for both. But this explanation will not cover the following third case; multiply both by 0, and the product in both cases alike is 0. It cannot be denied that the operation of taking a magnitude no times has but one meaning, and does not as in the former case depend upon the value of the magnitude to which it is applied: on the other hand it may be remarked with justice that in this case the meaning of the like condition or like operation X is precisely of such a peculiar kind as to annul the unlikeness of the magnitudes to which it is applied. Take a fourth case; if we square the unlike magnitudes a and b, the meaning of the condition to which we subject them is again dependent upon the magnitudes themselves as in the second case, only with the opposite result; the squares a^2 and b^2 are unlike. Fifthly and lastly the results are once more like and both equal to 1 if we raise a and b to the 0th power. In this case the condition to which we have subjected the unequal magnitudes a and b seems to be independent of their value; but in fact the raising to the 0th power is a quite inconceivable operation; we must remember that in general a^{m-n} is ns are true when the condition is of such a nature as not to affect unlikeness of the unlike things: but that like assertions are true them when the condition is such as to annul their unlikeness. t these two propositions are mere barren tautologies: they do not ble us to decide even so much as whether the assertions to be de will be like or unlike without a previous analysis of each case to ch us what is the general rule MP under which a and b are lly to be subsumed here, and what are the definite predicates P^1 d P^2 which attach to them in virtue of the special sense in which y, as unlike kinds of M, partake of this universal P. When we ve found these predicates P^1 and P^2 we see whether they are like unlike; it is not by analogy therefore, but entirely by subsumption t the conclusion is arrived at.

215. To the third principle, that of like things under unlike conions unlike assertions are true, a higher value may be assigned; it ould in fact be inconsistent with the law of identity if an identical ject under really different conditions showed no trace of the inence of this difference, and I shall have occasion some way further to make use of this proposition as a not unfruitful maxim in the atment of philosophical problems. But for the present what ikes us is the number of apparent exceptions. How could the gineer solve the problem of constructing a machine which under nging conditions regulates itself and maintains a uniform motion, he same subject or material substratum under different conditions solutely must exhibit different effects? A closer examination reoves this objection; it teaches us that in the cases here concerned her the unlike conditions are not simple but go in pairs, or that the subject is not simple, but a whole of various parts. But two rs of conditions may with regard to a definite effect be equivalent, cause the unlikenesses of the several members, in virtue of the inite relation which subsists between them, annul one another till ders are like; on the other hand various unlike the simultaneous influence of a and b it may be moved at the same speed and in the same direction as under the combined influence of c and d: if these four forces operate in the same straight line, the equality of their algebraical sum, i.e. the condition that $a \pm b = c \pm d$, is enough to give a like motion to the body; or in more general language, every motion m may be conceived as the resultant of a countless number of different pairs of components.

Now this result may be exhibited in various ways. If we regard the *sums* $a \pm b$ and $c \pm d$ as the conditions to which the body is subjected, then the conditions themselves are like one another, and the case comes under the principle that of like things under like circumstances like assertions are true: but if we leave the several forces separate, the case seems to make an exception to the third principle. Nevertheless I should like to maintain that this third principle is universally true; for its true meaning plainly is that the sum of *all* the effects experienced by the same subject or substratum under different conditions will always be different. And so even if two pairs of conditions are equivalent in respect of one kind of effect which they produce on the same subject, it does not follow that they are also equivalent in respect of all their effects, and it is not proper to attend to the former only and neglect that part of their effect which is unlike. If a and b work upon a body in opposite directions, and c and d also in opposite directions, and if their sums or differences $a \pm b$ and $c \pm d$ are like, the body certainly experiences the like motion m, and remains at rest if $a = b$ and $c = d$; but it obviously experiences very different pressures according as it is two large or two small forces that hold it in equilibrium. Though a self-compensating machine continues to act alike under constant and under varying conditions, yet the position of its parts changes as the conditions change, and it wears out faster when it is obliged to exert its compensating powers than when it leaves them unused, the conditions remaining uniform. If full sunlight falls upon one scale of a balance suspended in a vacuum, while the other is in shadow, the equilibrium is not disturbed, but the first scale is warmed and expanded more than the other. Lastly if we multiply a first by $a b$ and then by $b a$, these conditions are certainly quite equivalent in respect of the magnitude of the resulting product, but not in respect of its structure, and $a a b$ is in any case a different combination from $a b a$. It would be easy to add to these examples, already sufficiently various, and thus to confirm the universal truth of the third principle; but after all it is but of very little use for a proof by analogy; it never like in this effect, and what unlike, we can never tell without an enquiry of another kind.

The fourth principle needs but the barest mention; that of unlike things under unlike conditions unlike assertions are true is, after all that has just been said, so evidently unfounded or ambiguous, that no useful application of such a statement is conceivable.

I will only add in conclusion that the trains of thought to which the title of proofs by analogy is supposed to be appropriate do not even proceed directly from these principles, though they must be traced back to them. The presupposition on which they rest is rather that of similar things under similar circumstances similar assertions are true. Now similarity[1] is always a mixture of identity[2] in one respect and difference in another; if therefore it is difficult to base any valid inference upon the foregoing propositions which separate the mingled elements, it is still less possible to do so when the two are indiscriminately fused together in the resemblances to which appeal is made. I think therefore that I have sufficiently shown that there is no such thing as a proof *by* analogy; though I do not by this intend to deny that the observation of even remote resemblances is of great assistance to the discoverer both in detecting new truths and in finding grounds for proving given truths; for, to sum up my meaning briefly, there is no need to impugn the abstract validity of these three principles, but only their fruitfulness for demonstration. We cannot on the ground of the unanalysed similarity of two subjects transfer the predicate of one to the other, but only on the ground of their demonstrated likeness, likeness at least in respect of the conditions upon which the predicate in question everywhere depends; and this always brings us back to setting down a universal proposition MP and *subsuming* both subjects under the determining conception M.

216. We have still to consider those mathematical arguments which are commonly called proofs by *strict analogy*. As the name analogy originally meant proportion, every procedure that leads back to proportion has a reasonable claim to the title; the effect of common usage however is such that when we hear of an inference by analogy we expect an argument which reasons directly from similars to similars, without needing to take a circuitous route through a higher universal. But the methods employed by mathematicians cannot be thus opposed to proof by subsumption. A proportion between four determinate magnitudes, $a : b = c : d$, is merely the expression of a fact; it only becomes a source of fresh inferences when the last two members are left indeterminate; but in this form, $a : b = m : n$, it is the expression of a universal law; it asserts that the magnitudes yielded by the problem now before us at the moment are connected together in pairs in such a way that in every pair one member is to the other as $a : b$. If we give any definite value to m and n we get a syllogism in Darii,—all the pairs of magnitudes which the problem yields (M) have the ratio P, viz. the ratio $a : b$; but m and n (the S or subject of the minor premise) are such a pair; therefore m and n are to one another in the ratio $a : b$. No doubt this reduction to the first figure is very tedious; but we deceive ourselves if we fancy, because of the shortness of the formulated expression which the nature of the subject-matter makes possible in mathematics, that the train of thought also in a simple proportion is something shorter than that here stated. Even the simplest example of the rule of three is worked in this way. We say, if 1 pound costs two thalers, 10 pounds cost 10 × 2 thalers: here we assume, what seems to us self-evident, that the ratio between the quantity of the article and the price is *always* the same; accordingly we take the ratio of the one pound to its price as a general rule and bring the ratio of the 10 pounds to its price under it as a particular case of the rule: but the dealer perhaps sells the 10 pounds for 18 thalers and thereby shows that what we assumed is not self-evidently true in all cases, but that we really had to *make* the assumption for the purposes of our calculation: further it is evident that we tacitly conceive m and n as standing for quantities of the same article and of the same unit of currency as a and b, and so in this respect also take the first case as the general rule and *subsume* the second case under it. Every general equation which exhibits one and the same content under two different forms is equally a general *rule*, which holds good only for that kind of magnitudes which, by a convention which finds no expression in the formula itself, we intend to denote by these particular letters, and for which we originally showed the equation to be valid. It is not ι and v can be subsumed under the species m and n of which the equation has been proved to be true. Suppose we have proved by actual multiplication and by the argument from n to $n+1$ that

$$(1+x)^m = 1 + \frac{mx}{1} + \frac{m(m-1)}{1\,.\,2}x^2\ldots;$$

hat does not give us the right to infer also that

$$(1+x)^{\frac{1}{m}} = 1 + \frac{x}{1:m} + \frac{1\left(\frac{1}{m}-1\right)}{m\,1\,.\,2}x^2\ldots;$$

For in the first formula m stood only for the class of positive whole numbers, for which alone the proof by multiplication was feasible, and a fraction cannot be subsumed under it. If on the other hand we had found means to prove in the first instance that the binomial heorem *in the first case* holds true for the fractional exponents $\frac{m}{n}$, whatever positive value may be assigned to m and n, we might have deduced the first formula directly from this, since every whole number m may be expressed in the form of an improper fraction.

217. In conclusion I should like once more to connect what I have said with the *dictum de omni et nullo* or the law of disjunction. f S^1 and S^2 are two species of the genus M or two particular cases of the universal M, and if P may be predicated universally of M, we know that P may be predicated of S^1 and S^2 not in this universal orm but in the modified forms P^1 and P^2. Now in a special case it may happen from the way in which the various predicates $P\ Q\ R$ are onnected in M, that the various groups of characteristics $p^1\,q^1\,r^1$, $^2\,q^2\,r^2$, $p^3\,q^3\,r^3$ which they form in the several subjects $s^1\,s^2\,s^3$ must be dentical with one another; they then make so to say a secondary predicate Π, which may be ascribed to M itself, and which equally attaches without modification to every species of M. Thus the coneption of the triangle M requires three angles $p\,q\,r$, but the various alues of these angles in the various kinds of triangles always make up the same sum $\Pi = 2$ right angles; this identical characteristic Π herefore attaches to all triangles and we may at once ascribe it to ny single triangle when we have simply subsumed it under its genus. But apart from such special cases the p^2 or q^2 that will be proper to diminish to nought, in which case this nought must be capable of explanation. If this q^2 is to be determined, there must be a rule according to which the specific peculiarity of S^1 (which makes it not only a kind of M but *this* particular kind) helps to determine the modifications of the general characteristics of M,—in this case the modification of Q; and we must assume that the peculiar nature of S^2 will follow *the same* rule in determining q^2, the modification of the general characteristic Q which is appropriate to it. If we know this rule we can determine q^2, and this is precisely the case which is called inference by *strict analogy*, though as we have seen this rests upon nothing but the subsumption of a case under the like universal rule. But when this rule is not known, we still feel inclined to find out q^2 by considering the resemblances and differences in the relation of S^1 and S^2 to each other and to M, and the procedure based upon this we usually call inference by analogy; but it only enables us to *guess* the right result, never to *prove* it. It was known by the forty-seventh proposition that for right-angled triangles the square on the hypotenuse h is equal to the sum of the squares on the sides a and b which enclose the right angle. As this relation can depend upon nothing but the general properties of the triangle, the right angle, and the length of the sides, it is a quite justifiable impulse which bids us seek an analogous proposition about the square of the subtending side for other values of the subtended angle. If we simply put the formula in the general form $h^2 = a^2 + b^2$ there is no longer any mention of the right angle; but the formula we are seeking must mention the subtended angle, for it is evident at a glance that, a and b remaining the same, h gets longer as the angle increases and shorter as it diminishes. Accordingly to make the Pythagorean formula complete we must add another term which will become nought when the included angle $\phi = 90^\circ$: and as we cannot measure h by the angle itself, but only by a length dependent upon it, or by a numerical coefficient dependent upon it that determines another length, we may set down tentatively $h^2 = a^2 + b^2 \pm m \cos \phi$. The alternative sign $\pm$ is seen at once to be needless when we reflect that when ϕ increases beyond 90° h still increases but the cosine becomes negative; we only need the minus sign therefore in the formula. In order to find m which is as yet indeterminate we turn to the two limiting values of ϕ, $\phi = 0$ and $\phi = \pi$. In the latter case h^2 becomes equal to $(a + b)^2$ and $\cos \phi = -1$; in the former case $h^2 = (a - b)^2$ and $\cos \phi = +1$; both cases alike give us $h^2 = a^2 + b^2 - 2ab \cos \phi$. Now this formula is in fact correct for all values of ϕ, but it is as yet it would be easy to find another formula, e. g.

$$h^2 = a^2 + b^2 - 2\,ab\cos\phi\,.\,\cos^2(\pi - \phi),$$

which would also cover them; which of the two is also satisfied by all the intermediate values of ϕ remains unsettled, till by an easy geometrical construction, with the help of the forty-seventh proposition, we decide that the formula we first took is universally true. I have dwelt upon this simple example in order to show how many subsidiary considerations are necessary before our efforts to discover new truths by the analogy of given truths can even be put into a path which promises success.

CHAPTER V.

The discovery of grounds of proof.

218. In any demonstration of a given proposition *T* the most important thing is to find the major premise G, from which by appropriate subsumption *T* is to follow as necessary consequence. This problem, obviously a problem for the discoverer, does not admit of any logical rule by which the solution could always be found with certainty, without counting upon the free co-operation of the sagacity of the individual enquirer. We must suppose that previous reflexion has already supplied a number of general truths, which are related to the content of the given *T* in such a manner as to be serviceable for the purpose in hand, and which, recalled to consciousness by the similarity of the matter in question, suggest themselves to the seeker as grounds for explaining the given proposition. But over and above this he must have the keenness of mental vision which detects among these truths the appropriate ground of proof, and sees the changes which perhaps are necessary to the subsumption of the given proposition under it, and this we must allow is to a large extent matter of native talent and not even independent of the moods of the moment. The logical relation however which subsists between the parts of a true and therefore demonstrable proposition must be able to give us at any rate such a clue as may save us from groping entirely in the dark and to some extent put us into the way of finding, after further search of course, the ground of proof. This clue lies in nothing else than the fact which we remarked some time ago that every true universal proposition *T*, when we supplement and complete its subject and its predicate by all the subsidiary characteristics which are hinted at or implied though not expressed, must become an *identical* proposition. If then for the conception *S*, which occurs as subject in the proposition *T*, we substitute this completed sum of the several ideas which it contains in the forms of combination proper to them, this must include the ground which justifies the predicate; on the other hand if we substitute for *P* in its completeness the sum must subsist between S and P.

219. Suppose first that we have to prove the given proposition T, that the angle in a semicircle is a right angle. By analysis of the subject we find that by the angle in question we have to understand one whose enclosing lines start from the extremities a and b of a straight line $a\,b$ and intersect each other at a point in the circumference of a circle described about $a\,b$ as diameter. Now if the second part of this definition, which determines the position of the point of intersection e, is to be satisfied, the distance of e from c the point which bisects the straight line $a\,b$ must be equal to half this line, i.e. to $a\,c$ or $c\,b$. This requirement which follows from the definition of the subject suggests at once the one slight subsidiary construction that we need: we must draw this line $e\,c$, in order to bring before our eyes the relations upon which depends the necessity of the given proposition T. When we have drawn $e\,c$ the triangle $a\,e\,b$ which we already had is divided into two isosceles triangles $a\,e\,c$ and $e\,c\,b$, while the angle at e is divided into two angles α and β: from the fact that both triangles are isosceles this follows, and so far this alone, viz. that the angle $e\,a\,c = \alpha$ and that the angle $e\,b\,c = \beta$; but from the way in which these two triangles make up the triangle $a\,e\,b$, $e\,c$ being common to both, and $a\,c$ and $c\,b$ falling in the same straight line, it follows further that the four angles α, α, β, β, are together equal to the sum of the angles of the triangle $a\,e\,b$. We have then $2\,(\alpha+\beta) =$ two right angles, and as $\alpha+\beta$ is the required angle in a semicircle, we have found that it is equal to a right angle.

It is not always that we can get what we want by such an easy analysis as in this very simple case: let us therefore take another case to illustrate an artifice that is very frequently applicable. We may perhaps already have got a proposition T which teaches us what is proposition *T* if it is true, or find the true proposition *T* if the given proposition is false, or if none were given at all.

Suppose the question to be what is the sum of the angles of a triangle. Assuming that the propositions concerning parallel lines and their intersection by a straight line have been established without taking triangles into consideration, we take two straight lines *a d* and *b c* parallel to one another, and intersected by a third straight line *a b* in the points *a* and *b*. These three lines thus form no triangle, but an unclosed space; but we know *S* the sum of the two angles *d a b* and *a b c*, and know that it is equal to two right angles. If we now make the line *a d* turn about the point *a* so as to incline towards *b c*, there is formed between its new position and its old one an angle ϕ, which is taken away from *S* the sum of the interior angles; but at the same time there is formed between *b c* and the line which has been deflected to meet it a new angle, the third angle which together with the remainder of *S* the sum of the original angles makes up the three angles of the triangle now formed, and which by the propositions about parallels is equal to the angle ϕ which was excluded from *S*. Thus therefore in the passage to a triangle from what is not a triangle the sum of the angles contained by the three lines loses ϕ and gains ϕ; it is therefore equal to two right angles in the triangle as before.

220. Suppose we want to prove or to find the conditions of equilibrium for a perfectly free and absolutely rigid body, operated upon at various points by various forces in various directions. In the conception of a body here employed perfect freedom needs no further analysis; as absence of every conditioning relation to others it is quite clear as it stands; only if the relations were present should we have further to determine their import: the absolute rigidity of a body means that the distance between any two points in it is unalterable.

Now if no force were acting upon this body, we should be able to say of it that it either was at rest, or was continuing an original motion at a constant speed *c*: we should therefore only have to set down $c = 0$ in order to express the conditions of the equilibrium intended, the equilibrium of rest. But in order to decide how the body maintains equilibrium when forces are acting upon it we must adopt the same method as in the preceding case and first see how it would move if it did move, and then negate all the conditions which would be inseparably bound up with this motion. This is not merely a useful contrivance without any logical basis; for the equilibrium we are now seeking must be conceived not as mere rest but as the negation of the movements which tend to disturb it. Now as the only kinds irst-named kinds of motion; negate them and the possibility of the hird kind is gone.

221. If we first consider only movement from place to place or novement of translation, expressly excluding all rotation, it follows rom the definition of rigidity that all the parts of the rigid body must nove onward in rectilinear and parallel paths and therefore with the ame velocity. In whatever way a force acts therefore, if it has given o *a*, one part of the body, a velocity *c*, it must always, provided the novement be one of translation and not of rotation, have given the ame velocity to *b*, any other part of the body. Hence we are able, to ur great convenience, in estimating the movement of translation vhich finally results from all the forces acting upon a rigid body to eglect the fact that they act upon different points: we may treat hem all as acting, in lines parallel to their given directions, at an aritrary point in space, at which we suppose the mass of the body to e concentrated, and then by the known rules for the composition of orces determine the resulting movement R which they would impart o this point; the magnitude and direction of this resultant R are then dentical with the magnitude and direction of the motion which the ody receives under the united influence of these forces, and it renains at rest when $R = 0$. If we express this by saying that the ody rests when the effects of all the impulses to motion which are rought to bear upon it annihilate one another, the proposition is an dentical proposition for which no reason need be sought: our exlanation however further states the condition under which that nnihilation takes place, viz. the very same condition as that under vhich it will take place when all the forces are acting upon the same oint.

222. In mechanics however it is usual not to state this condition nder this form $R = 0$, but to break it up, for convenience in applyng it to calculations, into three equations, which I proceed to mention, ince the feasibility of a logical precept is certainly one of the quesions which applied logic ought to consider. If the number n of the relations to a single common standard, instead of measuring the divergence between every two. The usual proceeding then is to lay down three axes $X\,Y\,Z$, at right angles to one another, and then to determine the direction of each force P by the three angles $\alpha\,\beta\,\gamma$ which it makes with these axes or with lines parallel to them, at the same time conceiving each force as resolved into three components parallel to these axes, which forces will according to a familiar proposition be $P \cos \alpha$, $P \cos \beta$, and $P \cos \gamma$. The three sums then made by adding together all the components of like direction, i. e. the sums $\Sigma\, P \cos \alpha$, $\Sigma P \cos \beta$, $\Sigma P \cos \gamma$, will be the resulting forces which tend to move the body in directions parallel to the axes $X\,Y$ and Z respectively: if each of these sums as they stand be equal to nothing, the body does not move from its place in any of these three directions, and therefore does not move at all, for any movement in an intermediate direction would include a simultaneous change of place in the direction of two of these axes at least, and this has just been denied. So instead of $R = 0$ we have these three equations, $\Sigma\, P . \cos \alpha = 0$, $\Sigma\, P . \cos \beta = 0$, and $\Sigma\, P . \cos \gamma = 0$, to express the condition which annihilates all movement of translation.

223. We have still to look for the other conditions which make the rotation of the body impossible. Suppose now that a straight line rotates about one of its points; then with the exception of this one point which we regard as fixed (thus making it impossible for the whole line to have any movement of translation) all the other points of the line alter their co-ordinates. The line therefore cannot rotate if two of its points have unalterable co-ordinates. But though the line be fixed along its whole length, a plane which contains it may rotate about it: then all the points in the plane which lie outside this axis alter their co-ordinates: the rotation of the plane therefore becomes impossible if any point in it outside the axis be fixed, or in general if the three angular points of a triangle drawn anywhere in the plane be fixed. The same condition is obviously sufficient to make rotation impossible for a rigid body, every point of which is at an unalterable distance from every point in a fixed plane taken at will in it.

prove it by applying the previous three equations to each of these three points we must be able to prove what is the resultant effect at each of them of all the forces acting not at this point but at other points: but this, as will easily be seen, is the very thing that we are still trying to ascertain. We must take another course therefore, and, since the position of the triangle just mentioned is perfectly arbitrary, the course which most naturally suggests itself is to dispose its three angular points in the three axes XYZ, by reference to which we have already determined the directions of all the forces in operation: but the position in each axis of the angular point which we place in it is also perfectly arbitrary: we may therefore regard every point in each axis as a point whose position is unalterable, i. e. we may regard the three axes themselves as three fixed lines, in relation to which, if rotation is to be excluded, no point of the body can change its position and distance. If finally we consider the three axes as three dimensions which lie within the body itself, or as identical in position with three series of points in the body at right angles to one another, it follows from the definition of rigidity that the fixity in space of these series of points is all that is required to make any change of place impossible to the remaining points of the body. The problem therefore reduces itself to showing that all the forces in operation are unable to impart a rotatory movement in any direction to any of these three series of points, or to any of the three axes XYZ now conceived as capable of moving out of their previous direction.

224. This last way of treating the matter however would not serve as a convenient basis for calculation except when the directions of all the forces concerned passed through the three axes. This will not generally be the case: in order to take into account those forces which when produced go past those series of points without cutting them, we must substitute for the three lines three planes intersecting each other at right angles, each of which will therefore include two of these axes: the direction of each force produced if necessary must cut one of these planes. The problem now is to show that all the forces in conjunction are unable to cause either the planes XY and XZ to rotate about X, or the planes YZ and YX to rotate decomposed into three forces $P \cos \alpha$, $P \cos \beta$, $P \cos \gamma$, parallel to the three axes. The last of the three we need not consider here; it could only cause a movement of translation in the direction of the axis Z, which is already excluded by the equations of § 222, or a rotation of the plane XY about X or Y, which also need not be considered at present. Of the two other forces $P \cos \alpha$ is perpendicular to the plane ZY and $P \cos \beta$ to the plane ZX; the two tend, as is shown by an easy construction, to cause the planes ZX and ZY, and so the body in which these two planes are immoveably united, to rotate in opposite directions: the direction of the rotation which actually results would therefore depend upon the difference between the two forces. Not simply upon their difference however, for a proposition which at present we will only allude to, teaches us that the rotatory effect of a force which is perpendicular to a line is to be measured by the product of its intensity into the distance of its point of application from the axis of rotation. For the force $P \cos \alpha$ this distance is y, and for the force $P \cos \beta$ it is x: the difference of the products $y\, P \cos \alpha$ and $x\, P \cos \beta$, or the difference between the two momenta, must be equal to nought if P is to cause no rotation about the axis Z. We must repeat the same considerations with regard to all the forces concerned, and we finally get, as the condition which prevents all rotation about the axis Z, the equation

$$\Sigma\,(y P \cos \alpha - x P \cos \beta) = 0.$$

The other equations which make rotation about the axes X and Y impossible, will obviously, as the three directions are perfectly homogeneous, be of the same form; and, since even artificial aids to memory are not beyond the province of applied logic, I will remark that the equation for non-rotation about an axis never contains the elements which refer to this axis, but consists of the sum of the differences of two products, each of which unites a component force in the direction of the second axis with that co-ordinate of its point of application which is parallel to the third axis. The formula $\Sigma\,(z P \cos \beta - y P \cos \gamma) = 0$ annihilates rotation about X; the third formula $\Sigma\,(x\, P \cos \gamma - z\, P \cos \alpha) = 0$ annihilates rotation about the axis Y.

225. The proposition about the equilibrium of rotatory forces which we made use of in the preceding discussion is easily arrived at in the domain of statics by a slight device which reduces the question to the composition of motions. If I now select another mode of proof, I do so of course with no idea of improving the science of statics; I only adopt a treatment which is as far as possible independent

If now a force W acts upon the point b, and causes b, in whatever way, in the time t to pass through the segment $n\,\omega$, it must likewise have compelled any other point in the line at the distance ρ to describe in the same time t the segment $\rho\,\omega$: and conversely, any force which applied at the point ρ has caused this point to move through the small segment $\rho\,\omega$, has necessarily compelled all the other points in the line to describe segments corresponding to their distance from a. We now ask what must be the nature of the two forces P and Q in order that when they are brought to bear at the points p and q respectively they may produce precisely equal results, and accordingly when acting in opposite directions upon the line $a\,b$ may prevent each other from making it rotate. Now the conception of rigidity, i. e. the conception of the simple immobility of a, is too far removed from conceptions of movements to tell us how the latter would be affected by the former: we should have first to conceive rigidity itself as the result of movements, in order to make it homogeneous with the other movements upon which it is to exercise a restraining influence. Further it is impossible to compare P and Q so long as they act under different circumstances whose modifying power is yet unknown: we can only estimate them by velocities ϕ and ψ which they would impart under perfectly similar conditions to a perfectly similar moveable object: and lastly though P and Q may be applied at the single points p and q, they cannot operate upon them alone; in order to set up or to hinder a rotation, the effect of each must spend itself over all the points in the line $a\,b$, and we must know the mode of this distribution before we can understand how the effect of the one can annihilate the simultaneous effect of the other at every point in the line.

226. These requirements we may satisfy in the following way. Suppose that $a\,b$, which is equal to n, is first of all a perfectly free rigid line, consisting of an infinite number n of homogeneous points in parallel directions. This movement of translation passes into a rotatory movement when we give to the various points of the line various counter-velocities, which must be conceived as at right angles to $a\,b$ not only at the beginning of the rotation but at every subsequent moment. To the extremity a we assign a counter-velocity $-\omega$ by which it becomes the fixed point which our problem requires; to the point b we give a counter-velocity which $=0$, so that it maintains undiminished the velocity ω imparted to it by W; the intermediate points must meet so much resistance as will leave to each point ρ, whose distance from the fixed point is ρ, a residual velocity whose amount is already known, viz. the arc $\frac{\rho}{n}.\omega$, whose length is to ω, the path of the free end, as ρ is to n: the sum of the velocities of all the points ρ, from $\rho = 0$ to $\rho = n$, must be equal to $\frac{n.\omega}{2}$. Now a force P, which would give to a free element the velocity ϕ, would give to an element p in our rigid line the velocity $\frac{p}{n}\cdot\phi$, if p were subject to the above-mentioned resistance but able to move by itself; but as it cannot move by itself, the impulse imparted to it must distribute itself over the whole line. However this distribution may be effected, we already know the result; it can produce nothing but a rotation of the whole line, in which every point ρ receives a velocity proportionate to its distance from the fixed point and the sum of all the velocities is $\frac{p\,\phi}{2\,n}$. Every point ρ therefore receives the velocity $\frac{\rho}{n}\cdot\left[\frac{p}{n}\cdot\frac{\phi}{n}\right]$. Precisely similar statements may be made about a second force Q, which would give to a free element the velocity ψ, but to an element q of the line which is fixed at one end would give the velocity $\frac{q}{n}\cdot\psi$; when applied at q it would give to any other element ρ of the line the velocity $\frac{q}{n}\cdot\left[\frac{q}{n}\cdot\frac{\psi}{n}\right]$. Now if these two forces operating at p and q or the two velocities produced by them are to be such that when acting in the same direction either would annihilate one and the same third movement of the line, or that when acting in opposite directions they would counterbalance each other, then for any point ρ the two expressions which we have just found for their effects must be equal to one another, and so therefore $p\,\phi = q\psi$, and $\phi : \psi = q : p$. In other words the length of leverage must vary inversely as the strength of the force.

at under these conditions equilibrium will be the result. Now
as is commonly done, we imagine Q to be a weight, suspended
a hook or cord at m, and P as a strain exerted from above, we
citly assume that it is indifferent whether of the infinite number of
finitely thin perpendicular strips into which Q may be decomposed
thought each severally grapples the point of the lever which it
ould touch if produced, or whether all these several forces operate
on the lever only through a single representative which unites
em all, viz. the cord. Once assume this, and it must also be
different whether we conceive Q as *one* body, or as divided per-
ndicularly by a geometrical plane into two halves which touch
e another at the surface of section, and each of which is attached
the lever by a separate cord which unites all its forces in one
sultant. If then m was the distance from the fulcrum of the
iginal point of attachment, $m - x$ and $m + x$ are the correspond-
g distances of the new points of attachment of these two cords. In
her words equilibrium is preserved when two forces each of which
equal to $\frac{Q}{2}$, and whose sum $= P$, are applied at equal distances
ht and left from the attachment of the opposite force P: for the
rds themselves, or their tensions, are now the forces which are
rectly applied. Now so long as these tensions are the resultants
the forces of gravity united in the two bodies $\frac{Q}{2}$, it is evident that
is quite indifferent how these bodies $\frac{Q}{2}$ are shaped in other respects,
different therefore whether they still touch one another as before,
whether by increase of their length and diminution of their thick-
ss they become two separate bodies with a space between them.
we follow out this line of thought we see that it is quite possible
carry the displacement of one $\frac{Q}{2}$ to the left and of the other to the
ht by equal distances x as far as we please, till at last x becomes
ual to m: when that is done one $\frac{Q}{2}$, say the one that was displaced

it is nevertheless absolutely inconclusive. So long as x was less than m, the $\frac{Q}{2}$ that was moved away to the left had still a recognisable and intelligible influence upon the equilibrium of the lever; we could still see plainly that it together with the other half that was moving away in the opposite direction made up the force that was sufficient to counteract P: but so soon as x becomes equal to m, and the effect of this $\frac{Q}{2}$ altogether ceases, there is a break in the thought: for one of the points of relation has vanished, and our whole reasoning was founded upon its relation to the other. For when we first applied Q at the point m itself, and then disposed the two halves of Q symmetrically on either side of m, what we inferred held good in the first instance for the free line $a\,b$, which was supported at m by the force P: the fixing of the end a was not contemplated at all: though of course the same inferences held good also for the case when a was fixed, so long as it could be proved that, irrespective of this, equilibrium was maintained by the way in which the weights were distributed; for if equilibrium was maintained thus it could not be disturbed by the fact that a was over and above this regarded as fixed. But so soon as the influence of one half of Q vanishes, we no longer have equilibrium on the same grounds as before, and it is by no means self-evident that the vanished condition is exactly replaced by the fixing of the end a. We should in fact need for this special case to find a subsidiary proof which should show that a being fixed the effect of the half of Q was all along getting less and less as it approached a, and that equilibrium was nevertheless maintained; therefore it would continue to be preserved when the influence of this weight was reduced to nothing, while the other was removed to a corresponding distance. But if we examine it, we see that this subsidiary proof would in reality be the proof of the main question, i. e. it would be the proof of nothing less than the proposition that the power of equal forces to move a lever varies as their length of leverage. This mode

y directly compounding all the forces in operation so as to arrive
their final resultant; we often have to state certain universal con-
itions which it must satisfy, or certain limits within which it must
eep : with these assumptions then the several data of the given case
pply means for the complete determination of the result. These
ethods, among which we need only mention the application of
Alembert's principle, are quite invaluable and cannot be dis-
ensed with: but as they do not clearly show the history of the
sult which we calculate by them, we still feel a wish to employ
rect constructions so far as possible. I will mention in connexion
ith the preceding problem of the equilibrium of rotatory forces that
the motion which they generate when they are not counteracted.
he rule for calculating it is reduced to these two very simple pro-
ositions: (1) if a force acts upon a body that is able to move freely,
s centre of gravity takes the same rectilineal motion which the whole
ass of the body would take if it were concentrated at the centre of
avity and there acted upon by the force: (2) at the same time the
ody takes the same rotatory motion which it would receive from the
me force if its centre of gravity were fixed. Now in this very neat
ivision of the result there lies a paradox. For if the direction of the
rce passes through the centre of gravity, there arises according to
e second proposition no rotation, but only a rectilineal movement
translation, and yet we should suppose that in this case the force
as acting upon the body under the most favourable conditions: but
the direction does not pass through the centre of gravity, in which
se the force would seem to act under less favourable conditions,
ere follows not only the entire previous result but also a rotation,
hich strikes us as an addition without any obvious reason. If the
mpound velocities of the various parts of a body which is at once
oving onwards and rotating be decomposed into velocities in the
rection of its rectilineal course and velocities in the directions
erpendicular to this and to the axis of rotation, the sum of all
e former components, each multiplied into its differential-mass,
equal to the product of the whole mass multiplied into its recti-

actually occur, and we are forced to ask where they come from.

229. It is sufficient to answer this question in the simplest conceivable case. Let a and b be two equal masses, which we conceive to be concentrated at their centres of gravity: suppose that they act upon each other so as to remain always at the same distance $a\,b$ from one another: we may say then that a and b are united by a rigid unchangeable line $a\,b$ which has no mass. In order to simplify the figure to be drawn, conceive ab to be so fitted into the angle of two rectilineal axes which intersect at O that a lies upon the axis X and b upon the axis Y: at starting then we have for the mass a $x = \mathrm{o}\,a$ and $y = 0$, and for b $x = 0$ and $y = O\,b$, while for the centre of gravity of the system $a + b$, which lies in the centre of the line $a\,b$, we have $x = \frac{Oa}{2}$ and $y = \frac{Ob}{2}$. We will now suppose that a certain velocity is imparted to the mass a in the direction of the axis X, and that $a\,\alpha$ is the path which it would traverse in an indivisible moment of time under this impulse if it were free. As no force is acting directly upon the mass b, it would then remain at rest, and the line $\alpha\,b$ which expresses its distance from a which has moved away would be longer than the original line $a\,b$. But the forces in operation between a and b, which according to our assumption maintain the distance $a\,b$ unaltered, oppose themselves at every moment to the beginning of this elongation the measure of which would be $\alpha\,b - a\,b$, and prevent it, by making the two bodies approach one another in the direction of the line at the extremities of which they would be found if the elongation actually took place. Since neither of the two masses can one-sidedly compel the other to follow it, but both masses, being assumed to be equal, must by the principle of the equality of action and reaction displace each other to the same extent, we shall find their new positions α^1 and β by cutting off from the line $\alpha\,b$ the length $\alpha\,\alpha^1$ equal to $\frac{\alpha b - a b}{2}$, and from the line $b\,\alpha$ the length $b\,\beta$ also equal to $\frac{\alpha b - a b}{2}$. If from α^1 we let fall an ordinate, which we will call dy, upon the axis X, and from β let fall a perpendicular, which before any velocity was imparted to it: the centre of gravity therefore has received an impulse to move in a direction parallel to the axis of X, i.e. in the same direction in which a would have been impelled o move if the force had been brought to bear directly upon it. At he same time we have for the extremities α^1 and β the abscissae $Oa + a\alpha - dx$ and dx respectively, and thus for the new position of the centre of gravity we have the abscissa $\frac{Oa + a\alpha}{2}$; therefore, since the abscissa of its original position was $\frac{Oa}{2}$, it has received half of the velocity $a\alpha$ which the force applied to a tended to impart to a, and this is precisely the velocity which the same force would have imparted to the whole mass of the system (which is $a + b$ or $2a$) if that mass had been concentrated at the centre of gravity and the force applied to it there.

These considerations apply to the first instant of the whole motion, in which (as is usually assumed) the force applied to a, working instantaneously, gave it a certain velocity without any lapse of time, and in which the corrective reaction of the forces at work between a and b also took place without lapse of time. Since from this instant no external force any longer operates, all the motions produced will simply continue according to the law of persistence, only the internal forces that act between a and b have to be continually at work in order to prevent a and b from flying off at a tangent, and to maintain them at a constant distance from their centre of gravity; they thus generate a rotation which is circular in relation to this point, and since they are continually diverting the two masses from their momentary direction into another without any breach of continuity, the rotation takes place uniformly in a circle and with the same constant velocity with which both masses were impelled in a straight line at the first moment must impart when it has at the same time to move the mass *b* in the contrary direction. From this we see that a momentary external force, whether its direction pass through the centre of gravity or not, always produces in the body the same sum of movements of translation: the rotation which is added in the second case is due to the internal forces which act between the parts of the system moved. But these forces are by no means inoperative even in the first case where no rotation occurs: but in the first case their only effect is to cause the several parts of the mass, which are arranged in a straight line at right angles to the direction of the motion imparted, to maintain this order during the onward movement, an effect which reveals itself in no relative movement of the parts about their advancing centre of gravity so long as we proceed upon the assumption that the body is absolutely rigid; but it would at once announce itself in such movements if we conceived say three equal masses *a b c* united to one another by *pliable* cords and then imagined an impulse to be brought to bear upon the centre of gravity of the whole system which lies in *b*.

230. In the analysis which is required for the discovery of the grounds of proof we try not only to bring out the elements which are essential to the truth of the consequence to be proved, but also to eliminate those that are unessential for that purpose. For instance it is not uncommon in answering statical and mechanical questions to start from the supposition of a rigid line without mass. Now it may be granted that in the conception of a finite straight line the characteristic of finiteness implies the constant contact of each point with two neighbouring points, and the straightness implies that the line is rigid and cannot bend: only as a mere geometrical line it is not an object that could be set in motion by forces at all; the capacity of being affected by forces belongs to the lineally arranged *mass* only, and it is only the forces exerted upon one another by the minute components of the mass that actually give to this material line the rigidity and unalterable length which is merely demanded in the geometrical conception.

build in carrying out such enquiries. A line must undoubtedly have mass if forces are to cause it to rotate about its extremity, but with a view to the laws which regulate the effect of these forces it is only necessary that the mass be the same at any cross-section of this material line; any irregularity in its distribution would constitute a special case, in determining which we should have to apply with reference to these special data the laws of that simplest case when we have the problem in its purest form; on the other hand it is perfectly indifferent for these laws how great this mass is; the proportions between the forces and the leverages necessary for equilibrium are precisely the same whether the lever be thick or thin, whether its specific gravity be greater or less. When we speak of a line without mass therefore we do not strictly speaking set down its mass as nothing but rather as a unit, and further as a unit to which any value great or small may be given, and which disappears from our further calculations just because as an equal factor of all the terms that stand in proportion to one another it does not in the least contribute to determine or to alter the relation which subsists between them. This was the thought upon which the foregoing exposition rested. The line $a\,b$ was conceived as a line of mass, and every one of its points as a differential of the mass: it was only this that made it possible to speak at all of a force W acting upon $a\,b$, and to set down this force W as equal to $n\,\omega$, equal to a sum of individual forces each of which was such as to give the velocity ω to the differential of the mass. But we should have gained nothing by constantly taking count of the mass in our calculation; only the value of ω would have come out differently according as the mass of the line or of every one of the n parts of it which we distinguished was conceived as greater or smaller; the relations between P and Q would have undergone no change so long as both were always related to the same mass. The division of the labour of proof therefore which is here introduced does not consist in first putting mass altogether out of sight and proving the law in question for the line without mass, and then enquiring in the second place what becomes of this law when mass is given to the line; on the contrary we took count of this mass at the first step, but found that its magnitude has no influence upon the general form of the law: upon this ground then we may proceed in a second enquiry to ask how differences in the magnitude and distribution of the mass affect the absolute values of the magnitudes which are to be determined by the law. As soon as we take this line without mass literally and think of its being moved, we one extremity *b* of such a line receives a velocity *c*? It cannot separate itself from the rest of the line, for then it would not be the line, but only the free point *b* that was moved: but as the line has received no motion how can it follow the point? It may perhaps be supposed that this line would rotate: then the point *b* would have to communicate its velocity to the other points, and that in degrees, more to the nearer and less to the remoter points; but we cannot see how this is to be measured, for all the forces are absent here which operating between the minute parts of a mass might cause the impulse received by one part to extend itself to the rest of the series, so that every member of it might at every moment receive a definite proportion of the impulse. Finally as there is here no reason for such an apportionment of the effect we might instead of this come to regard the whole line *a b* as a unity so closely bound together that every part of it, separable only to our thought or sense, immediately assumes the same states that are set up in any other part: setting aside the question whether every part of the line would then receive the whole velocity c or only $\frac{c}{n}$, the result would at all events be that the line *a b* remains at rest when *b* receives the velocity c and the other extremity *a* receives an equal velocity $-c$. All these absurdities are avoided by the admission that only a line that has mass can be moved, not a line that has no mass.

231. In the subsidiary processes also, the substitutions and transformations by which we endeavour to make the given circumstances accessible to our judgment, we have to avoid suppositions to which, however much they may help the imagination, no real meaning can be given. To illustrate this I will mention a proof which is often employed to demonstrate the *parallelogram of forces*. The body is supposed to move in a plane from *a* to *c*, and at the same time this plane is supposed to move from *a* to *b*; and in this way it is fancied that the course of the body from *a* to the end of the diagonal of the parallelogram *a b c d* has been ascertained. This involves two assumptions which are not expressed but to which expression must be given; they are first the assumption that the motion of the plane will not but will part company while the table flies away from under it. But if we supply this necessary condition, i.e. if we say that the body continues to move undisturbed towards *c*, while *a c* at the same time is compelled to move towards *b* and to take the body with it, the whole proposition becomes an empty tautology, and that which is assumed is precisely that which was to be proved. It must rank then only as one of the means which may be employed to give us a picture of an already demonstrated truth.

232. Among the numerous other proofs of the same proposition several proceed from a common starting-point which is of interest for the logician. They begin with a statement of the special case in which two equal forces *a* and *b* impel the body in two directions, and it is regarded as self-evident that the direction of the resulting motion will bisect the angle between these two directions. But this assumption includes the further assumption that if the forces be unequal the resultant will divide the angle into two unequal parts, and since it is impossible that the kind of this inequality should be independent of the relation between the magnitudes of the forces, seeing that the fact of the inequality depends upon it, this assumption rests on a more general assumption, viz. that if two conditions *a* and *b* tend to give each a different form to a result *c*, the recognisable influence of the two in the actual form of the result will be proportional to their magnitudes; if then *a* and *b* are equal, *c* will be as far removed from the result which would have followed from *a* alone as from that which *b* alone would produce. Now I cannot see why we should appeal to this proposition once only when we are introducing the proof, and then conduct the proof itself by other complicated considerations: whatever be the forces *a* and *b* and the degree of their inequality, we may say universally that the extent to which the moved point is deflected by the force *a* from the path of the force *b*, and by paths are considered not as empty directions in space, but only as loci which would include the successive situations of the moved point.

The following treatment is the only one suggested by this last remark. Let α and β be the two points in the paths of a and b respectively which the moved body would have reached in the same time t if it had followed the force a only or b only, and let ρ be the point in the resultant at which the body arrives in the same time t under the combined influence of a and b: then $\rho\alpha$ represents the deflection from the path a effected by the force b, and $\rho\beta$ the deflection from the path b by the force a, and $\rho\alpha : \rho\beta = b : a$. Since we can only estimate the magnitude of the forces a and b by the space which they cause to be traversed in the unit of time, the ratio $a : b$ is also for the unit of time the ratio of the spaces traversed in the direction of a and b respectively; but it must also have this meaning for any time t and for any part of t; for since a and b are regarded as forces that operate for a moment only, the movement in the direction of the resultant must take place with constant velocity and in a straight line: the length which is traversed in the direction of the resultant therefore will always be proportional to the space traversed in the directions of a and b within an equal time t, and the lines $\rho\alpha$ and $\rho\beta$ which represent the deflections will form the third sides of triangles whose two other sides increase in the same constant ratio.

233. But this proportion tells us nothing about the absolute magnitude of $\rho\alpha$ and $\rho\beta$; they satisfy the proportion so long as they are mb and ma; the value of this m would still have to be ascertained. Now there is nothing in all the data of the problem that can help us to determine this: none of them could have any influence upon it except the magnitude of a and b, including the ratio of a to b, and the size of the included angle; but the suppositions already made seem to have taken full count of the influence of these elements; and it is quite impossible that anything outside the data of the problem can contain the grounds of something that is to flow directly from the problem itself. In cases of this kind the logical course must always be to search for the *most probable supposition* that satisfies the requirements. The meaning to be attached to this expression would be very hard to define in general language; and my sole purpose in treating this problem is to make up by an illustration for the want of a precise determination of the general conception. The most probable supposition will set down that which in virtue of its nature or magnitude is the minimum that makes possible the relation which we know must subsist, and which, if it were to subsist under other conditions or with other subsidiary characteristics than those we take, would necessarily furnish special reasons for inferring them, which reasons are here absent. In the case before us the proportion $\rho\alpha : \rho\beta = b : a$ must always subsist; therefore m cannot be nought; but in order that it may subsist it is enough to set down m as equal to 1; and this value of m may be regarded as by its nature the minimum that satisfies the requirements; for any greater or smaller value, as $m = 2$ or $m = \frac{1}{2}$, may be treated as $m \,.\, 1$, i. e. as so many repetitions of the unit with the vanishing of which m itself vanishes and with it the whole relation. Unity is the only value of m which affirms that the required relation actually subsists in such a way as to enable the other special values of m to be effectively introduced as further specific characteristics, in case there be any reason in the nature of the content under investigation for preferring one of these values rather than another. Where as here there is no such reason we fall back upon the supposition that $m = 1$, a supposition which in any case is necessary, and therefore is the most probable supposition; for under all circumstances, even if m had some other value, it would hold good at the same time with that value and equally satisfy the required proportion. Let us then make the assumption and construct the figure accordingly; i. e. let us from α, the extremity of the path traversed in the time t in the direction of a, describe a circle with radius equal to the path traversed in the same time towards b, and from β describe a circle with radius equal to the distance traversed towards a; then these circles will cut one another in the diagonal of the parallelogram formed by a and b, and the direction and length of the resultant are both determined at once.

234. But even when analysis has failed to detect any grounds in the data of a problem for any other than this most probable supposition, it is seldom possible to be absolutely certain that such grounds are not there, and might not be revealed by a more careful analysis. And so no pains must be spared either to confirm the supposition adopted by subsidiary proofs upon a different line, or to establish it indirectly, i. e. to exclude all other suppositions by showing the contradictions in which they involve us. We will take this further step then.

It seems self-evident that the resultant of two forces can never be greater than their sum; it attains this maximum when they both act upon the body in the same direction, and when the included angle therefore is nothing. It has been objected to this proposition also that it is after all not self-evident that when a second motion b is joined to a motion a in the same direction b is simply added to a;

for it is conceivable that the nature of motion or that of the bodies subject to it involves conditions which might even in this case make the resultant greater or less than the sum of the two. This objection seems to me unfounded, especially as applied to the case before us. In the first place when two motions in the same direction are given at the same time to one body, we may continue to regard them as two separate motions, but it is only because we choose so to regard them. They were two motions outside the body: they may have been imparted to it for instance by two other different bodies. It may be also that in the physical act of transmission from one body to another the motions may lose or gain something: but we are here speaking not of the mode of transmission, but of the velocities, so far as they already *have* been transmitted to the body in question. In this body, here considered simply as something moveable, without regard to all its other peculiar properties, the two do not need to be combined into one, but they are absolutely *one* from the beginning, and the resulting velocity is the sum of the two as surely as any velocity is what it is. But suppose the body already has a motion a when the second b supervenes; this could not make any difference unless the body violated the law of persistence and altered its motion every instant: for if it does not alter its motion, i. e. if at the time t it is in precisely the same condition as at the time t^0, the motion b which supervenes later must combine with the still subsisting motion a just as it would have done at the time t^0 if both had begun together. We may regard it as established then that the resultant R of the two forces a and b acting in the same direction can only be $a + b$. Of course this does not directly help us to estimate the result of forces whose directions diverge and make an angle ϕ. Meantime however it is at all events evident that the resultant cannot increase with the divergence; for then it would be least when the directions are the same, whereas we have just seen that it is greatest then, and greatest when they are opposite, whereas it is evident that it is least then. But it is equally impossible that it can be independent of the magnitude of the angle ϕ; and so it must necessarily diminish as ϕ increases, and we may now say that for forces of any direction the resultant R is either equal to or less than $a + b$.

This conclusion again which is still indefinite may be brought within narrower limits. When various momentary forces to any number we please are brought to bear at the same time upon a moveable point, the total result which actually arises can only be one, and therefore cannot alter with the various arbitrarily chosen series in which we in our same in the end therefore whether we first get the resultant R out of a and b and then try to get a second resultant out of R and $-a$, or whether we combine a, b, and $-a$ so that, a and $-a$ obviously cancelling each other, b is left as this second resultant. The conception of R therefore as the resultant of a and b implies that if we again take as components R and a with its original direction reversed and calculate their resultant by the same law by which we get R from a and b, we must come back to b; and so R and $-b$ combined will bring us back to a. And this consideration holds good universally, and quite independently of the still unknown law which regulates the dependence of the magnitude and direction of the resultant upon the magnitude of the component forces and the included angle. From this then it follows that each of the three forces or motions a, b, R is under the circumstances stated above the resultant of the other two, that each is therefore less than or at most equal to the sum of the other two; whence it follows that the three may be combined in a triangle, which contracts itself into a straight line only in the limiting case where one is equal to the other two.

But as thus obtained this familiar proposition only expresses a relation between the lengths of a, b, and R; we must also make out the relations between the angles for which this relation holds between the sides. If a and b and the included angle ϕ be given, the length of R, as yet unknown, is completely determined; for these given elements therefore there is only *one* possible triangle to be made out of a b and R. Conversely, given a triangle with a b and R for sides, there is but *one* angle ϕ which the forces a and b can make so that R shall be the length of their resultant. Geometrically R in the triangle increases, if a and b are constant, as the opposite angle ρ increases; mechanically, as the resultant of a and b, R diminishes as the angle ϕ increases; between the angle ρ in the triangle therefore and ϕ the angle at which the forces diverge from one another there must subsist some definite relation which we want to ascertain. In the triangle made up of a b and R, R has not the system of the three lines a b and R so that the vertex A falls upon C, and then turn it, in the plane in which it lies, about C so that the forces a and b, which in their new position may be denoted by a^1 and b^1, proceed from C in directions parallel but opposite to their former directions. Then evidently the resultant R^1 of these forces a^1 and b^1 must be both in position and magnitude identical with R, only opposite in direction. Thus then the direction of the resultant is determined; it must be the diagonal of a parallelogram formed by the intersection of the forces a and b^1 on the one side and the forces b and a^1 on the other, or by their meeting in a common extremity, or by their being produced to such an extremity. But if the lengths of a and b are given, the length of R is also determined, it must be the third side of a triangle whose other sides are a and b^1, which $= b$, or b and a^1, which $= a$; it is therefore the diagonal of the parallelogram formed by the lengths of the forces themselves. The figure then shows that the angle ρ subtended by R in either of these triangles is the supplement of the angle which the forces make with each other, i.e. that $\phi = \pi - \rho$ *.

235. We may further confirm this conclusion *indirectly* by showing that any other supposition as to the relation between components and resultant is impossible. Let us first assume that a supposition which we wish thus to test agrees with the foregoing so far as regards the direction of R, and only makes the length of R exceed or fall short of the diagonal D. Let us suppose then that the first resultant R_1 obtained from a and b is greater than the diagonal D_1 of the parallelogram obtained from a and b with the included angle ϕ, i.e. that $R_1 = p \cdot D_1$, where p is an improper fraction. Now if we combine this R_1 with the force a turned in the opposite direction, the angle between the two being $\pi - \phi$ †, the new resultant R_2 deduced from them according to the same supposition must be greater than the diagonal got from R_1 and a with this same angle, still greater therefore than the other diagonal D_2 which would be got by combining D_1 which is less than R_1 and a at the same angle $\pi - \phi$ †. But we know upon purely geometrical grounds, which are quite independent of all mechanical assumptions, that this diagonal D_2 is nothing else than the given force b; R_2 then would be greater than b, whereas we know for the reasons lately stated that it must be equal to b. If now once more we com-

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* [See Preface]

† [$\pi - \phi$ obviously should be $\pi - \phi +$ the angle between R_1 and b]

— $qp \cdot D_1$, i.e. R_3 is q times as great as R_1 was. Thus the suppo-
on that the resultant is greater than the diagonal leads to the absurd
ıclusion that it becomes greater and greater every time that we
eat this manœuvre in its calculation. The other supposition that
s smaller than the diagonal, i.e. that p and q are vulgar fractions,
uld lead to an equally impossible diminution. In order to make
indirect proof complete it would be necessary to show further
t the supposition of a resultant of the same length as the diagonal
making different angles with the given forces would involve a
ıilar absurdity, viz. that its course would be more and more de-
ted the oftener its calculation was repeated; and lastly it would
indispensable to prove that there is no combination of these sup-
itions in which the false consequences of the one would be
ınteracted by those of the other. But as the matter stands it is
ugh to state what the requirements of logic would be; we may
re ourselves the trouble of carrying them out at length.

236. Operations of synthesis or combination may always be carried
to some end, viz. to the result obtained in each case; but opera-
ıs of analysis on the other hand presuppose an end which we
ire to reach, though it is yet uncertain whether the subject we are
ıting is produced by a combination which makes this reverse process
ınalysis possible. Even in pure mathematics therefore the inverse
rations lead to difficulties from which the direct are free; and
ıilar doubts are suggested by the common practice of *resolving*
en forces into components, though if the components were given
doubt would be felt about combining them. As any force may be
it up into countless pairs of components, how, it may be asked,
we entitled to expect that any division which we arbitrarily choose
l have a real validity in the complex tissue of facts present in the
blem before us? In general terms this doubt is easily removed.

or resistances W with the given force F, the resultant thus got is identical with that which would be obtained from the uncancelled remainder of the one component of F and the whole of the other component which would meet with no resistance. But a real difficulty arises when the direction of the resistance itself is not immediately given and an attempt is made—in a manner that seems to me hardly convincing—to arrive by an application of the law of resolution at the principle itself which is here to be followed. I allude to the supposition that a plane resists in the direction of its normal only the imparting to it of a motion which makes with it any angle ϕ. It is quite easy to see that this motion *may* be decomposed into two, of which one parallel with the plane meets no resistance because it does not act upon the plane at all, while the other perpendicular to the plane is annihilated by the resistance of the plane, or at any rate is resisted by it. But how little right we have to carry out this decomposition here as one allowed by the nature of the case will appear from the following considerations.

Let the moving body be a perfectly smooth ball, and let it move at an angle ϕ against a perfectly smooth plane E which offers an absolute resistance; contact then will take place only in the geometrical point p, to which we must ascribe the same power of absolute resistance as to all the other points of E, however this may be brought about. Now what all these other points of E have to do with the result which follows, it is impossible to imagine; we think of them indeed when we speak of the plane E; but as they are not in contact, they cannot directly contribute anything to the resistance, and in deducing the result we may set them entirely aside without altering the conditions on which the result is to depend. But if we do this and retain the point p alone, the proposition about the resistance being at right angles becomes impossible, because it becomes meaningless; for to the point p either no line is normal or any line drawn from it in any direction is normal. But another principle seems evidently to apply here: surely p, *if* it resists, will resist in the direction from which comes the motion to be resisted: there is in the first instance no conceivable reason for action in any other direction. If then in our example p were perfectly fixed, and if at the moment of contact the line l drawn through the point p parallel to the direction of the motion did not pass through the centre of the ball, p would entirely annihilate the motion of that thread of the mass which lies in this line l; then for the rest of the mass of the ball, whose motion would not thus be annihilated, there would arise a movement of rotation, which would hat part of E which is in contact will only be able to resist the point f the cube's side which it touches in accordance with the foregoing rinciple, i e. in the direction p; before we could say that it would ot be so we should have to prove that the presence of the adjacent oints $q\ r\ s$ of the plane E helps to determine the direction of the esistance offered by the point p: only this could render possible in act that co-operation of the plane which we have hitherto spoken of, hough we have not made use of it in deducing the result.

And now surely it is clear that we shall never succeed in proving his so long as we regard E as a geometrical plane without physical nass and yet with power to offer resistance. It is not even enough o regard E as the limiting surface of an inert mass; we are obliged o add a physical hypothesis about the forces with which the mass esists encroachment upon the space it occupies. We must give the plane E some thickness therefore; contact will not take place at one point nerely, but the moving body will in fact either penetrate to a certain lepth and then be thrust back by the resistance of other displaced oints of the mass, or without coming into contact while it is still at a listance it will be affected by the repulsive forces of the masses united n E. And then we should have to prove with regard to these forces f all the points of the mass that in all the other directions they annihilate one another, but in the direction of the normal to the limiting urface are added to one another and combine to make the resistance vhich annihilates that component of the body's motion which lies in his normal but in the contrary direction. And indeed it is not at all urprising that we should be obliged to come back to an assumption f this kind: motion altogether can only take place in a real thing, ot in a point or a line: still less can we hope to calculate resistances to determine the value $F(x+h)$ which Fx, a function of x, assumes when the variable quantity x increases from the limiting value which it had in Fx to the new value $x+h$. To make the statement as simple as possible I will subject the problem to certain limitations: it would take us far too long to enquire here whether they are superfluous or not. I conceive Fx to be given in the shape of an analytical expression which indicates the mathematical operations or relations from which for every definite value of x flow definite values of Fx; I assume that these values of Fx remain finite for every value of x from o to $x + h$, and that they increase continuously as x increases continuously between these limits. In propounding the problem in this form, as one capable of a universal solution, we directly assume that the growth of the function from its value Fx to its new value $F(x + h)$ will follow precisely the same law which the former value Fx itself followed as x grew from o to its former limiting value x, and further that this sameness of the generating law will hold good for each infinitely small increment dh by which the function now increases precisely as for each infinitely small dx by which it formerly increased. From this it follows that it must be possible to express either value of the function, and in the first instance to express Fx, as the sum of an infinite series, each member of which indicates the increase which takes place as x increases by the addition of each successive dx. Now if it were the nature of Fx that for every smallest increase of x, i. e. for every dx, it increased by the same constant quantity $m \,.\, dx$, its total value at the end would be the sum of an infinite series of similar members of the form $m\,dx$: the number of these members would be just as infinite as the number of dx into which we conceive the final value of x to be divided, or by the accumulation of which we conceive it to be formed; the sum of the series is the integral $\int m\,dx = m\,x$. If on the other hand the increase of Fx for every dx depends upon the value which the growing x has already attained at the time when this dx is added, then, if the formula we are seeking is to hold good for every finite x and h, the series we now have to take must consist of nothing but similarly constructed functions of x, relative successively to the continuously increasing values of x; if we call this function fx or $f^1 x$, then $Fx = \int f^1 . dx$. Now there is no reason why we should not repeat with regard to $f^1 x$ the same considerations which we have already applied to Fx; if x in $f^1 x$ now denotes a definite value out of the many values which x may assume, $f^1 x$ may also be conceived as the sum of a series whose infinitely numerous and similarly constructed

39. These preliminary remarks really contain the solution of the
lem; nevertheless I will proceed to trace it back to the following
le train of thought which may serve at the same time to illustrate
her logical method.

Evidently $F(x+h)$ is equal to the sum of its former value Fx
the positive or negative increment R_1 which Fx has received in
equence of the growth of the variable x from x to $x+h$. In
r to determine the value of R_1 we make the simplest supposition,
that for each of these increments dh whose aggregate amounts
Fx increases by the same quantity $m_1 \, dh$; then $m_1 \int dh$ which
qual to $m_1 . h$ is the value of R_1, or is the total increase of Fx.
m_1 is not incalculable. For if, as we throughout assume, the
ease of Fx is to depend solely upon the nature of this function,
given value Fx must have originated in the same manner in
h its further growth is now to take place; i. e. while x was
ing through all values from 0 to x the function then in course of
ation must have exhibited for each dx the same increase which
function thus formed now exhibits for each dh, for dx differs
dh in name only. Now Fx may be universally described as
sum of a continuous series, whose general term is represented by
$. dx$ and its last term by the same expression if x stands for the
ite limiting value which the variable x attains in Fx. For each
this series increases by $f^1 x . dx$; this quantity $f^1 x$ must be
stant and be equal to m_1 if the growth of Fx up to its given limiting
e is assumed to have taken place in the same way as the growth
this point up to $F(x+h)$. For every dh therefore Fx in-
ses by $f^1 x . dh$, and the sum or the integral of these elementary

positive or negative supplement still necessary for the complete measurement of the true value of the function. As this further addition can only be required because $F x$ does not increase by the same amount for every $d h$ or $d x$, i.e. because $f^1 x$ is no constant quantity, but dependent upon the value which the variable x has attained at each stage, it is plain that $f^1 x$ in the second term, $h f^1 x\ (= R_1)$, of our formula still denotes only the fixed particular value which the general function $f^1 x$, now to be conceived as variable, assumes when the variable x assumes its limiting value x or when the variable h is equal to 0. We cannot therefore retain this second term $h . f^1 x$ unless to each of the terms $f^1 x . d h$ of which it is the sum we add the further increase exhibited by the limiting value of $f^1 x$ contained therein for each increment $d h$ of the variable h. For this increase again we make the simplest supposition, viz. that it is the same for each $d h$ and is equal to $m_2\, d h$. This m_2 is also capable of determination. For once more if our supposition is to hold good it must react upon $F x$ also; the same law by which this function is now to increase must have regulated its origin; the increase of $f^1 x$ must have been the same for each $d x$ and equal to $m_2\, d x$. Now $f^1 x$ is the sum of a continuous series whose general term is $f^2 x . d x$; this then is the very increment by which this series or its sum $f^1 x$ continuously increases each time that x is increased by $d x$; our condition is fulfilled therefore if we put down $f^2 x$ as constant and equal to m_2: then the growth of $F x$ beyond its given value follows the same law which regulated its formation up to that point. Its total increase therefore is the sum of two series; the first of these consists entirely of similar terms $f^1 x . d h$, and its sum $= R_1$; the second represented by R_2 contains increasing terms, the first term $f^2 x . d h$ represents the first new increase which $F x$ exhibits when the former limiting value x of the variable x is increased by the first $d h$, or when the variable h, growing from 0, attains its first value $d h$; each successive $(n + 1)$th term is formed by adding the same increment $f^2 x . d h$ to the value of the nth term; $h . f^2 x . d h$ therefore is the general term of this second series, and is what we must add as supplement to the general term of the first series. The total increase of $F x$ is therefore the sum of the continuous series $(f^1 x + h f^2 x) d h$, or $h . f^1 x + \frac{h^2}{1 . 2} . f^2 x$; the second term of this expression is the required value of R_2.

3. If a given function $F x$ were of such a nature that even this second supposition was not enough to exhaust its growth, we should

still be always able to retain the terms of the formula already found if we added a fresh R_3 to supplement them. And to determine this R_3 we should repeat the same process as before. We could only require it because $f^2 x$ also is not constant, but is dependent upon the value which x has attained at any point and increases with it. Let us assume that these increments are at least constant for each dh and equal to $m_3 dh$. If then we express $f^2 x$ as the sum of a continuous series whose general term is $f^3 x \,.\, dx$, we have but to put down $f^3 x$ as constant and equal to m_3, and we thereby make sure that our general condition is satisfied and that Fx has grown to this its given limiting value in the same way as it is now to grow beyond it. Now R_2, the third term of our formula, was the sum of a continuous series, whose general term is $h \,.\, f^2 x \,.\, dh$; if then we form a second series containing the additions by which R_2 is to be supplemented, $h \,.\, f^3 x \,.\, dh$ will be the amount by which each $(n+1)^{\text{th}}$ term of this series exceeds the n^{th} term; $\int h \,.\, f^3 x \, dh$ therefore or $\frac{h^2}{1\,.\,2} \,.\, f^3 x$ is the general term of this series R_3. We obtain the second and third increment of Fx therefore by summing the continuous series whose general term is now

$$\left[h f^2 x + \frac{h^2}{1\,.\,2} \,.\, f^3 x\right] dh,$$

and the result is that

$$R_2 + R_3 = \frac{h^2}{1\,.\,2} \,.\, f^2 x + \frac{h^3}{1\,.\,2\,.\,3} \,.\, f^3 x.$$

4. It would be useless to carry this process further; it will readily be seen that if we constantly repeat the assumptions here made the required formula will assume the familiar shape of the Taylorian series, viz.

$$F(x+h) = Fx + \frac{h}{1} \,.\, f^1 x + \frac{h^2}{1\,.\,2} \,.\, f^2 x + \frac{h^3}{1\,.\,2\,.\,3} \,.\, f^3 x \ldots\ldots$$
$$+ \frac{h^m}{1\,.\,2\,.\,3 \ldots m} \,.\, f^m x + R_{m+1}.$$

But this formula would be of little value if the very assumptions on which it rests could not be shown to be the only admissible assumptions. It would be beyond all doubt logically correct, but only in the sense in which the barrenest of tautologies is correct, if it only meant that any quantity M might always be expressed by a series of quite arbitrary terms provided that we reserved the right to add a remaining term R intended to make good all the errors which we had committed by making M equal to the series. The formula has a case? From the fact that for a given function Fx one of its derivative functions $f^m x$ turns out upon actual calculation to be equal to 0, that the series therefore breaks off before the term which contains it, we plainly can infer nothing but that there is no further increase of Fx that can be got by the further development of the series we have taken; the inference that no other increase can occur at all would imply that we could prove that this very mode of calculation must include all increase of which Fx is by its nature capable. Now this point we think no longer needs special demonstration; it is contained in the assumption which we made that Fx does not increase under any other condition than that of the continuous uniform increase of x, and that its mathematical structure remains the same for every one of the values of x which have been reached. If then a function grows in such a way that for every dh it exhibits the same constant increase, while at the same time every dh that thus enters into it becomes the starting-point of a new constant increase, we get as the expression of its total increase through the interval h an infinite series, in whose terms the one set of factors h, $\frac{h^2}{1 . 2}$, $\frac{h^m}{1 . 2 . \cdot\cdot m}$ depend for their form simply upon this universal form of growth and are therefore similar in form for all functions. But in order that this series may give the specific growth of each particular function in distinction from that of any other, the other set of factors $f^1 x$, $f^2 x$, $f^3 x$ are added to these universal factors in such a way that each of them indicates the particular magnitude, dependent in each case upon the nature of the given Fx, of the first, second, third, or mth increase which occurs for each dh; the series, as the complete expression for $F(x + h)$, closes when one of these factors vanishes. The developed terms of the series above given were therefore not arbitrarily assumed; what we meant to do with them was to measure $F(x + h)$, not by a standard foreign to the nature of this function, but by the standard supplied by the function itself and by the nature of its assumed growth; if by this standard the value of $F(x + h)$ can be expressed in a finite number of terms or in a number which though infinite admits of being summed, there can be no increase derived from other sources which would have to be added to this. For however a function may grow,—provided only that it is subject at no stage of its growth to the introduction of new conditions from without,—the continued repetition of the assumptions above made (first of a constant increase, then of a constant positive or negative increase of this increase, then of a fresh constant positive or negative increase of this second increase, and so on) will enable us to exhaust the total value of the resulting growth just as certainly as we are enabled to express any curved path by properly chosen epicycles, or any irrational number by an infinite series of positive and negative powers of ten. Taken in this sense, as a mere definition of growth, the series remains *logically valid* even when it is rendered *mathematically useless* by divergence for a demonstrably finite increase of the function. If it were not so, then, even if it were possible to restore convergence by transforming the function without altering its content, the result it yielded could only be regarded as correct in fact, supposing it could be shown to be correct,—it could not be regarded beforehand as obviously and necessarily correct: such transformation only serves to bring within the limits of calculability what holds good as it stands.

CHAPTER VI.

Fallacies and Dilemmas.

240. TRUE conclusions, as Aristotle has observed, can be correctly drawn from false premises. Every Laplander is a born poet, Homer was a Laplander, and therefore—by the first figure—a poet. All parasitic plants have red flowers, no rose has red flowers, therefore—by the second figure—roses are not parasitic plants. Metals do not conduct electricity, all metals are non-fusible, and hence—according to the third figure—non-fusible substances exist, which are non-conductors of electricity. Alter Laplander into Greek, plants which have red flowers into plants which have exploding seed-vessels, and write glass for metal, and in each example one premise will be true, while by inserting a new middle term in each case you may make both premises true, but in every case the conclusion follows with neither more nor less validity. Let T be a perfectly true proposition, S its subject, and P its predicate; then a middle term M may be chosen at random so long as the terms are arranged in both premises on the model of an Aristotelian figure: if this is done the conclusion T will always follow according to the figure.

We shall see why this is universally true, if we take as our middle term an abstract symbol M, instead of a concrete term: thus, all M are poets, Homer was an M; all parasitic plants are M, roses are not M; all M are non-conductors, all M are non-fusible. What these symbolic premises tell us is the relations in which S and P must stand to *some* middle term, if their conjunction SP is to be valid in the conclusion: and conversely these premises tell us that given *any* middle term M to which S and P are related as required, then the proposition SP must be valid. If the M is found and so both the required premises established, then SP is valid not merely in fact but now also of necessity; on the other hand if we could show that there exists no M to which S and P can stand in the requisite relation, we shall know that *SP* was impossible, for no experience could give us *SP* as a fact: but if we have merely chosen a wrong *M* then the case is different. The premises we have chosen will not do, but that is no reason why there should not be some other *M*, the insertion of which will render the premises correct and so necessitate the conclusion *SP*. If again we have correctly drawn a conclusion *SP* and that conclusion is unsound, there must be something false in the premises, from which it follows. In a word in all cases where *T* is not given in direct perception, but deduced from premises, what really depends on the correctness of those premises is not the truth of *T*, but only our insight into that truth. Without correct premises *T* cannot indeed be *proved*, but nevertheless it can be true and its truth is independent of any errors we may commit, when reflecting about it, and subsists even when conclusively deduced from premises materially false. This point deserves notice, for it is a common mistake in reasoning to take the invalidity of the proof which is offered for *T* as a proof of the falsehood of *T* itself, and to confuse the refutation of an argument with the disproof of a fact.

241. A proposition *T* is valid if it is rightly drawn from valid premises, but it is not proved unless these premises are valid independently of itself. If *T* itself or any proposition T^1, whose validity presupposes the validity of *T*, appears disguised in the premises, *T* is correctly deduced, but is not proved at all. This fallacy is called *petitio principii* or *circulus in demonstrando*, and in its naked form seems easily avoided. Yet it is frequently committed, especially where the conclusion is reached by a long chain of deduction and depends on the constructions of the scientific imagination as well as on the relations of abstract ideas. In such a case we are often able to deduce *T* with formal accuracy by first presupposing some indirect and distant consequence of *T*, which consequence of *T* is then taken as an independent truth from which *T* follows. There are no rules which will enable us to avoid this mistake, but it may be well to remember that we are peculiarly liable to it when we attempt to prove by a direct and progressive argument propositions which contain some final and underivative element of our knowledge. In such cases, whether the element be a necessity of our thought or a fact universally valid in our perception, the apagogic and regressive methods alone are applicable.

242. The second kind of fallacy is called *Hysteron Proteron.* It is so like the first (the argument in a circle) that we often have no reason to distinguish it therefrom. It consists in using a proposition, which both calls for and admits of proof, to demonstrate another, which not only needs none, but is itself actually the proper ground from which to prove the first proposition. We are told for example that God's will is holy, that the moral dictates of our conscience are the expression of the divine will within us, and *therefore* they too are holy and binding on us. But we cannot help objecting that if the holiness and binding force of our moral dictates were not felt by us as an independent fact and irrespectively of the origin of those dictates, the argument would fall to the ground. Upon other grounds no doubt we might continue to believe in a mere supreme being, but the idea of holiness would not and could not suggest itself to us, and hence the major premise of the argument proposed could never exist. The transition from God's will to our conscience is therefore no proof; but although inadmissible as a sequence of thought it is perhaps the right way of giving expression to the truth. For in a great many cases that which is in fact the consequence or *principiatum* may be for us a means, and often the only means, of knowing that which in itself is the *principium* or real ground of the possibility of the former. When we have acquired knowledge, by way of induction especially, and are exhibiting the result systematically, it is evident that we always take the universal statement, which we really know only from the particulars, and placing it at the head of them use it to prove those very particulars. Hence it is of importance alongside of this method to employ another mode of exposition which shall set out the items of our knowledge in the order in which they can actually be proved by the help one of another. We often allow ourselves to commit a *hysteron proteron*, when we are trying to prove a point, either in the course of conversation, or in the rapid reflexion by which we seek to assure ourselves of the truth of some proposition which we desire to employ in an enquiry. The inference in these cases is *ex concessis*, from premises whose truth we presuppose but do not discuss. In an enquiry the implication of these premises with the rest of our knowledge is taken as a sufficient guarantee, and in conversation we may find it easier to get these premises admitted than it would be to gain acceptance for the truth from which in reality they follow.

They are classified in the Aristotelian work on the subject, but many of them have no value at the present time, even in the light of pleasantries. There are yet some which remain as abiding sources of danger, and among these we may signalise the double fallacy *de dicto simpliciter ad dictum secundum quid* and *de dicto secundum quid ad dictum simpliciter*. Two general modes of fallacious thought are developed by the habitual commission of these fallacies and illustrate them on a grand scale. The first is doctrinairism, the second narrow-mindedness. The doctrinaire is an idealist, who refuses to see that though ideas may be right in the abstract, yet the nature of the circumstances under which and of the objects to which they are to be applied must limit not only their practicability but even their binding force. The narrow-minded, on the other hand, can recognise and esteem no truth and no ideal, even the most universally valid, except in that special form to which they have become accustomed within a limited circle of thought and personal observation. Life is a school, which corrects these habits of mind. The parochially-minded man sees things persist in spite of himself in taking shapes, which he considers unprecedented, but he finds the world somehow survives it and learns at last that a system of life may be excellent and precious, but that it is rash from that to argue that it is the only proper mode of orderly existence. And the enthusiast for ideals, when he sees the curtailment which every attempt at realisation inflicts on them, learns the lesson which the disjunctive theorem might have taught him. Every universal P changes in the act of being applied from something that held *simpliciter* into something that holds *secundum quid*,—changes from P to p^1 p^2 or p^3 : to refuse to accept it in any one of these, which are its only possible shapes, is to ask that it be realised under a condition which even logic pronounces impossible.

244. One of these fallacies consists in our taking a P, which holds good of M in the abstract, and asserting it of M under new conditions which make it no longer applicable. The other and opposite mistake transfers to M taken absolutely predicates, which are only true of M under certain conditions. In both we have an ambiguity of the middle term, which wavers in its meaning between the unlimited M and the determinate conditioned M^1. Examples present themselves and could be given in great numbers, but there is one worth dwelling on from its own special interest. I refer to the question of the morality of lying. On principle we condemn all lies, but in practice almost everyone allows there are exceptions, a confession which points to some mistake in the way in which the principle is laid down. In fact the particular lies which, apart from the influence of education, we find hateful are those whose object is to make others chargeable for our faults, wantonly to do mischief, or to wound the self-esteem of another by entangling him in false ideas in order to exalt ourselves. It is these secondary features that rouse our indignation against an untruth, and it is only these that make us call an untruth a lie. The *secundum quid* and its influence on our judgment is quite plain in these cases, and on the strength of them apart from other considerations we should be wrong to conclude that every falsehood, when the intention is not bad, is immoral. Other considerations do however exist. We communicate with our fellows in order to waken in all alike ideas of the same reality, and our object in doing this is that when we work together our efforts may coincide, and when we work apart we may avoid collisions, and in general we desire to avoid undertakings which are not in accordance with the nature of things. But failure would be certain, if everyone made it the rule to lie; everywhere the truth is one, but possible falsehoods are innumerable. The interchange of falsehoods therefore ensures no meeting-point for common action, so that however good our intentions we should ever be missing the mark. False assertion is thus contrary to the essence of assertion, to the moral end which all communication aims at, and therefore we set it down broadly as a thing in itself reprehensible. The untruths of poetry, jesting and courtesy are exceptions, they are not real assertions, and on these points we make a silent reservation. Here comes in the fallacy. We think that we can now unconditionally assert the badness of falsehood; that we have got rid of the old *secundum quid* and have got the *simpliciter*. Unfortunately the *simpliciter* is ambiguous; it may mean that false-speaking is wrong *in itself* and can be justified only *secundum quid*, i. e. for special reasons in particular cases. But it may mean that falsehood is *universally* bad, so that no special considerations can ever justify it. These versions of the *simpliciter* practically collide in our consciences, and it is that which makes our opinions about falsehood so self-contradictory. The logical premises from which we here started justify only the first version. What we started from was that universal false-speaking would frustrate our moral aims and therefore we said it was bad; but where the aim is immoral it may be right to frustrate it, and admitting our premise it is therefore still possible that a lie may be justified. To prove the *simpliciter* in the sense of without exception other premises would be wanted. It would be the business of ethics to discuss them,

neral, but not always and in all particular cases. But it may
qually well be true *by itself* in the sense of being true independently
conditions and hence always and necessarily true in every particular
se; universally, that is, not merely in general.

245. We may consider here some examples, where an universal pro-
osition is extended to instances which can formally be brought under it
exceptional cases, but where the conditions which make it applicable
ve disappeared. If the terms of the proposition are variable quantities,
d if these are followed to their limits at zero or infinity, we get such
amples. With a lever the work done remains the same, so long as
e product $w\,l$ of the weight w into the arm l is unaltered. Thus
e more l is increased, the less weight w is wanted to do the same
ork; hence, it has been subtly argued, at an infinite distance from
c fulcrum, a mass $= 0$ would suffice to balance any weight what-
er on the other arm of the lever; and this conclusion has been
ged against the validity of the general formula. It is natural simply
dismiss the idea by remarking that the formula contemplates cases
here real forces are applied to the lever, and is not true where that
ndition is wanting. This removes our doubts on the question of
ct, but hardly settles the logical problem. For we do not always
smiss these cases; we have no hesitation in taking $\cos 0 = 1$,
though the idea of a cosine is in its origin without meaning except for
real arc ϕ, from the extremity of which a perpendicular may be drawn
the semi-diameter through the initial point, and we pass from this
se to the limiting value $\phi = 0$. Now since the law of the lever remains
lid at every stage of approximation to the values $l = \infty$ and $w = 0$,
would be well if it admitted of being interpreted for these limiting
ses in such a way as to show what is the second meaning which it

effect, while the effect is in fact nothing at all. But here we might answer, though more in jest than in earnest, that as a matter of fact it *would* need an infinite force to keep a geometrical plane, such as we have now reduced our wedge to, from penetrating a block of wood; and it might be proved with equal show of formal rectitude that this would not require the block to be cleft.

I cannot at present as decisively settle the doubts which some have about the lever; though I should deem it irrational to postpone any consideration of the principle of the lever till one had solved the problem which arises in connexion with an arm of infinite length, for surely it is intrinsically absurd to think of the mass $= 0$ at an infinite distance as having any effect at all; the very idea, as it admits of no interpretation in fact, must be self-contradictory. And so it is, for the law has no meaning except as ascribing to a definite mass w at a definite distance l from the fulcrum a definite effect, which alters as l alters. Now why should a mass $= 0$ produce an effect at an infinite distance other than it would produce at any finite distance we like to take? How would the case of a mass $= 0$ acting at the end of a lever of infinite length differ from that of a mass $= 0$ acting at any other point in the lever, or from a third case, which would properly always coexist with the other two, viz. that in which we suppose the nothing to be applied at all points in the lever and—what is more—to act in any direction we like. Thus the attempt to retain the law of the lever for $wl = 0 . \infty$ fails, not merely because it gives incredible *results*, but because the law loses all its *meaning* inasmuch as things become indistinguishable whose distinction is essential to that meaning. Other paths lead to the same conclusion. wl is no constant quantity, so that w should vary inversely as l, but the effect varies for every and any l with the variation of w, and w is quite free to vary as it will, consequently the effect becomes $= 0$, when w sinks to 0, no matter what l may be; it follows that $wl = 0 . \infty$ can only have the value 0 and no other.

246. There is another fallacy which is akin to that of too wide or too narrow definition and in general arises from it. T being the point to be proved, the mistake consists in proving too much or too little either as to the qualities which T includes, or the extent to which it is applicable. The conclusion which proves too much may be correct and may err simply in going further than was required. For instance you may prove correctly of all animals what you wanted to prove only of men, and in such a case the ground is valid and has simply been applied more generally than was requisite. But in other

1 in a particular form which will not do so; it is corrected ' by using the ground of inference in the general form in which ves *T*. But the second form of the fallacy is more serious. cial case of *T* may have been correctly proved from certain ses, but those premises may be quite inadequate to establish in universal. To sum up, any argument which does not exactly ith the proposition to be proved must leave us in doubt as to plicability; *nihil probat qui nimium probat* is as true as *nihil qui parum probat.*

'. It would be easy to supply illustrations, but I prefer to con- nother fallacy, that of incomplete explanation. This is often met with in the speculations of amateurs, but does not generally e form of demonstration. It consists in assigning a general for some phenomenon without enquiring if the cause assigned count for the particular modifications, to which that phenomenon ject. It is perhaps not possible to deduce the law of the per- e of motion from any more universal principle; but at any rate lgar opinion that every motion ceases with the lapse of time is sible in itself, and can be used to prove the law by a *reductio ad lum.* That diminution of motion arises from real obstacles g in time is true and is quite conceivable, but it is quite incon- le how *mere time* should cause a diminution. No doubt our odily movements are enfeebled as exhaustion increases, and this suggest to us the idea that mere lapse of time can destroy all n; yet we are undeceived when we reflect that if this really happens t happen either earlier or later, at some one particular time or er, but that there exists no law and no reason to connect it with shall we assume that the consuming force of time is proportional to the velocity to be consumed? Then the question remains as to the ratio. But I refrain from any further hypotheses. In the first place since time and mass are utterly disparate things one sees at once how hopeless it is to try to determine a unit of mass, for which q would measure the arresting power of a time dt. In the second place we can equally conceive of innumerable different ratios as existing between the velocity and rate of retardation, and it is evident that mere time can of itself afford us no reason for preferring one of these ratios to another. But apart from all this there is an objection which would render all such attempts idle. For supposing that a time dt removes some part of the motion, the question remains—whence comes the residue, the motion which has not been removed? It is clear that in case of this residue the law of the persistency of motion has been presupposed. If we had not tacitly presupposed the law to be valid for it, we should have to say that the whole motion was arrested by the first period dt. What it comes to then is this. Either motion does not take place at all but vanishes [1] the moment it gives a sign of intending to endure through a time dt; or if motion gradually diminishes, the law of persistence is the primary law, and the diminution of motion is secondary, the result of external obstacles. These obstacles we shall now naturally seek only in what is homogeneous with itself, viz. in opposed tendencies to motion.

I will merely call the reader's attention to the connexion of this proposition (that every ground of explanation must establish not merely T in general, but also the possibility of its modifications) with the doctrine of the disjunctive judgment. It would take us too far afield into mathematics, if we followed up this clue; it is sufficient just to notice that this logical requirement has found a special and fruitful expression of itself in the principle of the homogeneity of the functions to be combined in an equation.

248. *Incomplete disjunction* is a fallacy which often occurs in collective and indirect arguments. In order to prove T, these arguments attempt either to show that that holds in all particular cases of T, which it is desired to affirm universally, or to establish T as the sole remaining possibility by first disproving all cases of non-T. Neither task is very easy. In practical life especially we often find it very difficult in laying down a rule to examine beforehand all the possible cases to which it may be applied and to see whether the proposed regulation would always be desirable or tolerable; and it is common fallacy is the dominant influence of some one order of ideas. ead of setting out all the possible alternatives we are led silently he one which consorts best with our own one-sidedness. For ınce our sensations are subjective states excited in us; it is easy how this; and further according to a view now widely accepted forms of space and time in which we arrange the manifold matter ensation are subjective also, they are modes in which our minds cive. From this point we are easily led on to regard the idea of nown things and processes underlying phenomena as a creation ur mind, which is compelled by its constitution to adopt this ns of giving unity to its singular perceptions. The subjectivity *ll* elements of knowledge is thus established, and finally we venture he inference that therefore there is no objective real world an-ing to the world of our ideas. But the inference is false: for osing that this real world does exist, it is easy to see that things ld be just the same to us as we find them. If real elements exist can never pass into us bodily: they can do nothing but excite s sensations and ideas, and these though caused by external ressions and our reaction against them would still be a creation of subjective nature. An objective space may exist or may not t, but at all events the perception of it must be the product of our ective faculties. So too with causality: whether the law has objective lity or no, in either case equally we are forced to recognise it so far as we think it and perceive its content to be in harmony the laws of our thinking. Thus we see the complete subjectivity ll the elements of our knowledge proves nothing whatever as to existence or non-existence of an objective reality. The best pre-ative against this kind of fallacy is the existence of hostile opinion. indispensable, beside developing our own doctrine, to familiarise elves with ideas which proceed from points of view opposed to own.

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[1] [Compare Metaph. § 163.]

those real difficulties in the idea of motion which they touch upon, but will exhibit the arguments as examples of fallacies that are difficult to classify. The first argument tries to prove that an arrow in flight is at rest. It starts with the assumption that time consists of indivisible moments, and then infers that the arrow cannot move in any one of these moments. For if it moves, it must now be at one place and then at another, but in each indivisible moment there is no now and then, no before or after. Therefore the arrow is at rest in each, and if in each then in all; hence it is always at rest. To this we may answer simply that rest also is impossible unless a thing is at one and the same place both now and then, both before and after; and since in an indivisible moment there is no before or after the arrow can no more rest in it than move. And this conclusion accords with the ordinary theory of motion. So long as dt is a length of time the arrow passes in it through a small space $v.dt$, but as soon as ever dt ceases to be a quantity and becomes a mere dividing point in time with its definite position in the series of time—then no doubt in dt the arrow passes through no space at all; still it does not rest in it, but goes through it with a velocity v. But apart from this objection what right has Zeno to maintain that in each successive moment of the arrow's flight it is at rest in the *same* place as it was at rest in before? There is nothing, I admit, in the idea of a moment or an arrow as such from which the idea of a change of place could follow. But it surely is involved in an arrow's *flight*. No doubt there is a difficulty as to the nature of that impulse which makes a body in motion different[1] at every moment from a stationary body, even if the moment be conceived as indivisible. And this is the point against which a sophism might have been directed with effect. But, failing this and failing any proof that velocity could not exist, Zeno had no right to start with the flying of an arrow,—so assuming velocity,—and then in his proof to drop the idea out of sight. All his argument shows, if you take it as it stands, is that rest is not motion and that motion can never be compounded out of rest. Had he retained the idea of velocity Zeno could at least have deduced such a successive change of place as proceeded by jumps from moment to moment; the conception of continuous motion he could not get at so long as he held by the notion that time is made up of indivisible moments.

Another of his arguments was that if the snail has a start the swift-footed Achilles can never catch it up, inasmuch as before ever he can overtake it he must first reach the place it has just left before and so

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[1] [Compare Metaphysic, § 171.]

. To do so it must first accomplish half the given distance,
half the remainder, then half the remaining quarter and so
r ever, so as never to reach the end of the space. The argu-
assumes that the halving process can go on for ever *ad*
tum, and so presupposes that the space is infinitely divisible or
ly consists of an infinite number of parts. It further assumes
n indivisible moment of time is required for the transition from
point of space to another ; and so concludes that an infinite
er of moments must elapse before any space whatever is got
since any space contains an infinite number of parts. The con-
n, if we admit the premises, is so far quite correct ; but it is
an arbitrary assertion that this aggregate of infinitely numerous
ents would form an infinite length of time ; seeing that they are
sible each of them must contain no sooner or later, unless, as
does here, we foist in between every two moments a real lapse
e or represent these moments as following each other at definite
als in a sort of secondary time, which we imagine as filling up
ackground. It is not even necessary to object to Zeno, what
otle's remark on the subject amounts to, that (according to our
s of expression) the integral of an infinite series of quantities con-
usly passing into one another may still be a definite finite quantity,
hat therefore the aggregate of moments of time may be a finite
h of time. The indivisible moments of time are conceived by
not only as being each for itself without quantity, but also as so
hed from each other, that there is no question of a transition, in
of which they would become constituents of time at all. The
of all these nothings is therefore itself nothing ; it is only an un-
nted complaisance on the part of our better-instructed fancy
allows it to be passed off on us as a quantity at all and then as
finite quantity. Achilles requires no doubt an infinite number of

follow opposite conclusions, equally necessary and equally impossible. One dilemma nicknamed Pseudomenos dates from Epimenides, who being a Cretan himself asserted that every Cretan lies as soon as he opens his lips. If what he asserted was true, he himself lied, in which case what he said must have been false; but if it is false it is still possible that the Cretans do not always lie but lie sometimes, and that Epimenides himself actually lied on this occasion in making the universal assertion. In this case there will be no incongruity between the fact asserted and the fact that it is asserted, and a way out of the dilemma is left open to us. Not so however if we drop Epimenides and the Cretans and instead of these two subjects, one of which is only contained in the other but not identical with it, put an identical subject: 'I lie now.' If my assertion is true, i. e. if I am lying, what I assert is false, and I am not lying. But what I assert is that my assertion is untrue: if it is false to say that my assertion is untrue, my assertion becomes true again and I am lying, and the whole chain of self-destructive consequences begins afresh. The reason of them is easily detected. Logically of course what is asserted is true or false, quite apart from the fact of its being asserted; it may be asserted or it may not: but the only sort of truth or untruth which the assertion can have, is what it acquires through the truth or untruth of what is asserted, which is independent of it. Thus we get contradictions, some of which are formally insoluble, when what is asserted is such as to involve something in regard to the fact of its assertion which makes the assertion impossible or untrue. The difficulty vanishes if instead of saying: I lie, we say: I lied. Just as little can we say in the present tense, I am silent, though we can quite well use the future, I will be silent, for then our assertion refers to another fact than itself, to a fact which is not in conflict with it.

There are many other instances of the sort, though none so classical as Pseudomenos. If a person answers yes, when he is asked if he be asleep, he sets his assertion in the same sort of conflict with what he asserts; so does a person, who calls out to an unwelcome visitor, that he is not at home. Lastly there are other cases resembling these, cases in which one subject has in an impossible manner to form both terms at once of a relation, which can only exist between two different subjects: Jean Paul's dwarf for instance, who only reached up to his own knees, not to mention other people's; or the inscription over the elephant's booth: this is the biggest elephant in the world, itself excepted; or lastly we may instance Munchausen's kind service to him-

'e or not give it back. If she guesses *right*, there is
:vent the child being restored according to agreement;
it true, that the crocodile does not just then intend to
l, if her guess is that he does not intend, he may yet
'act by changing his intention or giving it up against his
;he guesses *wrong*, she loses all claim to have her child
For whatever may have been the animal's real intention
;uessed wrong—he need not in his actions bind himself
tention; he need only observe the terms of the contract,
that she has guessed wrongly forbids him to return the
ie question asked of the mother, whether she is going to
back or no, need not refer to the animal's intentions;
:ive of this future as a predestined to-come, so that in
ady a settled matter, which of the two possible events is
ur. Interpreting the question in this way, we get an
plexity for obvious reasons; for we cannot without ab-
an issue, which unconditionally impends, depend on a
ose fulfilment would necessarily be as ineffectual to bring
non-fulfilment to bar—the inevitable. There is thus but
of the dilemma. If the restoration of the child is the
; going to occur, and if the mother guesses this, all will
'et not because her assertion in any way *conditioned* the
her assertion in itself is quite ineffectual; it has only
ree with the inevitable result and the terms of the bargain.
iade any other answer, that would only have served to
plainly the utter impotence of a bargain, which because
dition the unconditioned must of necessity be violated.
rm of the dilemma starts from yet another supposition,
both of these: it supposes that it is not determined
hich of the two alternatives is going to occur, i. e. whether
oing to be given back or not; that is to be settled by

requirement is contravened. For the condition fixed upon here is not an assertion made by the mother but the truth of her assertion, and further not the truth of an assertion which refers to some third fact independent of the future result and which could therefore be true or untrue no matter what this result may be; on the contrary what she asserts is that this result will either occur or not occur,—a result which is connected with no other conditions at all,—and so its own truth depends on the very thing, which should depend on it. Consequently on this supposition as on the former there is only one case which logically admits of a satisfactory issue; the mother must answer, you are going to restore my child; and then its actual restoration at once makes her answer true and fulfils the agreement. In that case the issue is a happy one, but it was not conditioned by the answer she gave. Suppose after all the crocodile does not give back the baby, the very fact of his not doing so makes her answer a false one and at the same time the animal becomes justified according to the terms of the agreement in not giving it back. If however the mother is so unfortunate as to answer: you will not give it back, '*then*' the crocodile must say 'I cannot give it back; the agreement forbids me to, since if I did your answer would become a false one; no more could I restore the child even if your answer could be correct, seeing that by the very fact of my returning it it would become false.' The mother then objects: 'you must in any case give it me; on the score of the agreement, if my answer was correct; but no less if it was incorrect; for it would become a correct answer, if you refused to give it back.' There is no way out of this dilemma; as a matter of fact however both parties rest their cases on unthinkable grounds; for the answer really given can as little be true or untrue independently of the actual result as could the answer she might have given, an answer which only differs from this in being more fortunate.

The dilemma of Protagoras and Euathlus rests on a similar misuse of hypothetical conjunctions of thoughts. Euathlus is to pay for the instruction he has received as soon as he wins his first case; but as he engages in no suits, Protagoras gets nothing and sues him on that account. Now whether Euathlus wins or loses *this* suit, the verdict must in any case either oblige him to do that which the contract releases him from doing or release him from doing that which the contract obliges him to do. Various solutions of the difficulty have been attempted on the supposition that Euathlus is allowed to win this his first suit because he has won no previous suit, and so had not yet become obliged to pay. It was then open to Protagoras to institute a fresh suit, which must have this time led to his pupil being condemned to pay. This would be shifting an absurdity off logic, in order to make a present of it to jurisprudence. I will not anticipate the decision of the latter, but I suspect it would say that in acting as he did Euathlus had fraudulently prevented a certain condition from being realised, according to which he would have been forced to fulfil an obligation. If therefore it could fix a date, after which no other interpretation could be put on his conduct than that it was fraudulent, then though Protagoras no doubt could not base a suit on the contract, the law might well go behind it and taking its stand on the obligation, under which Euathlus really put himself by receiving the instruction, condemn him to pay, just as if the ambiguous agreement had never existed.

CHAPTER VII.

Universal propositions as derived from perceptions.

251. THE ideas which we ourselves have put together are completely open to our inspection and we can review their content and manner of conjunction. And hence the conclusions we draw from them are necessary and the process of conclusion is proof or demonstration, the essence of which is to descend from the more to the less universal and starting from a general truth to end with a particular application thereof. But the conjunctions of phenomena in the world outside us do not carry on their faces the universal laws and conditions of their connexion. They are individual experiences to be severally expressed in particular propositions, and though each embodies an universal principle, yet the path up to that principle must be a matter of search. The simplest form of this ascent in thought is a process with which we have become familiar as the inductive syllogism, and hence it is the custom in our day to collect into one body the numerous operations which assist us in ascending to generals and to call this inductive logic and to set it against the deductive or demonstrative logic along with much disparagement of the latter. Such disparagement rests on a mistake. The inductive methods it is certain are the most effectual helps to the attainment of new truth, but it is no less certain that they rest entirely on the results of deductive logic. It is the theory of the validity of syllogisms, the convertibility and contraposition of judgments, and of the forms of proof, that is the source of each provision and precaution by which so far as may be we secure each step of our paths as we ascend by induction from given perceptions to the universal laws of the real world.

252. The first step of this ascent is barred we are told by an insuperable obstacle. Experience we are told cannot give us universal cognitions, and in one sense no doubt this well-worn saying is true: but if we take it to assert a difference in validity between two sources of knowledge, experience on the one hand and an *a priori* certitude on the other, then the saying is true no longer and is the opposite of truth. Every experience, whose contents in their connexion can be expressed without deficit or surplusage in the form S is P, must *ipso facto* rank as an universal judgment, even if this experience stands absolutely alone. The law of identity guarantees that if the same S were once more perceived in a second experience it would be impossible that the same predicate P should be absent or should be replaced by some other predicate Q. On the other hand it is no less true that experience does not directly present us with perceptions which fulfil this condition. Our perceptions do not give us a subject S in conjunction with a predicate P and nothing more or less than this subject and this predicate. The real and true subject with which the phenomenon we observe is essentially connected is not S but Σ. It has elements s absolutely necessary to the production of the phenomenon and which notwithstanding we do not perceive. What we do observe, S, is a residue and what is more an impure residue, for it comes to us indissolubly joined with elements σ, which have nothing whatever to do with the production of the phenomenon. It is the same with the predicate. The true predicate which attaches to Σ, the true subject, is Π and it we do not perceive. It has features p which are invisible, and the residue P which we do perceive is bound up with other circumstances Π, the results of conditions which have no influence on the matter in question though they are operative at the same time. A complete expression of the actual fact demands addition and subtraction and would run thus $S + s - \sigma$ is $P + p - \pi$ or Σ is H, while our first defective observation set down S is P as the fact. Only for the complete proposition Σ is Π (supposing this were given in a peculiarly fortunate perception) would universal validity be guaranteed by the law of identity,—not for the incomplete proposition S is P, which puts together what is not really thus connected.

253. It is important to bear this in mind, for apart from it we cannot understand a right, which science claims and which is essential to her development. If the question is as to a predicate Π, which we do not yet know and which we expect to find in a subject Σ, then wherever we are sure that we perceive this subject Σ whole and complete, and nothing else but Σ, we are equally sure that a single observation, which acquaints us with Π, has an universal import and that in every possible case, where the same Σ is repeated, the same Π must unalterably present itself. When the chemist is instituting an experiment, if he only can be sure that he is dealing undertakes a measurement he takes care first of all to eliminate the sources of error, with which, as he well knows, he is beset, but when once he has purified his observation he never dreams of regarding the fact that it comes ultimately from sensible experience as a reason for accounting it valid merely for this one occasion. It never enters into his head that under similar conditions the same object might perhaps on another occasion have a different magnitude. We need not enlarge on this head. Once suppose that a single observed case is valid only for itself and not for its repetitions in like cases—that the record of an instrument is correct for the one occasion in which it is noted and not equally correct for a second occasion under identical circumstances—once suppose that with like object and like conditions a different result may be true, and there is an end to all possibility of developing universal truths from experience; there is an end not merely to the discovery of laws but to the use of the word 'law' with any intelligible meaning. The art of induction, which is to bring us to universal laws, rests wholly on the acumen shown in developing pure and self-connected propositions of the form Σ is Π out of the impure and confused material of our perceptions, which come to us in the form S is P.

254. Let us try to sketch in a connected series the steps essential to that development. The countless impressions which we receive in succession or together may be taken as an indiscriminate mass Ω. How do we come to distinguish in the mass of them groups A, B and C and to regard each group as a self-connected perception? It is because the impressions we so gather into one are permanently conjoined and thus raise themselves above the level of the shifting background, or again because by their simultaneous appearance at intervals they contrast with the uniform background formed by the others. This first step is a necessary one, but there is so far no act joined and mixed with that which in no way belongs to it. We are impelled in like manner to the second step which consists in splitting up the sum of impressions contained within each of the groups A, B and C and in taking one part of each sum as a subject and the remainder as a predicate attaching itself to the subject. It is our psychical mechanism which accomplishes the beginning of this step also. Thought indeed actively intervenes before long and intensifies this mere conjunction of two ideas—this mere adherence of one idea to another—by transforming it into an objective connexion and by establishing an opposition between the subject and predicate, between the former as essentially independent and the latter as dependent and simply attributed. Still it is the mechanical course of our ideas which always guides us in the application of this added principle of thought and which settles in each case which group of impressions is to rank as subject and which as predicate.

Thus articulated the whole content of a perception A or B might now be expressed in the form of a judgment,—but a singular judgment only. The subject which we here distinguish as s^1 or s^2 is nothing but a perfectly determinate group of single impressions, such a group being the sole possible object of immediate perception, and the thought that either s^1 or s^2 may be taken as the example of an universal conception S is an added thought. It cannot originate in an individual perception, but only in the comparison of many which soon begins. And here again, when out of the several subjects s^1 s^2 . . . the smaller group of attributes common to them all is forced upon our attention till it emerges as a general picture, which is now denoted by the name S and which takes the place of those several subjects in our memory,—this also is at first the result of the course of our ideas which is conditioned in a mechanical manner by universal laws. Here too thought adds a new depth, it transforms the general *picture*, which only represented what was found common to the various individuals, into a general *conception* which has the force of a law and joins what is essentially connected: but still it is the course of our ideas that determines the first applications of this added thought, and settles for us which elements of the subjects we compare are to be united in the general picture or again in the conception and which are to remain excluded. The elements which are felt to be modifications of one universal and are at the same time more lively as impressions are accepted, and those which excite our involuntary attention less strongly or reciprocally destroy one another are rejected.

And the result of this process is on the whole more often right than wrong. This is not the place to pursue in greater detail the psychological development of our conceptions, but there is one point worth mentioning. In the sensible impressions, which are the ultimate components of every perception, it is from the first not the differences of the actual impressions that are of prominent importance so much as the differences in their relations and in their manner of union. The mere child can neglect the differences of colour and sees that the characters shown him in red are the same as those he has learned in black. His general pictures of trees and animals are drawn as yet but from few examples, but they already comprehend the essential traits with such accuracy that when he afterwards perceives a new and unusual shape, it readily takes its place in the series. Errors however do occur: how these are corrected is what we have now to consider. We have tried in the foregoing to make clear our starting-point, which is this:—the inductive process deals with individual perceptions and its object is to establish a further connexion between them, but these perceptions are more than mere *impressions* that we *passively receive.* On the contrary they have already been so far worked upon by thought that not only have their contents already been divided by us into subject and predicate, but besides that we have already brought the subject under the head of a general conception S or at least meet it with a selection of such conceptions under one of which we try to bring it.

255. Let us take the last case first. Suppose that a singular subject s^m, new and not yet known, is presented in a perception through the sensible impressions $p^m q^m r^m$, and suppose we have a perfectly clear image both of these particular marks and also of their conjunction. The image if we do not go beyond it does not contain one doubtful element and yet it does not satisfy us. We cannot rest until we know what the new object may be called, whether animal or plant. Our desire to know this is based on a twofold interest. Pure thought is interested; for it is only by subordination under a general conception that the mere coexistence of the observed characteristics is transformed for thought into a well-grounded coherence. But what weighs much more with us is the practical wish to go beyond the observed fact and to justify ourselves by the general conception in filling in what we have not observed. For the name plant or animal would be for s^m a barren title had we not reserved the right to found upon it a claim to a number of properties as belonging to s^m, which no immediate perception has shown to belong to it. We constantly find ourselves in this position towards real objects. For every perception howsoever accurate, let it even apprehend every single mark that s^m now has, is limited as to time. No perception can tell us the future with the present; it can never say what our object will eventually do or become, and can only seldom and incompletely show us how it will alter with altering conditions. This gap that perception leaves in our mind we fill up by subordinating the observed s^m to the conception of plant or animal. These conceptions have sprung from countless observations and they comprehend the whole collection of coexistent attributes, which can scarcely ever, and the successive attributes which can never be exhausted in a single perception. But it is only by the union of all these that we can adequately determine the real nature of s^m, while a single perception of it gives us only an inexplicable fragment of a connected whole.

This process on its formal side is an *incomplete analogy*, and since so considered it argues *ad subalternantem* from the observed sameness of some attributes in S and s^m to the sameness of all, it must be pronounced invalid by the canons of pure logic. None the less our whole daily life depends on the application of this incomplete analogy. We rest secure on it in dealing with substances useful or deleterious; it alone persuades us of the existence of minds like our own and assures us that their actions flow from inner motives such as we feel. And in fact our dependence on it scarcely needs to be mentioned, so plain is it. The practical question is as to the means by which the bare probability of these inferences can be made to approximate in value to certainty. The sameness of all the marks is what logically justifies the subordination of s^m to S, and the natural conclusion from this fact is that the probability of a subordination being correct rises as the number of identified marks is increased. But it is evident at once that the value of this conclusion is much lowered by the necessity of taking into account the difference in value which exists among these marks. And this is not the whole difficulty. It is idle, it is a mere form to bid us direct our attention to the likeness above everything else of the essential marks, when as yet we have no means of distinguishing them from the non-essential. It is experience and experience alone which enables us to distinguish, and the few general rules we can lay down are all drawn from experience. There are attributes, which arise indifferently from the operation of widely-diffused causes on objects vastly heterogeneous, and consequently these marks cannot serve as criteria because not characteristic of the distinctive content, which any one conception connects, and again the modifications of these marks can produce no essential difference in any such content.

from experience should we know that mere quantitative differences in the marks are in the main but of slight importance and that diversities in the forms of conjunction and in the respective positions of those points in which relations centre are a matter of far greater moment? There are universal conditions in the world, which tend to produce similar alterations in different substrata which they encounter, and this gives us a test for the real and exclusive peculiarity of any kind of things. For each genuinely peculiar kind by reason of the specific mode, in which diverse centres of relation are united in it, exhibits unexpected forms of reaction against the universal conditions. Thus specific reaction against non-special conditions is the sign of a genuine kind—but this sign is the teaching of experience. And it is experience once more that informs us that these rules have their exceptions in the case of some object that we perceive. There may be marks that seem unessential and whose variations may appear of no moment, and yet there may be in such trifles the sign of a radical difference pervading the whole nature of two groups of attributes connected under two different conceptions.

In conclusion we must not forget that in trying to range new objects of perception under old universal conceptions we are not unfrequently driven by utterances of the same experience to quite an opposite result. These universal conceptions themselves—animal, plant, body—are altered, their boundaries are widened or contracted as our knowledge of things grows. On the one hand we may find in doubtful instances, which seemed to fall under one of these conceptions, points in which their habit permanently differs from that of the undoubted instances; or again instances which seemed to be excluded may be found to exhibit a gradual and uninterrupted approach to what is the character as a whole of the known and undoubted species. Thus it will be seen that we trust not to universal logical rules but to a knowledge of things for the correct carrying out of the imperfect analogy, by which we class a perceived object s^m under a general conception S. In fact we must distinguish applied logic itself as a theory of science from its application as a scientific activity. The theory can do no more than lay down general points of view, of which we should never have become conscious had we not exercised the activity. Logic therefore cannot step forward and claim to impose its rules on the whole domain of real thought, as if the whole work of the latter was about to begin for the first time; it is of no use to the mind, which has as yet no conceptions at all, but only to the mind, ɔw all these conceptions have arisen, but this task does not fall to ıe theory of science. *Its* rôle is only to establish what is true and ɜrtain in these ideas now we have got them; and in as much as many rors and defects must still cling to these rough and ready results of ng intellectual development, theory must also point out how these ıortcomings may in future be remedied and that which is still doubtful brought near to certainty.

256. Now if, as we at present assume, the individual perception is far logically formed, that we at once apprehend the particular object which it portrays as an example of a general conception S, it will ıgage our attention no further provided that we find in s^m none but arks which belong to S, no matter whether all or only some of them. e shall however be led on to take a fresh step, when we find bound with s^m in the perception a mark M, which does not belong to the oup of attributes conjoined in S, so far as we know S. Experience ere, as I said just now[1], the only authority) teaches us to discriminate ree possible cases. In the first place altering conditions or accidental cumstances may have temporarily invested the s^m of our perception th a property, real or merely apparent, which under other circumınces it would not display. Wise with the knowledge we have eady won we quietly neglect many points of this kind: the same ject wears different aspects according to its position, attitude, moveent, distance or illumination, but we do not allow such differences to ake us in our conviction of its identity and its agreement with the neral conception S; cases where there is more room for doubt we cide by trying to make observations of the same object under ferent conditions; it is only an M which adheres to it under all cumstances that is regarded as a permanent mark of its nature. ıt we still leave it unsettled, whether this M owes its presence to the dividual nature of this subject s^m, which after all remains a particular bject, or to the universal nature of the general concept S, of which observed s^m is a species or an example. To decide between this cond and third case we resort to what is called *imperfect induction*; ıt is we compare s^m with other examples s^1, s^2.. of the same universal with which we are familiar. In most cases what leads us to make induction is that a number of individual perceptions s^1M, s^2M, s^3M

a suspicion that the ground of *M* is universally to be found in the nature of *S*, in various examples of which we observe it. This presumption is rebutted, if we find a single subject s^q, which has not the mark *M* when placed under the same variety of conditions, under which that mark attached to the subjects of the other perceptions. On the other hand all instances of *S* which have so far been accessible to our perception may possess this predicate *M*, without our presumption in favour of the truth of the universal proposition, all *S* are *M*, being *ipso facto* corroborated. For when we argue that what is valid in a number of particular cases however large is therefore valid universally we always argue *ad subalternantem* and such inference is to the last unsound. Still placed as we are we must hazard such inferences, for even if perception could embrace all existing examples of a genus, those that are yet to be will always evade our senses. Here too then all we can do is to heighten so far as we can the probability of this imperfect induction. In order to this we shall find two rules of kindred import of great use. In the first place the individual subjects from the observation of which we start must be very numerous; the greater the number of such *s* the more manifold must the outward conditions be which act upon them, and of which we thus eliminate the force and influence. Any *M* which all these subjects retain in common in spite of such variety of conditions must owe its presence to no extrinsic causes but to the intrinsic nature of these subjects. Secondly we must so choose the subjects we observe that their specific or individual differences shall be the greatest possible within the limits of the genus or the species, the universal *S*; we thus eliminate the influence which particular resemblances between the observed subjects, which are independent of the nature of the universal *S*, might have in producing the common predicate. The *M* which they all unite in retaining in spite of these differences will have its ground solely in the character of the genus itself, and the universal proposition, all *S* is *M*, which we wished to arrive at, will thus be justified.

257. Pure logic raised a distinction between analogy and induction. If two subjects agree in respect of several of their marks, we gather that they will agree in all. This is reasoning by analogy. We make an induction on the other hand, when we argue that because several instances of a kind behave in a certain way, therefore all instances of the kind will behave so. We have used the words in the same meanings here, and it was accordingly an induction, by which we drew from the given premises s^1M, s^2M... the universal conclusion *SM*. But this procedure may be regarded more simply. Suppose them possesses all the marks of an S, possess also the particular mark M. By such an analogy is it that we supply the premises $s^m M$, $s^n M$... not given in the perception, the subjects of which premises together with the subjects of the $s^1 M$, $s^2 M$..., which we have observed, exhaust the whole compass of S. The business of induction then consists in merely summing up the single propositions thus either given or supplied in the universal proposition: all S are M. We see from this that it is hardly worth while to separate in such applications of logic the part played by induction from the part played by analogy; nor is it worth while to find fault with the loose usage which confounds the two expressions; it is useless in short to try to refer to simple types of pure logic all the operations of thought, which may be broadly included under the name of an inference. One who has time to waste may perhaps enquire whether a voyager, who has sailed all round a land, concludes by induction, analogy, or subsumption, that it is an island. What does interest us here is rather the question, how we arrive at any universal proposition T about triangles. We prove T by constructing the triangle s^1; but this triangle, which we thus set before our eyes, is always a particular triangle, never more. It would seem as if T can in the first instance be true only of it, and always true of it so often as we construct it in the same way. Now we may of course change our mode of drawing it; still even if we found the proposition T corroborated in a thousand different triangles s^1, s^2, s^3..., this number would dwindle to nothing, when compared with the infinite number of possible triangles, which we have no opportunity of testing.

It is not therefore by any summing up of particular perceptions, which we create for ourselves in these constructions, that we reach the conclusion that T is true universally of all triangles whatever. We must be justified in regarding each single triangle we draw as a *symbol* for all, so that what holds of it holds of all the others which it represents. And our justification does not lie in the peculiar nature of spatial perception; that merely supplies the content of T; it does lie in the fact, that we only pay attention in our reasonings to those features, plane figure, included by three straight lines. The figure actually drawn can never exhibit this property in the abstract and apart from other properties. It can only do so by having sides of definite length and a sum of angles distributed in a certain way. But we do not let these special qualities have any influence on our conclusions; suppose we have unintentionally constructed our triangle with right angles or equal sides, we shall set aside propositions which are valid because of these qualities and these alone, as having nothing to do with the subject of predication which we had in mind. Psychologically no doubt it is the unanalysed impression of resemblance, which prompts us at once to transfer to all triangles by analogy the proposition T proved of the particular triangle we have drawn. Our logical justification for doing so is twofold: first it lies in our consciousness that all triangles, whether already constructed or no, may still be classed under the universal conception of a triangle, which conception we have symbolised in our construction; secondly in the consciousness that in that single symbolic representation we saw the proposition in question flow solely from this conception without any appeal being made to any other conditions.

258. In attaining to universal propositions of the form all S are M, induction has reached its first goal, and it is possible to rest content with the result, especially when we are dealing with a question of practical life; for in such questions we can go without a reason, so long as we are certain that as a matter of fact M is really true of all instances of S, say of all men; we do not care so much to know why it holds of them, and why only of them and not perhaps of animals as well. The theoretic impulse however is not satisfied with merely joining M to its proximate subject; it would fain seek out within the limits of S the narrower group of attributes, which contains the ground of this conjunction, and which conditions the same attribute, wherever it may occur, perhaps even outside S. Then the induction is pushed further; we use a series of universal propositions of the form: SM, RM, TM... as our new premises and try to deduce from them an universal conclusion of the form all Σ are M. In this new conclusion we understand and denote by Σ the true subject or the conception of the genus, or, to put it in another way, that complex of attributes, on which the predicate M in all cases depends and from which it results. Thus in our first induction we shall reach the proposition SM: in all mammals an exchange of gas takes place in respiration; in a second induction, in which S is successively replaced by birds, fishes, and amphibia, we shall reach the conclusion ΣM, another way the phenomenon of a change of gas ; its conclusion $\Sigma^1 M$, all organic beings whatever find themselves in like case, shows us the phenomenon in question bound up with a still more universal subject; and lastly by comparing the behaviour of bodies which resemble organic bodies in structure towards the surrounding atmosphere we might be led to the thought that under the conditions prevalent on the earth's surface, such an exchange of material is absolutely necessary to the development of those interdependent processes of change, which make up organic life. In all this it is to be noticed that the further we advance these inductions, the less do we care to obtain as our result a *categorical* judgment of the form S is P; we are no longer seeking the highest general conception, to which a given phenomenon attaches as predicate; what we are in search of is a *hypothetical* judgment, which will acquaint us with the most general *condition* C, upon which the phenomenon always depends and of which it is the consequence whenever it occurs. And this new problem of discovering the conditions under which the content of a perception coheres is of such preponderating importance in applications of the inductive method, that we shall henceforth in our investigation of that method confine ourselves to the form, which it assumes in order to the solution of that problem.

259. Let C and E respectively denote two groups of observed events; we will suppose that something or other in the way in which they appear has awakened in us the presumption—to be subsequently confirmed or corrected—that the two groups are really connected, and that C is or contains the cause of E, and that E is or contains the effect of C; lastly, let us bear in mind the remark we made at the beginning of this chapter (252), to the effect that C will seldom really contain the full cause of E and nothing but it, E seldom contain the whole effect of C and nothing but it. We may then indicate our problem thus: to discover from the impure observations $C E$ the pure case $B F$[1] of two terms belonging together of their own nature and related as condition and conditioned; and if we have to define the we may abate somewhat of its rigour according to the different interests which from time to time rule our investigations. For instance we may be content to know that B as often as it is given produces F, no matter whether it conditions anything else besides F, or whether F can be produced by any other antecedent as well as by B. We shall only be content with such a result, however, where we are merely concerned to know the real *causes* which produce the effect in question. When the question is not as to the real causes, but a theoretical question as to the *ground*[1], owing to which these causes condition that effect as their necessary *consequence*, we shall always have to determine B and F with all the precision aimed at in the definition; even where F may be due to different but equivalent causes it is not that in which these differ that is the true ground of this consequence, for each cause has besides F other and separate effects; only what is common to all of them is the true ground B, and this B has then no other consequence than just this F.

260. From a single impure case $C E$ the pure case $B F$ can only be guessed by an accidental and happy inspiration: it can be demonstrated with certainty only by a comparison of several different cases. If we can observe a sufficient variety of cases we shall be able to detect elements, which do not really belong to each other and are merely accessory, by the variety and change of the relations they bear to one another. We can then let these unessential elements drop away and retain in its pure form the pure relation $B F$, which they all involve. These impure cases form the raw material, on which we go to work, and are mainly supplied by *observation*; but the course of things if left to itself presents us in but few fields of research with the full number of cases that we should require in order to complete that elimination. It requires long epochs far transcending a single individual's range of observation for many natural processes to unfold the whole series of aspects which one should know in order to grasp the coherence of their conditions. But there are other obstacles to observation besides length of time; in the case of many actual products it is impossible to see how they have become what they are, because they never of themselves permit of being regressively analysed into the conditions out of which they arose. It is not often that we find ourselves so favorably situated as in the case of astronomy. This science has never met with more than accidental obstacles in its vestigating we can remedy this want by *experiment.* We can institute will a certain group of conditions C, and so compel the causes hich are really at work to respond with an effect E, which would herwise perhaps have never come within the domain of our senses. y varying at will the quantity and composition of that C we can ing about in E a series of changes in quantity and kind, which were ill less likely to offer themselves unsolicited to our observation. gain we can break up C into its component parts, and in each xperiment allow but one of them, or a definitely assignable group of veral of them, to take effect, at the same time cutting off the rest om action. The constituent elements of the result E admit of being parated in the same way, so that we learn which of them depends n which element of the compound C. Thus experiment is the ractical means by which we furnish ourselves with observations such number and involving such mutual differences and affinities, is requisite in order to the elimination of what is unessential in em and the derivation from them of a pure case $B F$. Defined in is way it is clear that experiment only has an advantage over obser-tion in so far as it is capable of supplementing the usual deficiencies f the latter; its function is to furnish us with suitable and fruitful bservations instead of the unsuitable and unfruitful ones which offer emselves. But it would be perverse to ascribe to it any other and ystical use in addition to that; we cannot set it over against obser-tion as a new method of knowledge; it is merely a way of preparing d setting before ourselves phenomena which it is of importance that e should observe. And for the same reason experiment must not be qualifiedly set above mere observation. In our day it is a prejudice f half-culture to suppose that anything that can be observed in broad ylight, without any preparation, ready to hand and in the gross, quantities by such artificial means, never by coarse observation alone. We must grant all this, but conversely observation often acquaints us with broad characteristics of phenomena, which in experiment would have been obscured by special conditions.

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[1] [Cp. 'Metaphysic,' sect. 51, on the distinction between 'Cause' and 'Ground' or 'Reason.']

261. I shall now attempt to lay before my reader the various kinds of relation between *C* and *E*, with which observation and experiment acquaint us, not in the hope of exhausting them, but in order to illustrate by examples how many and various are the possible cases and the consequences to be drawn from them.

1. The case $(+CE)$. *C* and *E* may be continually present in reality and continually together; still their mere coexistence, however uniform, does not warrant our concluding that they are so related as to condition each other, though such a relation may perhaps exist between them. Iron and silver and all the other chemical elements are always present in the world together; but it does not follow from this that one of them is the condition of another's existing, or that all of them collectively are joint effects of a single cause. At best the philosopher, for certain speculative reasons, which we cannot enter into now, may question the possibility of there being a plurality of elements coexisting yet unconditioned in any way by one another. But the primary use of induction is to understand nature, and the scientific understanding refuses to accept mere coexistence as evidence of an ulterior connexion. We find, moreover, that in each single one of these elements various properties or modes of action are uniformly combined. For example, all have in common the property of gravitation, and each in particular has as well its special affinities to all the rest. This case is not the same as the last; here we have one and the same subject, with different properties coexisting in it as its own. This oneness of the thing forbids us to suppose that the several attributes found in it have no reference to one another, and there is thus awakened in us the natural tendency of thought to explain one of these properties by the other or both by a third original one, which under different conditions transforms itself into those two. We will not at present fix the goal to which this logical impulse may lead us in the future; for the present it remains just an impulse which leads to nothing so far; for so long as our observations reveal to us nothing else than the perpetual coexistence of both predicates, they do not supply us with the means of showing the nexus of conditions, which perhaps exists between the two or between them and a third.

2. The case $(+C+E)$. *C* and *E* are present together, not uninterruptedly, but in frequent recurrence. In such a case it may be

mischances which befall us on Friday, and with regard to countless other superstitions of the sort. But we do not acquiesce in such a conclusion, if we can conceive of any real connexion between the C and E thus found together, and if there is any hope of finding out their connexion. We never think of acquiescing in it unless we shortly afterwards learn from further observations that their association is quite exceptional and abnormal. In itself the hypothesis of mere coincidence is the least probable of any; whenever phenomena occur frequently and repeatedly together there is probably a causal relation; it only remains difficult to decide whether one of the two C and E is cause or effect of the other, or whether both are not mere co-effects of a third cause Z. This doubt remains even when C and E appear not simultaneously, but after one another in a definite sequence in all cases of repetition. In that case C may no doubt be the cause of E, but both may also be joint effects of a third Z, which is not uniform, but undergoes changes, which succeed one another in a definite order. Day and night always follow on each other in this way, yet they do not produce one another, but are successive joint effects of the earth's revolution on its axis. Lastly, it may happen that E has always remained unnoticed, and only meets our observation when C occurs; thus the heart always beats in a living person, but a healthy man hardly ever feels it, unless a special excitement C supervenes; then C is not indeed the cause of E, but the condition of its meeting our observation.

3. The case $(-C+E)$. Doubts are left unsolved by the last case, which can only be settled by further observations which present themselves or are procured by experiment. We may find that E also occurs without C, or that C may be experimentally suspended, without at the same time suspending E. In such a case we cannot of course conclude that C is not the cause of E, though we may conclude that it is not its sustaining cause. The former conclusion would be a hasty one; we should appeal in justification of it to the principle: *Cessante causa cessat effectus*, but the only clear meaning which can be

ceases when the supply of heat abates, which produced it and is required as its constant sustaining cause. A child does not die with the death of its parents, the sole causes of which its existence could be the effect; but the equilibrium of a supported weight is lost when the supports are withdrawn. We need not at present analyse these cases any further; we can refer them all to an universal law of persistency[1], which in reference to our present problem we thus express: every reality, which has once been produced, of whatever kind it may be, continues to last, unless counteracting causes annul it. The effects which a cause produces do not therefore stand in need of a cause to sustain them so long as they consist in states of a subject which are in equipoise not only with the permanent nature of that subject, but with the external conditions under which it exists. They do require such a sustaining cause however, if there prove to be either in that nature, or in those conditions, forces which by their influence tend to transform it. If therefore E lasts on after C has ceased to be, three cases are possible: either there is no causal interdependence between the two at all, or else C is indeed the cause which produced though not the cause which sustains the effect E, and in this case again C is either a productive cause alongside of other productive causes, or it may be the sole cause capable of producing C.

4. The case $(+C-E)$. C may be observed to occur without being followed by E, but there is of course nothing in that to attract our notice, unless it conflicts with our usual experience, i. e. unless C and E have been observed, as a rule, to occur in conjunction with each other. In such a case it may be that C is not the cause of E, and we then, by drawing this new conclusion, correct the earlier one, which we had formed from our observations to the contrary. The connexion of causes and effects in reality however is not the same as that between ground and consequent in the field of abstract thought. There every ground, which holds at all, bring about its consequence not partially but wholly, and also in such a way that the whole of it can be perceived in the result. Two grounds may be operative at once, e. g. a quantity g may have as its conditions two equations determining its relation to x and must satisfy both. The influence of the second equation will then always show itself in this way, that of the many values of g, which the first alone left possible, it will leave over but a single one or a definite number of these conjoined in a regular manner. A change E on the other hand, which must follow in reality from the cause C, can always be set aside by a counter-

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[1] [Cp. 'Metaphysic,' sect. 162.]

cause Z so that it is lost to perception. We cannot say that Z annihilates the capacity which C has of producing an effect, for C can only be restrained from producing its effect E, so far as it reacts itself on the restraining Z; in this Z it always brings about another effect E^1, instead of E, which we expected, or it assumes itself, under the joint influence of Z and of its own tendency to produce an effect, a state E^1, which it would not otherwise assume. But this E^1 is very often of such a nature as wholly to withdraw itself from direct observation; in that case E seems to be altogether absent, while C is present; as a matter of fact E has only changed its form. This is invariably the case when moving forces meet with a fixed obstacle; they then seem to have no effect, whereas they really exert a strong pressure on the resisting body. If then we find that E fails to follow on C, it may be of course that there is a want of any causal connexion between the two at all, and in that case we must put a different interpretation on the sequences of C and E, which we have observed. But C may also be one or even the sole cause producing E and yet be prevented from bringing about E by a counterforce Z. This shows how much need there is of being circumspect, of looking round in every case of the sort to see, whether in place of E we cannot discover an effect E^1, which but for the obstacle it reveals would be absent. Lastly, when we institute C experimentally, and do not find that E ensues, and at the same time can find no trace of a E^1 taking its place, we are justified in concluding that C is not a cause capable of producing E at all.

5. The case $(-C-E)$. So far as mere observation goes the simultaneous absence of C and E will seldom strike us as noticeable, and when it does it will be because it conflicts with what we remember experiencing in the past. If however C has been constantly present and we find that when it ceases to appear E vanishes as well, the most natural thing to suppose is of course that C is at least the condition which sustains if not the condition which produces E, or may be that C and E are both joint effects of a third cause Z, and they both vanish because it has ceased. If E ceases to appear when we suspend C experimentally, the former alone of the two alternatives seems possible; still it may be otherwise. When we talk of a cause which has been active hitherto being suspended we may mean something more than that it just ceases. To effect such suspension we often have to take positive precautionary measures, and the new influence thus brought into operation may, while suspending C, at the same time create new conditions paralysing the further action of

the suspension of E as well. There was a prolonged controversy between those who maintained that infusoria are generated from an infusion of organic matter without germs of their own kind being present beforehand, and those who contended that their generation was conditional on the presence of spores or seeds adhering to the organic substance itself, or conveyed by the atmosphere, or contained in the water. The only way to settle the dispute was to show that the generation E of infusoria ceases when all access C of spores or seeds capable of producing life is cut off. They cut it off by boiling the water along with the organic substances and introducing air through red-hot tubes. The use of such means no doubt ensured the absence of living germs from all the three bodies concerned in the result; at the same time they were so violent that in excluding the germs they might also have rendered inoperative the causes which the counter theory assumed, viz. the inherent capacity of organic matter of developing into living organisms. The experiment therefore required to be modified in such a way as to eliminate the doubt.

6. The case $(+E+C)$. In none of our conclusions thus far have we established more than that C is a cause of E; that it is the sole cause, so that the converse of the proposition is true, and every E is the effect of a C, could only be ascertained by some method of exclusion, by which we could make sure that no other conceivable causes have the effect E. This exclusion is never possible with regard to the countless *proximate* causes, which are to be found at work in nature. We could not think of it till the elaboration of our perceptions was so much advanced as to have revealed to us a number of universally operative forces, which could be exhausted in a complete disjunction, to some modification of which forces every result whatever would be wholly due. Nevertheless inductive science frequently arrives at such convertible propositions; when in several cases it has found C to be the cause of E, it assumes that an E, of which it does not observe the cause, is to be referred as an effect to the same C. Logicians cannot be gainsaid when they declare it to be wrong to do so according to the canons of formal logic. For it is quite clear that the particular judgment, many E are effects of C, in no way warrants our concluding *ad subalternantem*, that all E are effects of C. Nor can the hypothetical judgment, if C exists, E exists, be converted simply into the judgment, if E exists, C exists. But we would remind those who would lay too much stress on this fact that the

tined in making good any short-comings, which such conversion .y have in respect of formal logical validity. There *might* be in :ure, he would say, a hundred similar effects due to a hundred ferent proximate causes, only it is not so in fact; as a matter of t similar effects flow from causes which do not resemble each other ·rely in being *able* to produce these similar results, but this ability ɜlf depends upon an ulterior similarity between the causes.

We hardly need dwell on this any longer. In order to make up what our conclusions lack in point of mere logical cogency, we peal to the fulness of the knowledge we have already actually quired, and such an appeal must obviously carry with it enormous ight. On the other hand we must bear in mind that the justifica- n so derived has its limits. Newton has expressed the principle question in his second rule as follows: 'effectuum naturalium ısdem generis eædem sunt causæ.' I think we may without lessen- ; our respect for his immortal genius confess that this formula by no :ans fulfils from a logical point of view those requirements of pre- ion, which as a mathematician he knew so well how to satisfy. e do not forget that this rule is not put forward as a logical law, t just as a rule or practical maxim of natural philosophy, probably lled forth by the brilliant discoveries which it preludes. But even such it is not a little indefinite and every single one of its terms eds to be explained. To begin with, the words *idem genus* require be defined, so that we may know what *effectus naturales* belong to ɜ same and what to a different genus. I do not lay much stress on that, nor can we even in logic altogether dispense with some ch vague expressions; for the rest we interpret it in this connexion the sense that merely quantitative differences would not make a ference of kind between processes resembling each other in form. ıt what are *effectus naturales*? If by these words we understand ery natural event so far as it is referred as an effect to any cause ıatever, the whole proposition which ends with the words *eædem usæ* is evidently untenable, so long as this last conception is left the necessity of their being *ejusdem generis* is rebutted by the most common experiences, which teach us that causes may often differ widely from each other and yet be equivalent and bring about one and the same kind of effect. Suppose the velocity with which a body *B* approaches a point *C* to be uniformly accelerated, this much is of course clear and necessary, that some force must act on it able to produce this and just this effect; but of how many different kinds may the forces be which do this! They can act as a pull *a fronte* from the point *C*, they may also act on *B* as a push *a tergo*, so as to drive it towards *C*. The former mode of effect may be due to the forces of electricity or gravity, the latter to a series of self-accumulating shocks. If we persist in regarding all these causes as *easdem* or as *ejusdem generis*, because in spite of their essential differences in other respects they all agree in producing just this one effect, we not only use words in a very improper way, but we turn the rule into a trivial tautology. For it is obvious that all causes, which are to have effects of the same kind, must at least be so far themselves of the same kind as to be each and all of them capable of producing these effects; they must therefore be equivalent as regards this effect. This is a mere deduction from the law of identity and as such requires no special maxim of physical science to enforce it; such a maxim should it is evident represent something as in point of fact true, which on formal logical grounds is *not* necessary; that is to say, in this particular case, such a maxim will assert that the causes of similar natural processes are not only similar in reference to these events, but are also similar independently of them. But as we have just seen, there are experiences which prove that what *this* maxim asserts cannot be universally true.

There is still another sense however in which the words *effectus naturales* may be used; they may mean not so much natural processes as processes in nature, that is to say not such events as incidentally arise on a petty scale out of the application of physical laws to fortuitous groupings of conditions, but such processes as have their abiding place in the grand theatre of nature, processes which would be considered ends of nature by anyone, who felt himself justified in using this conception. There is nothing in Newton's language to force us to interpret it in this way, but that something of the sort was before his mind is probable from the prominent position he gives his rule in the introduction to a work, which was intended to embrace in one vast intuition those very abiding, all-embracing, and all-determining events of nature as a whole, which we have described,

—we mean the revolution of the planets, the path our earth pursues, and the unceasing tendency of bodies to fall or press toward each other. Viewed in this light the above proposition would not be a direct rule to guide us in our investigations, but the expression of an actual fact, of which the existence has indirectly a controlling influence on the path investigation will strike out; we mean of the fact that there are at work in the world not an infinite number but a very small number of highest and most universal mutually independent causes, to one of which every group of interrelated effects is in the gross to be referred, though in detail one and the same effect is not always due to the same cause but may be due to very diverse equivalent proximate causes. It would still be difficult to fix the line of demarcation separating those great causes from these petty ones; nor would it be less difficult to make out what part of the proposition thus interpreted most deserves to be insisted on, that which points to the sameness in kind of the highest or that which points to the difference in kind of the proximate causes. Anyhow the scientific praxis of Newton is so admirable, that we do better to try to emulate it than to make a superfluous parade of its general maxims.

I will return to an instance. A chemist observes that a particular element C yields the reaction E; he then finds that a strange body, which he is examining for the first time, exhibits the same reaction E; he infers from this that the body in question is C, and this inference so far from being based on the simple conversion of that observation rests on the consciousness, which he has, of having already tested all the elements to be found on earth, and of having got this particular reaction E from none of them except from C. Formally this proof by exclusion is not in a formal sense absolutely safe, but yet carries with it great probability. If a new element C^1 is discovered, which gives the same reaction as C, he is so much the wiser, and forthwith looks about for some other test, by which to distinguish the two. Not quite the same amount of probability attaches to the conclusion drawn from spectrum-analysis. It is argued that materials, which in the spectra of the heavenly bodies produce the lines E, are identical with those terrestrial elements, which in a gaseous state display the same lines E in their spectra. Now we have not experimented with those non-terrestrial substances, and so we cannot be sure as we were sure in the former case that there are not several elements, differing in other respects yet agreeing in having this one reaction E. It is very probable there are not, because we know of no instance of one terrestrial element having the same lines in its spectrum as another them, and this proves that the capacity in question, i. e. the capacity of reflecting, absorbing, and transmitting different waves of light, does not cohere quite simply with the chemical nature of substances. On the other hand two elements are not attended with peculiar effects E or E^1, merely because the one is called or is Potassium, the other Sodium. The truth is that the only reason for their being or being called the one or the other is that the universal forces, with which bodies assert themselves against one another, occur in the two bodies in question with specific coefficients of different magnitude. But, it may be objected, there are conditions, which we cannot reproduce in any experiment. Under such conditions,—e. g. in the temperature prevailing on the sun's surface,—might not one of these coefficients, by the combination of which one element is characterised, assume a value which under terrestrial conditions it would only exhibit for another element? The result would be that different elements might occasionally exhibit the same lines in their spectra. All this is not so utterly unlikely, and so we cannot banish all doubt from our minds as to the glimpse into the constitution of the heavenly bodies, which spectral analysis has vouchsafed to us.

7. The case $(-E-C)$ would agreeably to our use of symbols mean, that we argue backwards from the absence of an effect E, which in other observations we found to follow on C, to the absence of C. There is no need of further explanation; all we can correctly infer from the absence of E is this, that although there may possibly be many different causes C^1, C^2, C^3 . . . , all capable of producing E, no one of them has been actually operative, either because no one of them existed or because each and all of them met with obstacles, which rendered the production of E impossible; the latter alternative is settled as before, according as traces are or are not to be found of another effect E^1, which takes its place.

262. Now supposing that in one of the ways described it has been set beyond a doubt that C either is or contains the cause of E, this last question can only be answered by repeated observations and experiments, by which we shall test one after another the several elements of C and see what is the effect of each. We may have no difficulty in distinguishing these elements, or we may only be able

t. The case $(C-a=E)$. The material analogy conveyed by
se symbols is plain. They signify that the absence or experi-
ntal suspension of one part a of the cause C produces no change
the effect E. If this be really the case, if, that is to say, the E
v observed be exactly the same as the E formerly observed, we
ll naturally conclude and shall be perfectly justified in concluding
t a has nothing to do with producing the effect. But this is just
at we do not always find; we are now dealing with all these cases
ply with regard to the manner in which they appear to our obser-
on, and we must remind the reader that very often the effect, so
as we can observe it, remains unchanged, whereas in fact the real
ct has through the suspension of a undergone a change into E^1.
pose six cords of equal length are fastened to the corners of a
ular hexagon, on which is slung a weight. If we then remove the
t, third, and fifth cords, the weight will, if the remaining cords
strong enough, not only remain hanging, but will appear to main-
its absolute place in space. Yet the latter is certainly not the
e; the tension of the three remaining cords is increased, and as
y have stretched a little the weight itself has sunk slightly in a
ical direction, and herein consists just the new effect E^1, which
taken the place of E; the difference between the two is lost to
perficial observer; who is led to conclude wrongly that the three
r cords contributed nothing to the original effect E, whereas in
t of fact the work which they did before has but been vicariously
ertaken by the other constituent parts of C. It is hardly requisite
otice how common an error it is to suppose, because an effect is
minute as almost to escape our notice, that it therefore does not
t at all. Such an error always avenges itself on us later, and
risk we run of falling into it is so obvious that all kinds of
hods are resorted to in order to magnify these slight effects and
g them within the range of our perception.

. The case $(C-a=-E)$. It is found that on a vanishing in

Every complex machine, every living body affords an example of this; in both there are many parts the lesion of any one of which is enough to put an end to the motion of the one and the life of the other, although no one of them by itself would have been able without the co-operation of the rest to produce and sustain motion and life. The fact that with the destruction of a single part of the brain a a definite spiritual function ceases is no proof that just this single part was the organ, which produced that function; even the counter experience, that no lesion of other single parts has the same result, does not render this conclusion perfectly certain; it always remains possible, that a was no more than the indispensable part, in which the effect of all the rest took just this form E. The function must then cease just as much when a is hampered as when all the remaining parts besides a of the brain are hampered in the discharge of their functions. In order to settle whether it is so or not, we must try to observe the changes of E into E^1, which arise when a is left undisturbed, while the functions of the remaining organs are checked in this action.

3. The case $(C - a = E + \alpha)$. The part a disappears in our observation from C or is by experimental means made to lapse, whereupon the effect E acquires a new element α which it had not before, or anyhow an effect α now arises for the first time; we may then conclude that the remaining parts of C involve the ground to which α is due, but that a hindered that ground from taking effect in such a way that on the removal of a, α can for the first time exhibit itself. But the observation does not entirely justify the conclusion; for it remains open to doubt, whether when a disappeared a new and hitherto unnoticed condition Z did not enter, which alone has to do with the production of α, a being capable neither of producing α nor of arresting it. We set aside this doubt by an experiment, which makes us sure that the means we took to suspend a really produced no other or further effect than this negation of a, and did not at the same time contain a positive influence Z, to which the appearance of α can be attributed. Whenever a state of equilibrium is destroyed by removing one of the forces which preserved it, we have

active before; in such a case we cannot help believing that the organisation is so planned that one function holds another in check, and we therefore refer the movements observed to the removal of the check. There is much in history, which, if we consider it, leads us to take the same view. Of course there are positive stimuli, which have driven mankind along a certain path of historical development, but the majority of great and violent revolutions are due to the removal of checks, which prevented ever present tendencies and feelings from unfolding themselves; and even those positive impulses for the most part only guide events for a while in their particular directions; after a time everything takes another turn, because unwittingly and unintentionally the given impulse has removed or weakened the checks which restrained forces of quite a different kind and of a different tendency.

4. The case ($C-a=E-\alpha$). This case does not require considering afresh, but has already been dealt with under the second and third heads. If the suspension of a part a of C occasions the disappearance of a part α of the effect, a and α must necessarily be connected as cause and effect, and a may be the exclusive cause of α and α the exclusive effect of a, so that a is the cause adequate to produce or sustain α; but it may be that a is only either the one or the other as combined with the remaining parts of C, and this last may continue to be the case, even when counter experiments shew that any other part of C may be suspended without α being annulled in the same way, for the parts which still remain may serve as substitutes for the parts set aside. And this is not all. It may be that a does but indirectly condition α, as in the third case; then another part of C, perhaps $d+f$, is the cause which produces and sustains α, only a third part b arrests the action of $d+f$, while finally this check in turn is balanced by a in such a way that the removal of a enables the counteracting force of b to suppress α. All the other conditions of life C may be left unimpaired, yet if only oxygen a be withdrawn, the living functions of the animal body are suspended, without so visibly altering its structure in other respects as a different cause of

having been before contained therein, the aggregate effect E' can only remain unmodified under certain conditions, which are the same as those under which, in the first case, the lapse of a constituent part a until then contained in C left E unaltered. Two cases are possible. Either there is to be found in the observation, while a is present in it, a Z which escapes our notice although it cancels the effect of a, or our experiment is at fault and we have not succeeded in introducing a in such a way that it can exhibit its effect. If however a is really active the aggregate E must be really altered into E^1, but this change may either withdraw itself from observation or it may not affect the particular part of the aggregate E, to which in our negligence we alone directed our attention, in which case it will equally pass unnoticed.

6. The case $(C+a=E+\alpha)$. A fresh element a, on being added to the causes C which have thus far operated, gives rise to the fresh element α in what was the effect E. It may then be that a is by itself the sufficient cause which would produce the effect α in the objects in question. But it may also be the case that a, like the last drop which makes a cup overflow, is no more than the cause which completes the tale, so that neither those previous causes without it, nor it without them, would have brought about this result. Lastly, it may happen, that the effect E or more generally the fact E, which by being augmented by α passes over into E^1, is neither a mere state nor an event ever repeating or continuing itself in the same way. It may itself be a process of development or growth, which once generated by a group of causes C is forthwith constrained by the nature of the objects, on which these causes act, to transform itself

without *a* at all. It is not altogether easy to settle such a doubt, because in this case the possibility of experiment is confined within such narrow limits. If we have once observed that in several cases the desired result has ensued upon the introduction of *a*, we shrink from the experiment of omitting what may be but is not certainly superfluous. On the other hand counter experiences may offer themselves unsought and seem to prove that *a* is not wanted, and yet not remove the ambiguity. The cases compared are seldom of quite the same kind, and it can hardly be proved that the *a* which is now left out has not found a substitute in some *Z* of equivalent influence. We meet with similar embarrassment in dealing with social and historical phenomena; it is difficult to decide whether a new turn taken by events is to be attributed to a measure or occurrence *a*, which is observed to precede it. Still harder is it to discover wherein he efficacy of *a* in all cases consists, and what collateral conditions nvolved in *C* favour it and render it possible.

7. The case ($C + a = C + b$). Two causes have an element *C* in common, but have also distinct elements *a* and *b* which differentiate hem. It is impossible that two such causes should have exactly the same effect, but it is quite possible that of their aggregate effect a certain portion on which we fix our attention is the same, or lastly hat so much of this portion as falls within our observation is the same. Such is the case which I denote by the above formula. The most obvious conclusion to draw from it is of course that both causes produce a like effect in virtue of their like element, and that so far as egards this effect their dissimilarity is without influence. I hardly need say that this conclusion is very often the correct one, even when wo or more causes have nothing else in common than just a few attributes, while *a* and *b*, in which they disagree, dilate into clusters of very many attributes. But it may be that *C* by itself alone never produces or sustains the like effect we spoke of; in order to do so it may always require to be backed up by *a* or *b* or *d*, in which case we must egard the latter elements as equivalent and companion causes of *E*. t may even be the case that the part *C*, which remains the same in different causes, is quite inoperative as regards *E* and that *E* is enirely dependent on the unlike elements of the two. Let three forces act on a particle situate in a fixed plane, the one of them *C* acting along a line vertical to the plane, the other two *a* and *b* along diver-

latter forces others, which give the same resultant. The first force d, the only one common to both systems of forces, is at the same time the only one which does not help to determine the direction and magnitude of the resultant. It is universally true that any balance of forces and any movement admits of being resolved in a thousand ways into very different combinations of particular causes. It may of course be objected that in all such cases a, b, and d are not so widely different from each other as to be disparate, that they still involve a common element x in spite of their differences. This x we shall be told must be reckoned as belonging to the common C, and then $C + x$ will always be the true cause of the like effect E. It may be answered that such an objection though true is yet irrelevant, for it amounts to no more than a restatement of what in the abstract is a truism, viz. that like *consequents* always have like *grounds*. In this connexion however we are dealing not with the abstract but with the concrete, and are concerned to know the guise in which these like grounds of like consequences appear in the actual observation, and we found just now that the like elements or attributes in two causes are not always the vehicle of these like grounds. On the contrary these like grounds are in fact often concealed in the combination of *prima facie* unlike elements, attributes, or conditions. These ambiguities then must be got rid of by means of collateral experiments. We must know whether C alone is able to produce or sustain E; if it is, then of course it does not necessarily follow that a and b are without effect, but they are anyhow elements in the cause, which we could do without, inasmuch as we then get the case ($C - a = E$) and its consequences as above considered. We must know furthermore if a and b alone produce E or no; if they do, then agreeably to the same first case C is not necessarily inoperative, though it will be no more than a contributing cause of E, which might be dispensed with. If neither the one nor the other is the case, then $C + a$, $C + b$, $C + d$ are pairs of mutually indispensable contributing causes of E, and it is now time, by new combinations of our perceptions or by varying the experiments, to find out what is the common element x in a, b, and d, and perhaps also what is the particular element c in C, which together constitute the true and sufficient cause $c + x$ of the identical effect E.

263. By means of the inductions which we have thus far passed in review, we shall not always be able to determine even the proximate and sufficient causes of an effect, much less to find out the sort of causal tie, which holds the two together. To do both is our end and aim and we shall come nearer to reaching it, if we can observe the equilibrium, but it may oppose a greater or less resistance to any attempts made to destroy it, or the force needed to maintain it may vary. As before, I group together the simplest cases we can observe by way of example.

1. The case $(mE = mC)$. Let us represent to ourselves once more the pure case, which we before denoted by BF and may now denote by CE. This formula means that C is the whole cause and nothing else than the whole cause of E, E the whole effect and nothing but the effect of C. Assuming then that both terms admit of direct quantitative determination, we regard it as a self-evident principle that like differences between two values of C will find a response in like differences between corresponding values of E, C and E thus standing in simple direct proportion to one another. Then mE will be $= mC$. This formula is no mathematical equation but a logical symbol and presupposes that the effect no less than the cause is capable of being measured by a standard of its own suited to its nature and permissible in its case. It asserts that if this is so the unit of the effect E is contained in any effect whatever exactly the same number of times as the unit of the cause C is contained in the cause which operates to produce that effect. This relation however is self-evident only in cases where a number m of particular causes C produce, each by itself, the same effect E, and where all we do is to add up the sum of these separate effects, which will then be proportional to the sum of the causes. Suppose we pay away the same amount C of money on m different occasions, and each time for the same amount of goods E, then assuming that prices remain stationary, the total bought will be mE when the total spent is mC. Let us take another example. Suppose m to be equal but separate impacts which act on the same number of different elements and give to each of them the velocity E, then the sum of all the velocities produced will be $m \cdot E$, and the amount of the motion generated will be $m \cdot E$ if we regard the

seller; in the abstract it remains true that each C is no more than the condition and adequate ground of a single E; this is the only consequent which in the abstract the ground justifies, it is only in the real effect that it is modified by those accessory causes. In the same way an impulse C may give a body the velocity E, but $m\,C$ if it acts on the body all at once is not unlikely to shatter it instead of moving it forward. $m\,C$ always remains the rational ground of the velocity $m\,E$, but the result is modified by other circumstances consisting in the texture of the body. There is only one condition under which it is self-evident that we can expect the cause $m\,C$ to be followed by its due effect, the motion $m\,E$, viz. that we may regard a material element as the mere substratum of motion, destitute of any native power of reaction of its own. We may put it in a general way thus. In applying our principle we suppose the m fold cause to be equal to m particular causes C, and assume that there are no circumstances of any sort present, which would compel a single element in this sum to take more or less or other effect than if it were present alone and the rest of the terms not there. The m fold cause will then produce the m fold effect, and conversely in cases where our observations reveal this relation approximately we may be pretty sure that we have before us a pure case $C\,E$, which is identical in the sense specified with a pure causal relation $B\,F$.

2. The case $(E = \mathfrak{E})$. It is often the case that a cause C acts on the same object t times, t being understood to mean either the number of times this action regarded as momentary is repeated, or the number of time units, in each of which the continuous force C produces a certain degree of effect. Now if this force is of such a kind as to allow the object exposed to its influence to remain identically the same, the same effect would take place afresh in the object every fresh time we let the same cause operate on it. Thus after the cause had operated t times or after the time t had elapsed there ought to be present in the body t times as much effect, provided that is that agreeably to the law of persistence every earlier given effect is preserved and not annulled by any counteracting force. This is the case with motion in space, in the case of which we presuppose that the causes producing motion either do not change the object moved or only bring about in it inner states, which throughout exercise no counteracting influence upon its assumption of new motions e. If by the effect E we understand the velocity generated, E will always $= e\,.\,t$ and depend on the time. Now for an opposite case. A constant cause C acts continuously on an object during a certain time t,

impossible for the particular impulses to accumulate, and thus would leave the effect E constant and independent of the time. One more example. A cold body grows warm under the rays of the sun, and is then found to maintain a constant temperature for any length of time during which it is further exposed to the same rays. The mere incidence of the rays cannot have caused this phenomenon; it is only accountable for on the hypothesis of a companion cause, namely the radiation which proceeds from the heated body: when it has once reached a certain temperature relative to its surroundings it is obliged by the law of radiation to give out in its turn just so much heat as it continues to receive.

3. The case $(dE = \frac{m}{E} dC)$. There is really no case except that of simple movement through space in which we can assume that the effect produced in the object a will not in any way prejudice the effect immediately to follow. In general this a is changed by the first operation into α: and this fact, that the object that receives the effect does not remain the same, constitutes a variable concomitant condition Z, which associates with each fresh impulse of the cause C effects of which each is more different from the first than that which it succeeds. Let us first assume that the change of a into α is of such a kind as to tend to thwart the next operation of the cause, in the same way as an already compressed body offers resistance to any fresh compression, as the mutual approximation of its elements increases the repulsions operative between them. The measure of this resistance cannot be a constant quantity independent of all the agencies, which here co-operate. It must on the one hand be proportional to the specific intensity of the inner repulsions, to which the resistance is due and which are different for different bodies; on the other hand it must be proportional to the amount of compression already effected, since it is this which by bringing the elements closer to one another in the manner

in case of two different causes C and C_n on their respective amounts. Now natural causes are never quite instantaneous in their action. We can analyse every C into a number of dC, which are successive, though for the rest their distribution in time is arbitrary. Each of these fractions dC of the cause would if it acted singly produce a corresponding and constant fraction of the effect $dE = m \, . \, dC$, but inasmuch as each of them acts on an object which is already modified by the action of its predecessor, the effect dE is altered for each of them. It therefore makes no difference whether we regard C and C_n as two different causes or as two different values, at which one and the same growing cause C has stopped in its growth or is for the purpose of our analysis supposed to have stopped. If we then signify by $E = f(C_n)$ the result already produced by n successive dC, we obtain for the effect dE which will result from the addition of yet another dC the following: $dE = \frac{m}{E} \, . \, dC$. Among pure quantitative functions it is the logarithm C, which shows this mode of growth, and so we come upon logarithmic expressions in calculating operations which by their own results create obstacles to their own repetition proportional to those results.

4. The case ($dE = mE\,dC$). We have just seen that a cause cannot when repeated have its effect diminished merely because it is not acting for the first time. Just as little can its effect be increased by the mere fact that it has already acted several times. Both effects are ascribed to habituation: we say 'practice makes perfect,' and also 'habit hardens.' An increase in the effect produced obliges us, no less than a decrease, to assume a contributing cause Z; this Z consists in such a modification of the object a influenced by the cause into α as facilitates every subsequent operation of that cause by continually opposing to it less and less resistance. Thus the first blow shakes a stone in such a way that the second blow has only to intensify the vibrations already going on within it in order to overcome the cohesion of its parts. If nothing else enters into the calculation we must for reasons of the same sort as in the above case reckon the magnitude of the effect produced as at any moment proportional to the aggregate result or to the integral of the earlier effects. In the case of pure quantitative functions of C it is the exponential function e^c, which[1] possesses this peculiar property of a differential quotient equal to the integral itself. Thus we shall often

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[1] [There is here an unavoidable ambiguity of notation. e which was before the symbol for *effect*, here stands for the base of Napierian logarithms.]

and decrease at the same time that their causes go on steadily increasing. Whenever therefore E periodically alternates from increase to decrease, while C changes in one uniform direction, there must exist besides C one or more companion causes Z, the relations of which to C are either in themselves variable or are so deranged because it happens that they operate together that the effects of all now accumulate upon and now cancel each other, and so pass through maxima and minima from the one to the other of these reciprocal attitudes. We can conceive of the combinations possible in this case being infinitely numerous; the formula I have used is no more than a very inadequate symbolical expression of these possibilities.

CHAPTER VIII.

The Discovery of Laws.

264. In the relations between causes and effects examined in the previous chapter lie the clues by which we are guided in instituting fresh experiments or seeking for fresh observations in order to exclude the possibility, which still remains, that different causes may produce the same effect. The general import of this procedure is always the same: from the impure observations SP or CE we have to discover the pure case $\Sigma\Pi$ or BF, by eliminating from the observation all that has nothing to do with the causal nexus before us. I see no reason to analyse this general precept any further into a number of separate methods. It is much more worth our while to point out that in elementary algebra we have already an instructive type of the very various modes of operation by which we may reach this end. We have given us equations which jointly determine the relations of two or more unknown quantities; these equations we transform in all sorts of ways by adding on new quantities, by subtracting others, by multiplication and division of the whole; and are thus at last able to compare the equations immediately with one another, and adapt them to the elimination of particular unknown quantities. The present problem is to be solved in a similar way, now by a timely addition of fresh conditions, whose influence we can calculate; now by a suspension, equally calculable in effect, of given conditions; or again by altering the relative position of the co-operating cause; or, lastly, by modifying our own attitude towards the material we have to observe. I will not stop to decide whether we shall ever be able to reach by such means a pure case BF; but even supposing we were so lucky as to have discovered the exact cause C of the exact effect E, we should yet in no case have completely satisfied our curiosity, save in the case of historical enquiry. For the only conclusion we could draw there really is a connexion between C and E, but to know the *law* according to which that connexion comes about and varies.

265. The term *law* has different meanings as we use it in connexion with different circles of human interests. Its logical meaning however never varies. Stated in its complete logical form a law is always a *universal hypothetical judgment*, which states that whenever C is or holds good, E is or holds good, and that, whenever C undergoes a definite change into C^1 through a variation of itself dC, E also becomes E^1 through a definite variation of itself dE which depends on dC. A law is hypothetical, because it is never meant to be a mere enumeration of what happens; its sole function is to determine what should or must happen when certain conditions are given. All laws are thus hypothetical in their import, and those which refer to permanently given or permanently presupposed conditions are no exception to the rule; they only seem to be so because they are not stated in the form of an hypothesis. Thus we enunciate the following in a categorical form as a law of nature: all ponderable elements attract one another inversely as the square of their distances from each other. Here we merely state the fact that in the case of such elements a particular condition is adequate to produce this consequence; this condition is perpetually fulfilled and consists in their simultaneous presence in the same world. Again the constitution of a state categorically maps out the relations which hold between the various groups of its members, but always under the tacit proviso that so long as the state exists at all, these fixed ordinances shall be constantly maintained and renewed as generation succeeds to generation.

But besides being hypothetical—that it is as a matter of course—every law is also universal, and must on that account be as strictly pronounces expropriation under certain conditions to be always legitimate, it may fairly itself assume the more pretentious name. It is implied in the idea of a law that it should pay such regard to variations or differences in the condition and consequent, only the idea cannot always be realised. The certainty that two bodies attract each other is in itself a fact which needs to be further determined; natural science does not see in it a law until it can assign the particular ratio in which the attractive force varies in its amount in dependence on differences and variations in the mass and distance of the bodies, or on some other condition of variable magnitude.

It is the same with moral and judicial laws also. A commandment so universal as that which enjoins love of our neighbours may fairly, as an expression of the deepest motive which can govern us, possess a higher value than can any law, yet in its form it lacks the precision of a law; for it is neither clear *prima facie* what result should follow from such love, nor in actual life can the commandment be fulfilled, without the love which it prescribes—whatever it may consist in—having a definite degree of liveliness, or without its force flowing along a channel in one case along which it does not flow in another. The general formula we have quoted gives no hint whatever as to what this channel shall be. Judicial laws, on the other hand, are based on the distributive *suum cuique* in its widest significance. Whether they prescribe actions or fix penalties, the predicate they attach to every case of the recurrence of what they bring under the general notion of any legal relation is not intended to be incapable of modification. Differences in quantity between various cases have a real significance in the eye of the law: it is only the defectiveness of our standards for determining those differences, which compels us in practice to be content with roughly graduating the scale of legal consequences, when we would far rather make it exactly proportionate to the individual differences on which those consequences depend. It would seem that none but purely negative laws and moral prohibitions ignore any such graduation of ground and consequent. I leave it to the reader however to judge, whether in a theoretical sphere negative separated from the predicate, which is simply denied of them all. As regards moral prohibitions, it is true that we do not find in them as such any reference to such a gradation or adjustment of penalty to guilt, nevertheless in passing judgment on a breach of them, we always make such reference. They prohibit beforehand any appropriation of another's property, but the commission of such an act is according to its particular nature subjected to very various degrees of disapprobation and punishment.

266. There is a difference of intention between a *law* and a *rule*, which may in most cases be easily seized, though it cannot be maintained in all. In practical life a law determines a state which is to be brought about by an activity or mode of conduct, and which is essential to the fulfilment of the ends of the political or social community; the rule supervenes as a practical ordinance, and since there are many possible courses of conduct, all in themselves equally contributory to the realisation of that state, the rule helps us partly to select the most advantageous of those courses, partly to secure, if only by fixing a definite mode of procedure, the requisite uniformity and harmony between individual performances. In theoretical investigations of reality, we mean by a law the expression of the peculiar inward relation which exists between two facts and constitutes the ground at once of their conjunction and of the manner of this conjunction ; and in every simple case there is but *one* law. The rule, on the other hand, prescribes a number of logical or mathematical operations of thought, by which we are so to combine our perceptions as to arrive at conclusions, which in their turn tally with reality, and there may be several such rules all equally sound for one and the same case. Thus it is only for the law that we claim an objective truth. The rule is merely subjective, and sums up the various adjustments of our reality, but they are free to make use of any roundabout method, which our attitude towards things necessitates, and to transform the inner relations of things as they like. This difference in intention between a law and a rule is no doubt a wide one, yet in making it we are hardly ever quite unbeset by doubts, least of all where we are concerned with the investigation of reality. It is clear at the outset that not a few of the methods of procedure at present in vogue are mere rules; but more than that, it remains an open question whether any one of the laws, which we believe ourselves to have discovered, really deserves the name in the special sense explained above. We are accustomed to use the name where we have reached very simple and universal propositions about the actual conjunction of phenomena. Thus we regard it not as a rule, but as a law of nature, that the force of gravitation diminishes according to the square of the distance; yet the inner nexus between the terms of this proposition is still undiscovered, and we do not know how it is that the quantity of space, which intervenes between two bodies, can cause their reciprocal effect to vary as it does. Ultimately, therefore, even this law is a mere rule, which teaches us how to calculate from given data of distance and mass the variations of their effects; it does not exhibit the inner connexion of these effects with their conditions. We shall have occasion to recur later on to this question. At present it is enough to notice that in the considerations which immediately follow we shall look on the law as no more than the simplest rule which conjecture has to guide it in getting at the genuine nature of things.

267. Thus far we suppose that the means specified have enabled us to discover as accurately as possible the pure causal connexion between C and E. We also suppose our experiments or observations to have supplied us with a number of pairs of values of this cause and its corresponding effect, these values being quantitatively determined and forming a double series. It is anticipating somewhat, yet we may suitably preface our attempt to determine the universal law of such a double series by a consideration of the various causes, which may produce a divergence between the quantitative relations which we find and the true relations of things which we are in quest of. In the first place let it be borne in mind that what we observe is not the things themselves, but the impressions, which things make upon us. We will not at present attempt to settle whether the impression produced in our consciousness *can* ever be like the things and their relations which produce it. One thing however is clear on the face of it, and that is, that it is not obliged to be like them, but ontent to understand by such truth or correctness of our observa- as we at present aspire to, their universal validity for all human vers, who are normally constituted and placed under similar con- ns. If it is asked how we can be sure that any particular obser- n possesses such universality, we can only answer that practically ultimate decision in every case rests with the overwhelming rity who agree in their views, as opposed to the minority who ree. If anything appears to me different from what it appears eryone else under exactly similar conditions, there must be some in my individual observation, an error which will vary and may t right by repeating the observation, if it can be traced to mere entary inattention, but which becomes a permanent, and in a wer sense *personal error*, when the anomalous organisation of ndividual's senses is to blame for it. How widespread is such tiveness of sensible apprehension in regard to the qualitative nt of sensation, is shown by the way people will differ in their nents as to the resemblance and contrast of colors, or the ony and dissonance of tones. Such disagreements, however, equally noticeable where it is a quantity which has to be ated. For all practical determinations of quantities given in y ultimately rest on the accuracy of our sensible impressions, ll that artificial methods and instruments of measurement can do transform what is too big or too small, the one by splitting it up, ther by somehow magnifying it, in such a way as to bring both n the sphere of more moderate intermediate quantities, of whose ity or inequality we can judge with sufficient accuracy by help of ensible faculties alone. And really it is to such a simple judg- as this last that all our measurements are reducible. Nature not endow us with a power of specifying offhand how great is ifference between tw al quantities of space time or together produce a length *equal* to *d*. But even if we use a microscopical standard we must admit that everything ultimately depends on the certainty with which sense-perception shews us that the extremity of the *d* to be measured exactly coincides with the extremity of one of these infinitesimal units of measurement. When intervals of time are equal we recognise them pretty accurately as being so in virtue of our feeling of the equality of one beat with another; but we can only measure unequal intervals against each other by dividing them into beats or equal recurring units; nothing but the immediate sensible impression, however, informs us of the equality of these units themselves. And when we use clockwork to mark the recurrence of these units with audible ticks, the accuracy with which it does this still rests ultimately on the certainty and precision of the visual impressions, which helped us to set out the spatial dimensions of the works and their parts in such a way, that their movement shall in fact give out those signals at equal intervals. Lastly, if this expedient is to serve to fix the times, on the expiration of which certain phenomena only observable by other senses, as by the eye, occur, nothing but the immediate impression can tell us that a phenomenon of this other kind exactly coincides in time with this audible signal, and it is just here as we know that our judgment is for physiological reasons not so acute as we could wish it to be—on the contrary, it needs the previous correction of our personal error.

In conclusion, I will but briefly mention, what is familiar to every one, the relativity of all our determinations of measure. There is nothing absolute except the numerals by the help of which we count the recurrences and specify the number of units found. The units themselves can only be determined relatively to each other, and there is no sense in asking how big anything is unless we measure it by a presupposed standard. To find those units, that is, to determine them in such a way that they may be fixed, useful, and unambiguous, is itself a problem, which the art of observation has to solve. It is enough at present to remark, that in unchangeable natural bodies we have a means of determining units of length, while we have exact periodic astronomical appearances whereby to determine units of time;

268. Supposing that this primary defect, the personal error, has een remedied, what we observe may still fall very far short of the uth, owing to the position, which either individually or as men enerally we occupy towards things themselves. We could illustrate is from other than spatial phenomena, still it is they which enable s most readily to appreciate the frequency with which the same rocess or the same object yields very different images according to e point of view of the spectator. I think I may hazard the asseron that every regular event gives a regular projection of itself for y point of view we like to take, but the rules by which we reason om one such phase of the object to another are framed in such a ay as to favor one point of view more than another, and on that count it is often exceedingly difficult to go back from the event as *rojected* to the event which *produces the projection.* A circular moveent will only appear circular to a spectator, whose standpoint is mewhere in the line drawn through the centre of the circle at right gles to its plane; to an eye situated anywhere outside this axis and is plane it will appear an oval; while if one views it from any point the plane of the circle but outside its circumference, it will appear an oscillation in a straight line. The synthesis of the times aversed by the moving point and the loci corresponding to the times ill form a separate series for each point of view, and each such ries will be regular in its formation, though one of them will have uch more value than another as an indication of what really takes ace. Now if this was all that met our observation and if we had ot already got a stock of other experiences in regard to what is true reality and of usual occurrence, we should have no reason to desire y other rule than that, say, which in our example, expresses the ctilinear oscillation. But in nature we are seldom left without condary features, which force themselves simultaneously on our servation and lead us first to doubt and then to correct our first pressions. That we observe that circular movement means not at we think or represent it mentally but that we *see* it and we only axis itself can fail to notice these variations and so feel no impulse to seek an explanation of them. Now let us place ourselves in the very plane of the circle, the body will then, as it travels from one extremity *a* of its apparently rectilinear path towards the middle of the same, wax in size and brightness, while after passing the middle it will wane in both respects till it reaches *b*; if it then recedes from *b* to *a* this decrease in bulk and brightness continues at first, reaches its minimum at mid-path, while from these onwards to *a* the body waxes afresh. If one takes it that all these appearances are real, one has many questions to answer. Why does the body reverse the direction in which it is moving when it reaches the extremities of its path, and why does its velocity increase as it approaches the middle and decrease as it approaches the ends? Either there is something in that middle point the effect of which is to draw the body towards it, or there must be present and at work in the prolongations of its path equal and opposed forces urging it in that direction. But why, if that be so, should it pass through both the minimum and the maximum of its size and brightness at the same middle point and yet the force or forces remain uniform? The easiest conjecture to make would be that the two appearances were merely coincident; quite apart it might be said from its movement along its path the body is subject to periodic increases and decreases of bulk, which however are merely functions of the time not of the place. Still as at any time *t* it must be in some place or other it may at the moment of its greatest bulk just as well be in the middle of its path as anywhere else, and as its bulk requires in order to reach its minimum the time *t*, which it takes to accomplish a half oscillation, this minimum too must take place just as the body occupies this same middle point.

But who would credit such an explanation as that? In the rest of nature such periodic enlargements are altogether unheard of, while changes, such as we have described, in the apparent size and brightness of bodies are quite familiar; we know that bodies are liable to them according as they alter their distance from our eye. Relying on such analogies then we shall try to grasp or apprehend the fact observed as the projection of other and truer facts. We notice no withdrawal of the body between the loci of the maximum and minimum, on the contrary both coincide with each other in the middle of the path. Moreover the ways by which it goes and returns appear coincident at every point. Taking all these considerations together we are obliged to suppose that the true path is a plane closed-up curve, one of the diameters of which must lie along our line of sight at the centre of its apparent path. By comparing the particular apparent loci occupied at successive moments of time we shall further discover whether the true path is a circle, an ellipse, an oval, or what. The mere mention of the name of Copernicus will be enough to make the reader understand how the accumulation of insoluble difficulties in the facts as observed impels us to transform our views of nature, and how much at once becomes clear when we grasp what is sensibly given as a mere projection of a reality beyond our observation. In order to that however we must already be in possession of a store of universal truths as well as of earlier experiences of facts; pure logical precepts may stimulate but cannot conduct us to the goal.

269. We must now go back a step. Before we try to interpret the observed facts in the manner specified, we must be in possession of the actual laws, which we think of reducing by means of such interpretation to a form at once simpler and more in correspondence with the real course of things. Nothing is given to help us in the discovery of these laws beyond the series of values displayed by the causes and their corresponding effects. Now even if we assume that these numbers before us are perfectly correct as a statement of what we succeeded in observing, still the transition from this series of isolated terms to the universal law of its formation is always a *jump* on the part of thought. How do we know that such and such a law is the only one valid for the series and true? There is no process of demonstration by which we can find such a law, none by which it could be shown to be what it claims to be. We can never do more than guess at the law and then by the help of innumerable secondary considerations heighten the probability of its being the true one. It is of importance to be quite clear on this point. If we have to start with a limited number, say n terms of a numerical series given in the order in which they succeed one another in the series, it will be easy to find a simple general formula, exactly corresponding to these given n terms and expressing their general term; but even then this formula need not necessarily be the only possible one: it may at least be apprehended in different ways. For example, let the given terms be 1, 3, 5, 7, 9; then if 1 denotes the place in the series occupied by the first of the given terms, $2n-1$ will exactly express the general term. But if we think the general term in exactly this manner it will hardly correspond to a real physical relation, of which it is meant to serve as the regular expression. The same given series may however be thought as an arithmetical progression with the initial term 1 and the difference 2, and besides that as the series of differences got by subtracting the square of a whole number from the square of the one following it in the numerical series. Both readings of the series may be expressed by the same general term, both determine every term of *this* series, but the genesis of each term is conceived differently in the one case and in the other, and this difference of manner is of importance, because it allows of our making different assumptions in regard to the physical relations of the phenomena expressed by this series.

Thus without going any further we here have unsolved doubts in plenty. But this is not all. The presuppositions we make in this case are not at all the same as what we make in the case of observations; a general term found in the manner just described holds exactly for only the n terms, from which it has been generalised. Not so with the laws which must be generalised from observations; we require these to hold good no less for the values of the causes and effects which we have not observed than for those which we have. We can of course interpolate terms in a given series; that is, we can calculate missing links in such a way that they will fit into a series agreeably to a law of its formation, which we have beforehand abstracted from the given terms, and which often proves to be not a little complex. But then we assume that the particular law developed from the given terms holds equally good for terms not given,—an assumption which is always permissible when we are merely concerned with the completion of a *conceivable* series, but which is altogether inadmissible when the question is whether this conceivable series itself corresponds to a something *real* even where this correspondence has not been observed. Thus before we demand that a law, which we have somehow got out of the given terms, should be extended to terms not given, we must have reasons, which justify our pursuing such a method of interpolation at all. We may illustrate this by a very simple example. Let us figure to ourselves the values of C as so many abcissae x, each larger than the last by Δx, and the values of E as so many ordinates y. Now if the given series gives the same value $y = B$ for all values $m \Delta x$ of x, it may of course be the case that the equation would hold good for all the unobserved ordinates, which correspond to fractions of a Δx. In that case the line joining the extremities of all the ordinates is a straight line and parallel to the axis in which the abscissae are taken. Still this does not follow as of course. Take any two Δx we like, the ordinate y between their extremities may have every possible value, and the curve which unites the various co-ordinates y may a minor rule for selecting observations, like that we have noticed above for imperfect inductions. The rule in question forbids us to form the series of pairs of values in such a way that *C* progresses according to a regular law, and none but the particular values of *E* are permitted to appear, which correspond to these symmetrical values of *C*. If we do so the chances are that we shall only get a series of singular values, of maxima or minima, or fixed values of *E*, which periodically recur, and which either give us no insight or suggest false surmises as to the intermediate course of the curve. A regular advance of *C* by equal increments no doubt helps us to guess the universal law of the series; but if we wish to confirm this guess we must make the increments of *C* change as unsymmetrically and irregularly as possible. To put it quite simply, a man who never observes a place of public resort but once in every seven days and that on a Sunday afternoon, has no right to suppose because it is crowded then, that it is as crowded on a week-day. A man who never looks at the moon but through a chink which only allows him to see it at its full height, cannot guess the path it pursues through the heavens for the rest of its time. If on the other hand we find that the values which y assumes for intermediate values of x, taken at random from between the values already considered, adapt themselves to the law derived from these latter, we have for the first time some justification for interpolating all the other y's in conformity with this law. Strict logic would not admit even this to be a complete justification; so long as it is impossible to observe *all* the successive values of *C* and all the corresponding effects *E*, so long we remain in doubt whether the law which holds good in the cases observed would hold in those not observed.

This doubt is narrowed in practice by collateral considerations or to sink to zero for particular values of C; lastly we shall know if they are likely to accumulate undiminished by the lapse of time, or whether we must assume that some counterforce is constantly annulling wholly or partially the results generated. It is these assumptions, which are grounded in fact, which justify our transferring the law for the pairs of values we have actually observed to values not observed, and of doing so with a strong probability of being right. There is still another expedient in cases, where there is no restriction to the number of possible expedients. By means of autographic arrangements attached to the apparatus in which the effects of the force are rendered visible, we can compel the force to register of itself the results which it produces at each moment of its continuous working. By help of such mechanical means our observations, which would otherwise be always limited in number, are so infinitely extended that they follow each other without any break, and the visible curve thus generated allows us to form as safe a judgment as can possibly be based on observation in regard to the continued or intermittent nature of the effect, the uniformity, retardation, or acceleration of its rate and its periodical or non-periodical increase. It is always of course open to those who are given to logical hyper-criticism to object that every curve drawn consists ultimately of a series of point-like deposits of pigment and that these only appear as a continuous line to the naked eye, which interpolates whether we like it or no. After all, we may be told, you have only got a number of particular perceptions and these do not allow you to infer the nature of effects, which found no pigment available to register themselves by and which therefore correspond to the gaps between the colored points, which make up the curve. Let us leave such objections to answer themselves; all I wish to do is to accentuate the truth that the discovery of an universal law is always a guess on the part of the imagination, made possible by a knowledge of facts. This knowledge is recalled to our memory by the resemblance of the given case to analogous earlier cases, and thus offers itself as an explanation. But a demonstrative method, or a method which involves no logical jumps, a sure logical receipt for arriving at the true universal law of a series of events, does not exist.

270. If we return to our series of values in order to see how far the problem in hand is successfully solved, we are confronted by numerous cases in which it emphatically is not. Among such are all those statistical calculations, which view a result E, which really depends on the co-operation of several conditions, by sole reference to

n ever make such an attempt unless, once more, he had exp
which taught him to put some trust in it. However much the
edure may lack precision from a theoretical point of view, he still
vs that as a matter of fact something comes of it, though not
what he wishes; and conversely it is the absence of all result in
r cases, which induces him to abstain from similar attempts.
t result we do usually arrive at in such cases is based on the fol-
ng considerations. Among the conditions on which the continu-
of a man's life depends, that which in estimating it we regard as
most important is beyond doubt the age *C*, which he has already
ned; for inseparably bound up with that age is a modification of
bodily system, which continues slowly to run its course and is
nately sufficient of itself, even though all other conditions remain
rable, to make death inevitable. During long periods of one's
however the action of *C* changes slowly and inconsiderably, while
ther sections of one's life it increases very quickly and significantly;
e it follows that the same outward conditions have an uniform
ence on the body during one period of life, and during another
qually uniform but uniformly different influence. It is upon this
action of the present stock of vitality and circumstances that a
's capacity of further life really depends, and so we may suppose
between certain fixed ages the expectation of life decreases
rding to one tolerably constant law, between certain other limited
according to a different but equally constant law; we cannot
ever conceive of an universal law which should determine the
ctation of life universally for the whole of life, and so for any
a man may have reached. In such investigations therefore partial
or formulas are usually laid down, which are only meant to hold

our calculations; if they are, to start with, of a complicated nature it is at best empty affectation to lay them down at all; in such cases it is more useful to go back to the original form of a table containing in its simplest form the mass of observed facts, from which they were derived.

271. But matters may be less unfavorable, and we may be able to reckon on the presence of a universal law capable of being expressed by the help of two centres of relation C and E. The question then arises, which we are to choose of the many laws that may with equal truth or with equal approximation to the truth be supposed to underlie the series of pairs of values presented us. In raising this question we make assumptions slightly different from those we have hitherto made. The numerical terms of our series will not represent the observable facts with such complete accuracy as we supposed before; they will contain inaccuracies, but we are for the present content to believe that these are small, and that they are not all on one side, but exceed the truth about as often as they fall short of it. Accepting these conditions a doubt arises whether the particular formula, which fits in most accurately with the given values, is to be regarded at all as the law we want. The pure case $B\ F$ will hardly fall within our observation quite unalloyed; the result which the condition B would by itself alone involve will be somewhat altered by the simultaneous co-operation of other causes which we can never wholly eliminate, and this matter of fact, impure already, will be still further modified for the worse by the slight flaws which are inseparable from our observation. Thus the data from which we start involve what we want along with disturbing elements which we do not want; a formula, which was exactly adjusted to those data, would be a copy of this mixed matter of fact, but not a law for the pure case, which we sought to separate from its alloy of accidental and irrelevant circumstances. This consideration forms the general ground upon which we permit ourselves if at all to neglect the slight divergences, which still remain between the given values and a law approximately covering them; we then put down these differences as due to *unknown* disturbing causes. Cases however may arise, in which a law completely answers to the given values and must yet be regarded as not the true one, or anyhow as less true than another, which answers less closely to them; this will be the case when there are *known* disturbing causes, which must necessarily act, but of which we find no hint given in the former law. Let us assume that two bodies a and b revolve together on different planes and at different distances round a third c, which steadily attracts both; and that it strictly follows from our observations, that the two bodies describe two similar regular ellipses: then either the observation itself must be pronounced defective, or the elliptical orbit cannot be regarded as the law of these movements in the desired sense. For if we only admit first the attraction between *c* and *a* and between *c* and *b* and admit of none between *a* and *b*, *a fortiori* however if we do admit there to be attraction between these two, the path, which *a* would describe, were *b* not present, must be modified when *b* is present together with it. Either therefore the real paths of the two bodies, when they are moving together, diverge from a true ellipse, in which case our observations are inaccurate and fail to reveal to us these slight divergences; or the ellipse is the actual path of either body, in which case the path prescribed by law is some other one, which they would traverse, except for these disturbances. For after all in such investigations as these our aim is not merely to get an universal expression or copy of the facts as they result from the application of an universal law to the definite conditions of a special case; what we do want is rather such a general statement of the law as will allow us, just because these special circumstances are excluded, to judge of the results which would follow, though the collateral circumstances were changed, from the same or similar main conditions. In such cases then as this we shall be inclined to doubt the truth of an assumed law, when it fits in with a faultless and all too striking exactness to the given observations. If it be asked what other law should be held to be a truer one, we answer that that can only be conjectured according as the disturbances we disregarded can be estimated on other grounds. The doubt raised in us however may induce us to combine our observations in a new way, or to institute experiments which may throw light on the matter.

272. In case there are several laws, which all come about equally near to fitting the data before us, we are accustomed conformably to the above to prefer the simpler and to see in simplicity as it were a guarantee of truth. Against this view, which raises the *simplex sigillum veri* into a universal principle, logic must enter a no less universal protest. If what we have to do is to calculate a special case by the light of a general law, the simpler formula is of course to be preferred, because it is *more convenient*; but from a more general point of view its simplicity is no test of its truth or probability. We must carefully consider what we may generally expect in the particular field, which we would explore. If it is clear that in that field a result E depends on divers independent determining elements, then a *simple law* expressive of their connexion is of course not impossible, but extremely unlikely. Properly the first feeling we should have on finding such a law would be one of distrust in its validity; we should believe we had taken things too easily in our observations or in our reasonings and had left out of sight some of the essential conditions; we shall only be satisfied if a searching investigation shows that these neglected conditions really always cancel one another's influence in such a way as to justify our excluding any reference to them in the universal law. Say we have found from mere observation that a body starting from the surface of a sphere under the attraction of the centre of the sphere always reaches a certain other concentric surface with the same final velocity, no matter along what path it passes from the one surface to the other; such a remarkable discovery as this we could only credit on one condition, namely if it were shown that this remarkable compensation of different collateral conditions really takes and must take place in the case.

We are easily deceived in similar cases, when the result found is not so paradoxical as the above. The formula $T = \pi \sqrt{\frac{l}{g}}$ seems to unite all the determining elements, upon which the time of a pendulum's swing depends, for a superficial observation does not give any effect to the angle of vibration. A more exact theory however shows that this simple expression is only approximative and that the true law is far more complicated. A certain speculative principle which we may come across later on leads us to suppose beforehand that in nature there exists a variety of compensatory arrangements, in virtue of which certain types of resulting events are maintained in perpetual conformity with the same simple law, no matter how different the medium, through which in particular cases these types are realised. Nevertheless one should only count upon the presence of such arrangements, where observation beyond doubt reveals them; on the other hand, where we have no means of thus forecasting the limits, within which the result of imperfectly known conditions must confine itself, the supposition of simple laws and the predilection for them remains an error only to be guarded against by a thorough exploration of all essential details of the given object of investigation. The present state of natural science does not perhaps make these warnings so necessary as they were a score of years ago, when there was a strong tendency to explain such complex phenomena as organic life by very simple, but no less inadequate principles. Of course it is very different er of independent causes, but rather as themselves manifesta- of those fundamental forces, whose constant action under all of secondary conditions makes up the complex tissue of physical sses. For these cases which do in fact realise approximately or letely the type of the presupposed pure case $B\ F$, we certainly regard the simplicity of the law as a sign of its probable validity; t for the somewhat aesthetic reason that simplicity is in all cases acteristic of truth, but because for these pure cases only one of imple forms of regular coherence—already noticed (263)— en cause and effect is in fact *conceivable*.

. The reader will have noticed how much importance attaches eady acquired knowledge for the discovery of new laws, and e even went so far as to appeal to all manner of previous con- ions and accessory thoughts, through which alone the imme- ata of sense come to have a precise meaning. The usual way ting the necessity we are thus under is to say that we need *eses*, in order to make use of the results of observation. In e may be inclined to apply this term hypothesis to several of odes of thought, of which we have already availed ourselves. we may say it was to make an hypothesis to infer back as we om a periodical increase and decrease of an effect while the constantly increases to a shifting of the relative positions of the elements associated in the cause. I think fit however in the ts of logic to define terms differently and to distinguish between *tes*, *hypotheses*, and *fictions*. The regressive inference just men- is a postulate, that is, it expresses the conditions which must up, or the ground of explanation which must be given by some whether thing, force, or event, before we can think the nenon in the form in which it is presented to us; it thus s or postulates the presence of something that can account for en effect. The postulate is not therefore an assumption which n indifferently make or leave alone, or discard for another;

really happening may be something altogether definite. What *is* left indefinite is the answer to be given to an essentially different question, namely, the question who or what that is, which by its concrete nature introduces exactly those conditions, which according to the postulate must needs be fulfilled in order that the given appearance may be possible. If a body of known mass moves in a known curvilinear path with a known velocity, we can assign with perfect accuracy the sum of the conditions, i. e. of the resultants B, B^1 ... which must act upon the body at every moment if it is to move in this way. All that remains indefinite is the source from which B and B^1 come, whether they are both of them simple impacts of simple forces or themselves the resultants of several joint forces, whether in short they are effects of forces or communications of already existing movements. It is clearly an abuse of language at once to apply the term hypothesis to all such demands of thought. If someone merely tells us that this curvilinear path requires forces of a certain intensity and direction to divert the motion from the tangent just so much in every moment, we should answer him in some such way as this: you teach us nothing which we did not know before, you merely repeat the conditions, which it was evident from a bare analysis of the given appearance must be supplied by *any* theory, which could be brought forward in explanation of the facts.

But we mean something else by an hypothesis; we mean by it a conjecture, which seeks to fill up the postulate thus abstractly stated by specifying the concrete causes, forces, or processes, out of which the given phenomenon really arose in this particular case, while in other cases may be the same postulate is to be satisfied by utterly different though equivalent combinations of forces or active elements. Thus we may fix at once two characteristics of the hypothesis. Firstly it is far from being an empty surmise, which comes into our heads without any reason at all: it always rests on a postulate, which we must accept, and is designed to explain the contradictions or lacunae, which make the given appearance *prima facie* unthinkable. These it explains by assuming a secret inner organisation of real things and processes, in which these contradictions vanish, while at the same time it becomes conceivable how and why the said contradictions unavoidably arise for us in the outward appearance, which alone we can observe. The second characteristic of an hypothesis is closely connected with the former: every hypothesis is meant to be an account of a fact, and is no mere figure of thought or means of envisaging the object. A person who sets up an hypothesis believes of his observation. In such a case there is no need for the facts thus divined to be simple and ultimate facts; they in their turn may give rise to researches going still further back into the grounds of their possibility; it is enough for the hypothesis if the facts it *supposes* can be conceived as really existing, though we reserve for another time the question how they come into existence. Students of Optics found (to put it briefly) that observed facts make it necessary to postulate that rays of light act in the same moment in a different manner on their right side and on their left, and that this action itself alternates incessantly with the time, and that therefore there must be some cause capable of bringing about just this phenomenon. The physical hypothesis was that this postulate would be satisfied by transverse vibrations on the part of atoms of ether. What may be the source of these transverse vibrations, which form so indispensable a preliminary in the explanation of the phenomena, remains a question for the future to solve; in any case however it involves no contradiction, which would prevent our conceiving it as a process which actually takes place.

We have still to explain what we mean by *fictions*. These are assumptions made by us with full consciousness of the impossibility of the thing assumed, whether it be because it is self-contradictory or because for other than intrinsic reasons it cannot pass muster as a constituent of reality. Fictions are of use when there is no proposition T, under which a given case M can be logically subsumed as a case of its application, whereas there is a proposition T^1, from the actual applications of which M is only distinguished by a definite difference d. We then class M under T^1, draw therefrom the conclusions we want and correct them later on by adding on such modifications δ, as are rendered necessary by the distinction d, which still remains. The finding of the circumference of the circle by inclusion of it between an outer and an inner polygon may be regarded as merely a method of limitation, unless we like to consider the conception of the length of a curve as in itself a sort of fiction; but any-of ds ultimately independent of it. It is hardly requisite to remark upon the extraordinary importance of such modes of procedure for the intellectual process of discovery; but we also encounter them in other branches of knowledge, and the lawyers' custom of turning to the most nearly allied maxim of law T^1, when there is no special rule under which a particular case may be brought, is from a logical point of view to be classed as a fiction, though we generally apply the name only to cases of a peculiar kind. Jurisprudence must of course be left to shape its own nomenclature; still I cannot believe that what used to be regarded as a fiction was not something more than a mere transfer determined by a fresh act of legislation of all a man's legal rights and obligations to a subject, who *per se* stood in no relation to these. I think it depended on something further, and in the case of the Roman adoption the assumption of the father's name who adopted seems to me to prove that, as a psychological fact, an attempt was made to begin with to regard a relationship which could not be established in reality as yet after all established, while the corresponding sum of rights and duties was determined as the result and on the basis of this fiction.

274. So important are the results which we expect from hypothesis that we cannot blame the attempt so often made to subject to some sort of discipline the free course of the discoverer's imagination from which alone hypotheses can flow. But we must observe that though most of the rules laid down are truly excellent so far as they can be carried out, yet we must not regard a particular hypothesis as illegitimate because it disregards them; if we do we seriously curtail the utility of hypothesis. Let us illustrate our meaning. It is alleged in the first place that the hypothesis must satisfy the postulate, on which it is based, not by a fictitious representation, but by assigning a reality, and that it should therefore make assumption only of what may be thought as fact, not of what is inherently self-contradictory. This is obvious enough; still we go too far, if we require that the content of an hypothesis should always carry with it the possibility of being directly refuted by subsequent observation. We may look on this requirement as constituting an ideal, and it certainly is a very useful rule to observe, for it teaches us, where we can, to construct our hypothesis in such a manner, that its falsity, if it be false, transpires at once instead of being for ever proof against direct refutation by reason of its content being wholly inaccessible to observation. Still we should have to sacrifice many useful assumptions, if we pressed this demand in all cases. The teaching that the points of light, so conspicuous in the heavens of a night, are bodies of vast size, only very remote from us, is at the bottom only an hypothesis, by means of which we try to understand the otherwise inexplicable daily and yearly motions of these lights. However false this assumption may be, it is clear that no future advance of science can ever directly refute it; we must therefore abide content if our hypotheses are thinkable and useful, if they are capable of explaining all interconnected appearances, even such as were still unknown when we constructed them, if that is to say they are indirectly confirmed by the agreement of all that can be deduced from them in thought with the actual progress of experience. But if we would be so fortunate as to find an hypothesis, which will not lack this subsequent confirmation, we must not simply assume anything that can be barely conceived as real; we must only assume that, which besides being thinkable conforms so to speak to the universal customs of reality, or to the special local customs which prevail in that department of phenomena to which the object we are investigating belongs. We do proceed so in all fields of enquiry. For instance, if in the text of a legal enactment a particular phrase only admits of an ambiguous deduction being made in regard to a given case, we do not interpret it in an arbitrary fashion by simply allowing our wits to play freely on it; we go back to the *ratio legis* on which the formula is based, and by the light of that seek to interpret the phrase in a manner suitable to the particular case. It is the same in the natural sciences; there too a successful hypothesis is always due to the attention paid to analogies noticeable in the material world at large or in particular departments thereof. Nothing but the analogy of fluids and of the atmosphere could have originally suggested the hypothesis of the continuous filling of space by matter; there was nothing in solid bodies to suggest the idea, for most of them are not only divisible into parts but are composed of a number of actual parts. In the case of such bodies the notion of the continuity of matter was only applicable in regard to their minute parts, and so it became a scientific truth that they consisted of discrete atoms, each of which could just fill its own small space continuously and no more. Now when it was found that solid bodies became fluid and fluid ones solid, and that even gases assume solid and liquid shapes under certain conditions, the atomistic theory was fully justified from that point of view; it only transferred what was actually true of one part of the body or of certain forms of it, to other bodies or other forms, in the case of which the same state could not as a matter of fact be demonstrated to be real, though it could be shown to be possible, inasmuch as upon this assumption the appearances presented by them remained perfectly conceivable. As soon as it is found that certain groups of phenomena are readily explained on the supposition that nature habitually acts in such and such a way, fresh discoveries are made every day, because people at once try how far other facts may be referred to the same principle. Such was the case with the undulatory theory. On the surface of water, in strings, on resonant surfaces, waves could be directly seen and their shape rendered visible in particular cases by artificial means; there was no apparent reason for supposing these movements to be confined to certain materials and there was accordingly much to be said in favour of the hypothesis, which sought to explain on the same principle first the propagation of sound by the air, next the movement of the luminiferous ether, and lastly the phenomena of heat.

Similarly, in the organic world, people stumbled at a few points on a division of labour of which they had never dreamed; where before very different functions had been attributed to the same substratum, each of these functions was shown to have a special organ of its own, which did not do service for any other function. It was then suggested hypothetically that the same thing went on in regard to the nerves organic to the different sensations of colour and sound; whether the truth of the matter has been reached is still open to doubt, but from the point of view of logic there can be no doubt that the hypothesis is justified. Again, movements are often observed in plants, even contractile movements; still it does not appear that these are due to the contractions of living contractile tissues as are the movements of an animal's body; consequently, plausible as this hypothesis is in itself, it is not advanced in this case, because *prima facie* it does not seem to agree with the habits of nature in this region; on the other hand it is worth while enquiring whether this semblance is not a fallacious one.

275. There is yet another condition which a hypothesis must fulfil. It should be exactly adjusted to the postulate which it is framed to meet, and not contain either more or less than it must contain if it is to answer to the demands of the postulate. Hence a rule which must be carefully observed in constructing hypotheses. When we have to account for something which happens, we must not look vaguely about as if for inspiration; we must before anything else rigorously analyse what is given, and so lay down the exact postulate which the hypothesis must satisfy. When we have done so we may neglect for

which are not themselves mere consequences of other ele-
ust be accurately observed, for it is entirely from the way in
ey are conjoined that we have to conjecture the most suitable
the hypothesis we shall choose. We must then make a
our world, to see if it contains any elements, causes, forces,
nations of forces of such a kind as to satisfy the postulate
n; after the fullest survey of these and guided at once by a
and theoretical motive we shall make choice of those which
specified requirements in the simplest manner and in the
mplete accord with the ruling analogies of the particular
nt in question.

ample, a body is found covered with wounds; our first con-
be to settle whether the wounds must have been inflicted
man was still alive or after he was dead; we then try to
the magnitude, mode, and direction of the forces, which
ve caused the wounds. Having thus ascertained the con-
ve found a postulate on them, and enquire whether this
is satisfied by assuming a mere natural force to have acted
y presupposing a weapon to have been wilfully employed.
uiry may be said to settle the *form* of the hypothesis, e.g. we
to make the assumption that murder was committed, after
proceed to detect the agent not by the help of ill-founded
ut by asking ourselves what persons there are of whom the
ht be expected, partly because their relations to the murdered
ld have supplied a motive, partly because there is nothing in
racters to prevent our suspecting them without direct evi-
We have no space to give in all the necessary detail an
which would illustrate the extreme care which in judicial
ion is taken to satisfy every part of the postulate; a con-
unded on it is not regarded as a safe one, unless it accounts
single circumstance which because it is a violation of th

netism, and this without specifying the circumstances which need to be explained. They talk in a vague and general way of this fluid being emitted and immitted, forgetting that such barren generalities are perfectly useless as explanations of the kind, quantity, and sequence of the phenomena for which they are supposed to account. Natural science is not so liable to go wrong in this way because it must state its problems with so much mathematical precision even to render them intelligible.

276. I shall presently have to speak of individual facts, in treating of which the important point is not so much the simplicity of the hypothesis framed with a view to their comprehension as the completeness with which the hypothesis covers all that is contained in the facts. Experience teaches us in how many roundabout ways an event is sometimes brought about in a particular case, whereas in other cases it may arise from several simpler causes. But at present we are not concerned with individual facts; we are still trying to discover the matter-of-fact which is the common basis of a whole class of frequently recurring events; and here in deference to a sort of principle of 'the least cause' we must prefer the simpler hypothesis to the more complicated: not because simplicity in itself is any guarantee of truth, but because if we go out of our way to assume any datum whatever, which is not indispensable in order to the production of the thing, we make an utterly void supposition overstepping the given postulate and therefore unjustifiable in point of method. But our procedure may be logically correct without being endorsed by reality. Suppose we have selected our hypothesis and are trying to deduce from it the original appearance, we may find that our deductions do not agree with the data, either because our analysis of the latter was defective, or because fresh observations, which were impossible before, have brought out new aspects of the thing. In that case the hypothesis must needs be amended. This may be done in two ways. The hypothesis contains elements, which in themselves admit of being modified, and we determine these in a more suitable manner, so that as grounds for the deduction of the given fact they are no longer either too wide or too narrow but just adequate. The other way is to add on fresh subsidiary hypotheses in regard to a few theory either in everyday life or in science. We do not pull a house down and build it up anew just to get rid of a flaw, which a slight modification of its construction would remedy; we do not at once devise a brand new constitution when a few provisions of the old one begin to be oppressive; and widespread as is unfortunately the tendency to ride principles to death, the opportune adjustment of necessary changes to what is permanently good in old institutions has always been considered the true art of statesmanship. And if we look at the way in which the body of science has grown up historically we see that it too is very willing to essay new points of view under old and incommodious forms, if only not to lose any of the truths which have once been won through those forms. I do not mean to say that science should or will rest content with such methods; we all trust that the result of all our painful investigations may prove a simple and thoroughly consistent whole; but until we have arrived at that result we must not be deterred by the oddly complex and patchwork garb, in which our views must needs be clothed, if we are careful to adjust them to each freshly known or better known feature of our object by means of subsidiary hypotheses tacked on to our earlier assumptions in regard to it. This is the only way in which we can hope to reach the simple and plain result we seek. The more carefully we now proceed the more surely may we expect that in the course of our procedure (just as in any intricate calculation, which must yield a simple result in a foreseen manner) our manifold assumptions will spontaneously reduce themselves to simpler and more universal ones, so that in spite of all the circuitous reasonings employed a net result will remain with us, which is not only simple and synoptical, but completely covers every part of our postulate. In conclusion all will admit that a lucky gift of insight may make us able to do without all these roundabout methods; but logic cannot impart inspiration; the only method it can teach is what we have cited:—we must curb our impatience and steadily go on transforming a hypothesis once manner reflect the large outlines of a thing, but are quite inadequate to account for its particular features.

277. A nice point remains to be noticed. Nothing can seem more imperatively necessary than that a hypothesis, which is meant as a conjecture of something which really is or happens, must before all things allege nothing but what is in itself *possible*: and of course it must assume nothing which is ascertained to be impossible: but still there is a doubt as to where the possibility which is still admissible begins and ends. I have tried to solve the doubt by carefully choosing my words; I have said that the hypothesis may legitimately involve anything that can be mentally represented as given matter-of-fact, but nothing else, and I really believe not only that this is all we should require but that we may admit so much as this to be possible without coming into conflict with the idea of the hypothesis. The hypothesis intends to conjecture a fact, but it is also content that this fact should when conjectured just exist in the way in which facts really observed so often exist: viz. that while we can conceive it or picture it we cannot explain the manner in which it may possibly come to be. Nothing can warrant our assuming by way of hypothesis a circle, which is at the same time a triangle; it is beyond the constructive power of our fancy to frame a mental image of such a figure, nor could it ever present itself to our senses as a given matter of fact. On the other hand we may assume the existence of invisibly small yet extended atoms of unchangeable shape and size; there is no contradiction in the notion of them, which would prevent our conceiving them as possible objects of perception, were our senses rendered more acute by artificial means. There is no reason why we should not look on the existence of such atoms as real, and suppose that though they are inaccessible to our unaided senses they are yet the basis of the phenomena which we can observe. We may probably have to modify this idea, when we try to think it out and examine its possibility as an element in the system of nature; but still there is no need to do so, till we have availed ourselves of it as a preliminary principle and found it of permanent use in accounting for particular phenomena. In the same way the theory of transverse undulations of a luminiferous ether answers, we saw, the requirements laid down by a postulate of observation, and such undulations can no doubt be conceived as really taking place, though no light has yet been thrown on their physical origin. The entire supposition of an infinitely extended homogeneous or isotropic ether is indispensable so far as we can see to our theory of the propagation of light, but it belongs to the same class of ideas;

we are rather like men who are tubbing a well with masonry; like them we build from above downwards and so are we obliged to assume a substructure of hypothetical facts, which we trust will be sufficiently firmly upheld for a time by the unanalysed ground at the bottom to support our superstructure, until we can carry our knowledge a step deeper down and replace the hypothetical basis of our knowledge by a basis of facts, and then go through the whole process over again. It must be admitted that at this rate we leave a doubt as to where hypotheses and fictions, laws and rules respectively begin and end; I have hinted at this idea before and I shall recur to it again.

CHAPTER IX.

Determination of Individual Facts.

278. We cannot be certain about a matter of fact unless we have ourselves directly perceived it; and even then only on the supposition that our interpretation of the sensible impression, which is all that is originally given, is correct. We interpret this by combining it in the form of a judgment into a whole of interconnected parts. When our information comes to us through others, we can only be sure that our information is trustworthy when we can rely on the witnesses or reporters. There may be much to recommend and justify the confidence we repose in them, but nothing can ever demonstrate its necessity. Again we habitually argue back from given facts to facts not given, but only attested by the former as their causes. Every such inference is liable to be wrong, because although every consequent must have a ground and a single ground adequate to produce it, still there may have been several different but equivalent matters of fact, all equally entitled to be called the cause—because each of them involved the ground—of the given effect. Again we frequently argue forwards from observed circumstances or events to a future or contemporaneous fact, which however withdraws itself from our observation. There is an uncertainty about all such inferences, because every condition may in the actual course of nature meet with a counter condition, which though it never annuls the consequences of the other, yet hinders them from actually assuming the particular form in which, except for that hindrance, they would have manifested themselves. It follows that wherever anything is outside the range of immediate perception, we are in our judgments of reality limited to probabilities, and have to look about for means by which to raise these probabilities as nearly up to the level of certainty as is sufficient for our purpose.

279. In thinking about such matters we are swayed by two very general and somewhat antagonistic principles. In the first place there is no such thing as a train of events, causally related and belonging to the contrary, every such series of events goes on in one and the same world at one and the same time with numberless others. It always therefore seems utterly unlikely that any cause should unfold without a hitch the whole endless series of effects, which would have flowed from it if it could have acted alone upon the component parts of the world. A conviction that such is the case colours our daily life and conversation; it finds expression in the old warning not to moor one's ship by a single anchor, nor one's life by a single hope. If we are anxious to bring about a particular result we take a variety of precautions, each of which will effect what we want; if one miscarries, another will reach it; if they all come to nothing through the operation of external disturbing causes, we shall be able to console ourselves with the conviction that such a conspiracy of chance as would prevent a single one of the many causes on which we relied from producing the desired effect was quite as improbable as that they should one and all have succeeded. In the same way we distrust a historian who deduces mighty revolutions from mere trifles, or the doctrinaire who because some tendency really had a decisive influence in an earlier epoch, pretends to see in all the details of the history of centuries just its reactions and no more. The former loses sight of the innumerable collateral conditions, in virtue of which alone so trifling an event could even seem to be fraught with such vast consequences; nor do the reasonings of the other carry conviction with them; mankind is a collection of many heads, which for ever teem with unconnected and incalculable impulses. We cannot believe that these have been wholly without influence in determining the course of history, especially when conjoined with the influences of nature, which follow an arbitrary order or disorder of their own. We are æsthetically dissatisfied with any poetry which sets before us a human character which is unswervingly self-consistent in all its actions, great and small; such a character lacks the air of being a genuine creation of reality, because no trifling irrationalities of behaviour are ascribed to it, no venial but wayward likes or dislikes: such a mere personification of an abstract quality is wearisome in fiction, while in life, if such a man could live, he would be so repulsive that we should hardly feel towards so impersonal a being the moral obligations, which are only intelligible between persons. No less incredible would a story be in which all the endeavours and resolves of a thoughtful man were brought to nought by a constant recurrence of adverse accidents. Were such a work meant seriously it would shock us, and we could only endure it as a bit of comedy which awoke in us the soothing reflexion that the whole sphere of action was an insignificant one, as well as a happy disbelief in the reality of what was being tricked out before us as a possibility. Even music seems, not untrue indeed, but insipid and unmeaning, if the flow of its melody can be too easily discerned beforehand. It must not make the simple forward movement which answers to its initial strain; it must reveal its living elasticity by the suddenness of the turns which it seems constrained to make by obstacles, which encounter and thwart it. Lastly we distrust any practical project which instead of co-ordinating side by side, paratactically, to use a phrase of syntax, independent conditions of success, lets them depend hypotactically on a web of mutually conditioning presuppositions. Such schemes only provoke ill-success; for in multiplying the parts of the structure we only multiply the points of contact with hostile influences, and by making one depend on another perpetuate the effects of a check once received.

280. The second of the two principles mentioned is suggested by the fact, that although we can imagine several different groups of equivalent causes agreeing in the attribute of producing one particular effect, still each of these groups will have in addition to the common effect other and peculiar accessory effects of its own which will differentiate it from the rest. Now what we look upon as a single matter of fact is very often a complex whole composed of manifold effects all gathered into one. The different combinations of causes then adequate to produce just this complex effect will be very few, so much so that may-be only one of the many combinations we are accustomed to meet with in our experience will be really adequate. So long therefore as a given matter-of-fact is only known to us in its large outlines we are accustomed to suppose very various causes in order to account for it; as soon however as the finer side traits which characterise it come to be known, our choice of causes narrows itself considerably, till at last we find that there are very few facts, of which we can make hypothesis, which will satisfy all the requirements of the postulate founded on these data. Among these facts we then select that one in particular which at once is the simplest and presupposes the least number of mutually independent and co-operating elements. Nor is the above principle a ruling thought in science only; it governs the most various considerations: a whole chain say of simple facts is set before us in evidence, which taken in connexion with each other may be conveniently explained if we assume that a particular deed was committed; from such a hypothesis we can, we

ribe each link in this chain of incriminating evidence to a
cause compatible with his innocence, and will try to explain
conjunction of them all as due to a mere unfortunate coin-
; but the persons trying him will turn their attention ex-
to the assumption which explains them all in their connexion
h other, and are hardly likely to listen to his forced pleadings.
the same way a patient often consoles himself by referring
the several symptoms of his malady to a trifling cause of its
ut he does not for all that deceive the physician, who by his
s pitilessly exposes the serious complaint, which at once
the concurrence of all these accidents conceivable.

dly need add that these obvious principles of judgment
fice to recommend one preliminary conjecture in preference
ner; where we have important issues to decide we must
rget that what is improbable is still possible. It is not enough
merely to follow out to its conclusions the particular as-
which the evidence before us forces upon us as the most
For it even to come near to deserving belief, it is not
hat all the evidence should of itself converge in favour of it;
have carefully tested the less likely suppositions which the
the matter admits of, and have found that they leave just as
unae and contradictions in the facts to be explained as does
er. Besides this we must take care as far as possible to argue
positive evidence; negative evidence is ambiguous: whether
the omission of an action, or the absence of a state, it can
used to prove a matter-of-fact when what it denies may be
as being necessary under any other presupposition. All that
om a denial of anything is just the denial in turn of what we
hink without virtually affirming the thing denied. Lastly in

gradually disclose themselves; no doubt they agree with our conjecture, but we cannot use them to strengthen it. In conclusion, the observance of all these rules, of whose application it would be much more interesting to give examples than to formulate them in this dry logical manner, is compatible with much error; still we must not underrate their real value on that account. One practical maxim we may draw from the consideration of all these imperfections: where we must act, whether we would or no, and where we can never rise in our calculations to the level of certainty, there we may confidently trust to probabilities; where on the other hand we are not obliged to act at all, or at any rate not obliged to do anything extreme and irrevocable, the proper course is not to regard our personal convictions, which rest on mere probabilities, as sufficient warrant for carrying out our belief in action.

281. Where we have matters-of-fact given us, with whose inner coherence we are in a measure acquainted, and would estimate more accurately the probabilities based on such coherence, we trench on a field which spreads beyond the scope of the general precepts of logic, and in it we must rely only upon our actual knowledge of the particular case. In regard to future events, however,—and I shall consider no others in what follows—we often find ourselves differently placed. Of a number of mutually exclusive alternatives we may know that one or the other must happen; but not know of any ground for preferring one to the rest; nevertheless practical needs may force us to make choice of one, and to base our actions on the supposition that it will happen. Under such circumstances we can only regard all equally possible cases as equally probable in reality. There is no other rule by which we can be guided in our judgments. Now we disclaim all knowledge of the circumstances which condition the real issue, so that when we talk of equally possible cases we can only mean those particular cases which are co-ordinated as equivalent species in the compass of an universal case; that is to say, if we enumerate the special forms, which the genus can assume, we get a disjunctive judgment of the form: if the condition B* is fulfilled, one of the kinds f^1, f^2, f^3 . . of the universal consequent F* will occur to the exclusion of the rest. Which of all these different consequents will in fact occur, depends in all cases on the special form b^1, or b^2, or b^3 . . . , in which that universal condition is fulfilled. If we knew this particular form of B, say b^3, we should be able to deduce for

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* ['Bedingung' and 'Folge,' the initial letters of the English words not being convenient symbols.]

For our present purpose, however, we suppose that we are
of the special shape which B will assume if it does really
t follows of course that, if B be realised, some one or other
nsequents f^1, f^2, f^3 . . . must follow; but from our point of
v all remain equally possible, inasmuch as the only condition,
we know, of their being any of them realised, is the validity
general, and that holds equally good for all and favours none
ular. Let us assume for the present that the universal con-
can, if it assumes all the variations compatible with its
roduce n, say six different consequents $f^1, f^2 \ldots f^6$, then the
ondition B must be realised in n, i. e. six different ways, for
the equally possible and mutually exclusive consequents to
o realise itself. Thus we see that, assuming what is equally
to be equally real, the chance that a particular case will occur
f being mathematically determined; for each of these f has
share in the prospect of being realised in a particular case,
others which are equally entitled with it to be real; but the
ll these probabilities must be a constant quantity independent
number, for it must denote the certainty that some one or
of the particular consequents f, however many they be,—that
that F generally—must occur in each individual case so soon
eneral condition B is realised in any one of its forms. This
is equally absolute for every B and every F, and only in re-
t do the respective chances of the several cases admit of being
ively determined; consequently there is no reason for, or ad-
n assuming the constant in question to have any other value
y; the chance of any one particular case of the n co-ordinated
us becomes $=\frac{1}{n}$, and the sum of the n chances $=\frac{n\,.\,1}{n}=1$,
r words 1 is the exponent of certainty. Thus far I have used
ssion 'co-ordinated cases' without explaining it; I do so
der to prevent misunderstanding: a co-ordinated case is a case

merely the general expression. For example we may if we choose give the disjunctive judgment the form: if B holds good, then either f^1 or F^m holds good, by F^m being understood all the m or $n-1$ consequents f, which are not f^1; in such a statement f^1 and F^m are not co-ordinate terms; the chances of f^1 indeed remains $\frac{1}{n}$, but the chance of F^m is the sum of the chances of all the elementary cases which in thought we unite under this formula, and so it $= \frac{n-1}{n}$.

Now it often happens that we are led to institute an enquiry by the interest which attaches for us to some property which the different cases comprised under F^m have in common, and for that reason we separate them from the rest and denote them by a common name as *one* case, to which we oppose the rest. If we would then formulate the probability of this collective case F^m, we may say it is equal to the proportion, which the number of elementary cases combined in it bears to the aggregate sum of all possible cases; or we may state it more accurately, taking account of the connexion of the whole matter, thus: the probability of F^m is equal to the ratio, in which the number of the variations of B, which may issue in a case of the kind F^m, stands to the entire sum of all possible variations of B; in a simpler and more general form still: the probability of F^m is equal to the ratio which the number of chances favourable to it bears to the sum of all thinkable chances, $= \frac{m}{n}$. This fraction is what we understand by the mathematical probability of a future event, and is not at the bottom essentially different from, but only more accurately determined than the probability of common parlance. For usually we say vaguely a thing is probable without specifying the degree of probability which attaches to it; of two events that one is pronounced absolutely probable, whose mathematical probability is the greatest or at any rate usually if wrongly regarded as greatest, the other event only appearing improbable in comparison with it. In treating of chances mathematically we do not ordinarily talk of a thing being improbable, but if we did we could only mean that which is relatively less probable.

282. From small beginnings, which seemed at first to be useless except to satisfy scientific curiosity, the calculus of chances has developed in the hands of the greatest mathematicians into an extensive body of doctrine, bearing fruit in the most diverse fields of scientific

r. In this form it has outgrown the limits of this treatise, ugh every detail of it would always be more entitled to a ı a system of logic than those useless syllogistic subtleties, ı deference to our extravagant love of classical literature we be always repeating, still I am forced to confine myself to meration of the simple logical thoughts, which are merely ary to calculations into which we cannot at present go any But in doing so I am conscious that a gap is left, and must ıt that this gap needs to be filled, though I do not attempt to rself.

the first place we must make it clear to ourselves what we y the probability, which we have just learned to measure in plest cases mathematically. It does not imply any positive ı on our part touching the real future occurrence of the event, ı we attribute it; it does not express any objective property re belonging to the event, but denotes throughout what is subjective, viz. the degree of confidence, which we may bly accord to the future occurrence of a particular case, when we have given us to go upon in forming our judgment is the of cases possible under the particular given conditions and actual ground carrying with it the necessity of one of them exclusion of the rest. Let, in accordance with § 281, the ity that a particular side of the die will face upwards after the $^1/_6$, the probability that one of the five other sides will fall $= {}^5/_6$; then all that these two numbers signify is this, that e throw the trust we may reasonably repose in the occurrence rst case must stand to our trust in that of the second in the 5; they contain no positive prediction that the one or the l occur, or that on repeating the throw the one will occur more ly than the other. We postpone the question, how far such ence from the calculated probability to the real event is each in virtue of the constitution of its condition B or B^1. If two dice are thrown, the side which the one shows uppermost has nothing to do with the side which the other shows uppermost; but each die has 6 sides, each of which may fall uppermost, and each of these may with equal possibility coincide with any one of the six sides of the other; there are thus 36 possible cases and the probability of each single one of them is $1/_{36} = 1/_6 \cdot 1/_6$. If however we look upon it as making no difference, which of the two similar dice shows the one and which the other of two different numbers of points, the probability of any two in particular concurring $= 2 \cdot 1/_{36} = 1/_{18}$; for if we throw but one die or the die B there is of course but a single chance of any particular side falling uppermost, but if we throw two dice, that is in case of the combination $B+B^1$, there are always two chances in favour of any two differently marked sides falling uppermost together. On the other hand the probability that two similarly marked faces will fall uppermost together must still remain $= 1/_{36}$, for there is only one combination which can produce a particular doublet. Lastly, if our object be to throw both dice together and get a particular number of points between the two, the sum 7 has most probability $= 1/_6 = 6/_{36}$, for it has 6 favourable chances in the combinations $6+1$, $5+2$, $3+4$, each of which occurs twice; the smallest probability, viz. $1/_{36}$ attaches to the sums 2 and 12, each of which can only be produced in one way.

Again, suppose we put in an urn B 17 black and 3 white balls, in a second urn B^1 6 black and 4 white balls, and then ask what chance there is of drawing two white balls, one from each urn; it is evident that in this case as in the last what the one hand grasps is quite independent of what the other hand has grasped; but the probability of drawing a white ball out of the first urn is $m = 3$ favourable chances out of 20, the probability of drawing a white from the second urn is $m^1 = 4$ out of 10. Now there are 10 balls in B^1, and we may draw a white ball from B with any one of them; also there are four whites among these ten; consequently the chance of one of these four being drawn along with whatever we draw from the other urn would be $4/_{10}$; but as the chance of our drawing a white ball from that other urn was only $3/_{20}$, the chance of our drawing two whites together one from each urn will $= \frac{m}{n} \cdot \frac{m^1}{n^1} = 3/_{20} \cdot 4/_{10} = 3/_{50}$. We should get another result if we gathered all the balls into one vessel and drew twice out of it, taking care however to restore the ball first drawn before we drew a second. The result of the second draw would then as in the above case be independent of that of the first; for each draw the probability of a white ball being drawn would $= {}^7/_{30}$, so that the probability of two whites being drawn in succession would $= {}^7/_{30} \cdot {}^7/_{30} = {}^{49}/_{900}$, that is to say would be less than in the first case. The difference of the two results may seem strangely great, as without calculating it one would hardly suppose there was any essential difference between the two modes of proceeding; there is however, inasmuch as it is harder or easier to draw one of the white balls just according as there are more or fewer black balls mixed with them. The chance, ${}^7/_{30}$, of drawing a white out of the whole collection of balls amounts no doubt to ${}^{14}/_9$ of the chance, viz. ${}^3/_{20}$ that there is of drawing a white out of the urn which contains 20 balls; for the same reason however it amounts to only ${}^7/_{12}$ of the other chance ${}^4/_{10}$, which is the chance of drawing another white from the other urn, which contains 10 in all. Consequently the chance of drawing two whites by the second method is only ${}^{14}/_9 \cdot {}^7/_{12}$ or ${}^{49}/_{54}$ of the chance of obtaining the same result by the first method; we have in fact ${}^{49}/_{54} \cdot {}^6/_{100} = {}^{49}/_{900}$. It is better to be quite clear on this point, so I will take a still simpler example. Let us assume that the urn B contains but one white and no black balls, while B^1 contains one white and one black; then if we draw from B we are certain of one white ball, whose probability therefore $= 1$; and we may draw either a white or a black from B^1, with either of which it may concur; thus the chance of either of these cases, one of which consists in two whites following one another is ${}^1/_2 = 1 \cdot {}^1/_2$. Such is the result got by the first method, that of dividing the balls in separate urns. By the second method however, which consists in putting them all in one urn, we are certain of nothing; for the first as for the second draw the chance of a white is the same $= {}^2/_3$, and that of two whites in succession 15 $= {}^4/_9$, that is to say smaller than it is upon the first method.

3. Suppose the variations of a condition B produce a series of cases of the kind f, but the actual occurrence of one of these cases modifies the condition B^1 which leads to consequences of the kind f^1; the chance that a particular case of the series f will coincide with a particular case of the series f^1 is equal to the product of the independent chance of f into that of f as modified by the occurrence of the former. We get such a case by slightly modifying the last example. If we put back the first ball drawn into the urn, which contains 30 balls, we leave the second draw independent of the first; but if we do not restore it, the urn will only contain 6 white out of 29 balls; the chance of still drawing a white becomes ${}^6/_{29}$ and that of

alighting on two in succession $= {}^{7}/_{30} \cdot {}^{6}/_{29}$, and is only about 0·88 of the chance which there was of drawing two successive whites, when the ball first drawn was put back into the urn. This was to be expected, as the number of white balls is now proportionately less than that of the black, among which they must be sought. Under this head fall many of the problems to which the calculus of probability may be applied, and great care must be taken to discriminate them from the former class. We very often have to do with events, whose chance of recurring in the future depends on the number of cases, in which on previous occasions either they themselves or others standing in a definite relation to them, have been realised; and it is not always easy by analysing this interdependence to ascertain the influence, which the occurrence of one case exercises in conditoning the probability of the one to be next expected.

I have no space to illustrate this by examples, but shall give an instance of a different sort. An eye-witness imparts something he has seen to someone, who in turn imparts the information to a third person. Now we know from experience that the further news travels in this manner the more distorted it becomes, and accordingly it has been proposed to ascertain what degree of trust may be reasonably reposed in a statement in proportion to the number of people concerned in its transmission to us. I do not believe that any amount of calculation will really help us to answer the question. To begin with, it is not quite plain what we are driving at. An allegation is either right or wrong; but if wrong it deviates more or less from the truth; and we might assign to it a greater or less degree of credibility according as it deviates more or less, supposing it to be possible to measure against one another the different amounts of these deviations. But this we shall seldom be able to do; each term of a judgment, expressing an original observation, can be taken apart and falsified in a way peculiar to itself and when falsified can be variously combined with other terms; the aggregate of errors thus arising cannot be regarded as constituting a series of terms, which we can compare together, and we should thus have no available standard by which to estimate the objective credibility of the statement as handed down. But after all this is not what we really want; we want to ascertain the particular degree of trust which may be based on our knowledge of a single condition, which we have stated, viz. of the number of times a bit of intelligence has been handed on from person to person before it reaches us. But here the objection at once occurs that this condition of transmission does not in itself contain anything that could at all justify us in predicting a gradual falsification of the statement transmitted. When, as in the above example, we have drawn a white ball and removed it from the vessel, which has in it 30 balls, 7 of them white, we know that the conditions of a fresh draw are changed and we know exactly by how much; on the other hand if we restore the ball we are equally certain that the conditions are the same as before, that the second draw is a *res integra* so to speak and its chances the same as those of the first. It is to the latter not to the former case that the problem now before us corresponds; the mere fact of transmission, taken in itself, cannot cause me to transmit something else than I have heard; so far as the mere transmission goes there would be not a mere probability but an actual certainty that the last hearer will accurately receive the original statement. Thus the falsification of a statement depends not on the number of times it has been passed on but on the size and sort of errors made in it each time it has been passed on; consequently our knowledge of the number of times it has passed from mouth to mouth will only help us to estimate its trustworthiness, if the size of the various errors be either constant or a regular function of that number. There is not the least ground for assuming any such thing; we see it to be quite the reverse if we really reflect on the very various cases which may occur. The eye-witness A may or may not have wished to communicate aright what he has rightly observed; his hearer B has or has not understood him aright, or he may have understood him and yet desire to hand it on himself in a distorted form; a third person C, who intended to distort afresh what he already misunderstood, may chance to hit upon the actual truth in what he communicates. If we consider all these possible conditions we see clearly that the trustworthiness of a communication in no way depends in any regular manner merely on the number of times it has passed from mouth to mouth. We disregard these conditions because we are ignorant of them; but if we had the power of knowing them all the question would answer itself and we should not need to calculate it at all. All we can do in the matter therefore is to make utterly arbitrary assumptions in regard to all these conditions, which would be tantamount to bringing the whole thing down to the level of arithmetical examples, which had no bearing on or application to real events. Such would be the following sort of calculation: say that we hear and in turn report anything with such accuracy as to deduct one tenth of its credibility; then after the 20th repetition of the statement its credibility would only be $0{\cdot}9^{20} = 0{\cdot}1216$, only a little more than $^1/_7$ of what it was originally. Here all is arbitrary assumption;

it is arbitrary to assume that the credibility diminishes in geometrical progression, instead of arithmetical; the latter is quite as conceivable. No less arbitrary is it to suppose at all, that the exponent or difference of term from term must be equal; the result too which we thus reach has no meaning; it might perhaps be true of frivolous street-gossip, but as regards serious historical traditions it is a gross exaggeration of the rate at which their untrustworthiness increases.

4. Given certain facts we have to conjecture their true causes; we must calculate the probability with which the given effects would follow from the various possible causes, and select that cause as the true one on presupposition of which the facts would most naturally follow. I draw four times in succession from a bag, and draw from it 3 white and 1 black ball, restoring the ball each time, and the question is asked what number of balls of each sort must the bag most probably have contained in order to give this result. In order to answer the question we must know the whole number of balls in the bag, in order that we may be able to state the number of conceivable combinations capable of causing the given result. Suppose there were 4 altogether. Now to account for the result at all there must be at least 1 white and 1 black; how many more of each kind remains indefinite; there are 3 possible combinations, which we can assume: 3 whites + 1 black, 2 whites + 2 black, 1 white + 3 black. For these 3 combinations the chances of drawing a white are respectively $^3/_4$, $^2/_4$, $^1/_4$, of drawing a black $^1/_4$, $^2/_4$, $^3/_4$; the joint chances however of drawing in 4 successive draws 3 whites and 1 black become on these various assumptions $^{27}/_{256}$, $^{16}/_{256}$, $^3/_{256}$; consequently the first assumption, that 3 whites and 1 black ball were in the bag, is the most likely; at the same time the fractions got give the particular probability which each of the other two possesses. A very simple consideration confirms this solution. Had the bag had in it but a single white, according to the third hypothesis, we must in 4 draws have grasped it 3 times, while we only once grasped a black ball out of three which offered themselves,—a supposition obviously less probable than four draws in which each ball has his turn. It may be noticed that this calculation of course presupposes that the different causes, which we can assume in order to account for the given facts, in themselves possess equal probability; this was the case here so far as any distribution of the two colours among the 4 balls was in itself quite as possible as any other; where the probabilities of the causes are not equal, due account must be taken of the same in our calculations.

5. When we see the same result repeat itself under the same

general condition *B* we are led to expect it to occur again if *B* recurs. The chance of its really doing so admits of being calculated. A bag has in it two balls and it is found that so often as we draw we always get balls of one colour, say white, so that the colour of the other ball remains unknown to us. Hence we expect to get a white upon drawing a third time, supposing ourselves to have drawn twice. How shall we measure the probability of our expectation? One ball must be white, so that there are only two possibilities, either the other is black or both are white. Now two whites have already been drawn in two draws, and the probability that this which has happened would happen becomes on the first assumption $^1/_4$, on the second $= 1$; consequently the odds in favour of the rival assumptions stand to each other as $1:4$ and as their sum must be $= 1$, the first must be put $= {}^1/_5$, the other $= {}^4/_5$. In case we make a third draw the odds in favour of the white are $^1/_2$ on the first assumption and 1 on the second; the sum of the favourable chances presented by both assumptions taken together is thus $^1/_5 \cdot {}^1/_2 + {}^4/_5 \cdot 1 = {}^9/_{10}$. In this case the actual event had occurred and we only knew and calculated the *a priori* chance which it had of occurring under two rival presuppositions as to its conditions; but even where we have not this knowledge, we may draw an inference as to the chance that an event will recur from the number of times we have actually observed it occur. Suppose we are quite ignorant of its conditions and grounds and only know that an event *E* has once occurred under certain conditions, say at some critical moment of time *t*, it may at first sight seem as if the chance of its occurring a second time under the same conditions was exactly as great as its chance of not occurring at all. But this is a miscalculation; were it true the observed fact of its having once occurred would be ignored in our calculation, and as the same reflexion might be fairly made after the event had occurred for the m^{th} time, we might find ourselves in the absurd position of maintaining that the fact of an event having occurred even an infinite number of times did not make its occurrence next time any more likely than it would have been, if it had never yet occurred at all. This however would be evidently paradoxical; for every fresh repetition of an event is a fresh and additional testimony to the continuance of the unknown causes on which it depends, and so strengthens the probability of its occurring again. Our conclusion therefore in regard to the first case must be this; that *E* will not occur is in itself just as likely as that it will; but for the existence of causes which bring about *E* we have the testimony of this one observed case of its occurrence; for the existence of causes which prevent *E* we have nothing but the bare possibility. We consequently have two reasons for expecting *E* to recur, where we have only one for expecting it not to, as the two chances stand to one another in the ratio 2 : 1, while their sum must = 1, the chance that *E* will recur = $^2/_3$. In general therefore, if an event *E* or a particular cyclical course of like events *E* has been observed *m* times without any exception, the probability that *E* will happen again in the same way is $= \frac{m + 1}{m + 2}$; in this fraction the denominator represents the sum of conceivable cases, since after *m* real cases have occurred there are always two additional cases, which we can think of as occurring, viz. the repetition or non-repetition of *E*; the numerator as usual denotes the number of favourable chances. I think this simple deduction of the formula will satisfy the reader; it is to me as convincing as the more obscure analysis, by which it is usually obtained. One sees that as *m* increases the fraction approaches nearer and nearer to unity, and so it becomes more and more nearly certain that *E* will recur. The example usually adduced is that as the alternation of day with night has been now historically attested for 5000 years, the probability of the same alternation recurring to-day = 1,826,214 : 1,826,215; that is one may bet 1,826,214 to 1 on its occurring again. Now if it is true of calculations of probability in general that they do not express what will actually occur in the future, but only the degree of subjective confidence, which we repose in their occurrence; it is in a certain sense doubly true of these cases, as we clearly feel when *m* is a very small number. For then the assumption from which we start is that the number *m* of cases in which *E* has been observed to occur testifies with a certainty proportional to the magnitude of *m* to the *continuance* on the next occasion of the causes favourable to *E*: and this assumption is itself but a probability, the strength of which is somewhat arbitrary, and of which we only know that it increases with the increase of *m*. Properly therefore the formula would not directly measure the probability that *E* will recur, but the probability of this probability, which comes to this, that not only the value to be assigned to the probability, but also our confidence in this value approaches nearer and nearer to certainty as *m* indefinitely increases.

6. A future event may be fraught with good or evil for us, and it is usually a sense of these consequences to ourselves which impels us to gauge the strength of our confidence in its happening. We shall shape our motives and actions according to its strength, and these will therefore depend doubly on the likelihood of the event *E* and

nits of being precisely determined. Let a game be so arranged t the player gets two thalers if on the first toss he throws heads and : thalers if he throws first tails then heads. The probability of the mer case is $= {}^1/_2$, its expectation $= {}^1/_2 \cdot 2$; the probability of the ond case is $= {}^1/_4$, its expectation $= {}^1/_4 \cdot 5$; lastly the aggregate expecon of winnings when play begins can only be the sum ${}^9/_4$ of these) expectations ; for though the two lucky cases exclude one another :ording to the arrangement, yet the expectation of winnings must arly be greater when both the two prizes are offered than when y one, and the expectation of the one must be exactly left over, if expectation of the other is reduced to nothing by the gradual ninution of the prize assigned to it. The same reasoning would oly, if it were agreed that the player should receive two thalers if he ew heads the first time, and then another five in case tails followed. e two winning cases are then compatible with each other, but here all that can be won is either two or seven thalers, and the chance of nning either is ${}^1/_4$. In this case then as in the former ${}^9/_4$ of a thaler resents the aggregate expectation of the player and the utmost he reasonably stake upon it. Suppose again that of different events E^1, E^2, which we may expect, some are fraught with evil others with od consequences to us; in that case it is easy to see that the aggree expectation which we may entertain, if by our own actions we are ing to risk their happening, must equal the difference between the n of the mathematical expectations of the favourable events and the n of the expectations of the unfavourable ones. If this difference a negative quantity it expresses the magnitude of the risk we run more correctly speaking, the magnitude of the anxiety we should . This principle is wide and important in its applications; by ins of it we are not only able to determine what bets and games of nce are fair and equitable—a sort of calculation we could as well without as we can do without its object—but it also assists us in nging the most serious public and private business, such as the nagement of finances, the undertakings of trade and the organisajoy to the miserable or a trifling gift to the poor is of greater value than a fresh triumph to the fortunate or splendid winnings to the rich. No doubt as a matter of fact one that has much is wont to desire so much the more, but in this respect logic takes the point of view of equity, according to which it ought not to be so: in assuming as a self-evident principle that the *relative* value of an advantage bears an inverse ratio to the advantages of the position to which it is added, it expresses the standard according to which a man seems to be justified in desiring to improve his condition, when the good things available for this improvement have also to satisfy the wants of others. This general law does not admit of being mathematically applied, unless all the advantages of a situation and all the good things requisite in order to better it admit of being mathematically compared; it is therefore chiefly of use in regard to the increase of a capital, which can be expressed in money. Let V be a capital which we have already got, and z the addition which it is to receive: then this increase of V may always be regarded as the sum of an infinite number of smaller increments each of the size dz; the relative value however of each subsequent $(n + 1)^{\text{th}}$ augmentation by a dz is in inverse proportion to the size of V as already enhanced by the preceding increments, that is to $V + n\,dz$, and would thus $= \dfrac{k\,.\,dz}{V + n\,dz}$. In this formula k is a specific coefficient, which differs with the different sorts of cumulative advantage, but is constant for all z of the same kind and does not admit of further determination in the abstract; and as it forms a common factor in all values which we can compare we omit it in what follows. The relative value of the aggregate increase by z is then the integral of this expression, in which we must replace $n\,dz$ by values of z ranging from 0 up to z: it is therefore

$$= \log (V + z) - \log V.$$

In accordance with this formula we should find that for a capital $V = 1000$ the relative values of the increase when $z = 1000$, $= 2000$, $= 3000$, $= 4000$, are approximately 1, $1\,.\,6$, 2, $2\,.\,3$: that is to say, they grow very much more slowly than do the increments themselves. For the different capitals $V = 1000$, $= 2000$, $= 3000$, $= 4000$, the relative values of the increase, when each is augmented by $z = 1000$, are approximately 0·301, 0·176, 0·125, 0·097. When we have thus calculated the relative values of the advantages which some event will bring us, we may multiply them by the chance there is of their ever being attained which we will call m: by this means we get $m \log \left(\dfrac{V+z}{V}\right)$

for our *moral expectation* of them, i. e. the mathematical expectation of these advantages reduced to their relative values; and this is what in all sorts of enterprises determines the amount of risk we may prudently incur in view of some prospective advantage. We have assumed the factor m to be constant however high z is; it may be so, but it may also be a function of z or of $V + z$, in which case of course it is to be included under the integral sign and brought into the integration. In point of fact there are many sorts of undertakings in which while the first success is hard to win, subsequent successes become easier and easier, or in which the possibility of further success diminishes with the increase of what has been won. Lastly the formulae do not help us to measure all that one may wish to measure. By treating z only as the sum of dz, without taking account of the time t which it takes to achieve the summation, they neglect the distinction between gradual and sudden improvements. The real, actual, or physical values of the two *may* be the same, but their psychical effect, or to put it simply the pleasure they occasion, may be different, and this after all enters as a factor into the idea of the comparative value of an advantage. Let us assume first that the extent to which a particular satisfaction admits of being further enhanced $= \frac{1}{V}$, if V represents the degree of satisfaction of the same sort to which one has already attained; and secondly that the increase in the satisfaction generated remains proportional to the size z of the sudden increase in the advantage; then $\frac{z}{V}$ will measure the pleasure due to the accession of z. But it is easy to see that these are not the only conceivable assumptions: it might even possibly be found that the eventual enjoyment is also a function of m, i. e. of the chance that z will occur: we might perhaps be more deeply affected on winning a satisfaction, of which we had almost despaired, than on winning one of even greater comparative value that was more probable.

283. The last observation just touched upon problems which have not as yet been brought within the range of calculation, though there is nothing to prevent their being so brought, if an advance in psychological knowledge should ever afford us starting-points from which to grapple with them. Other problems there are to which it is but an idle play of words to try to apply the calculus of chances. For although this method of inference does start from our ignorance of the special grounds, which condition a particular event, still it makes certain presuppositions, which we cannot neglect. In the first place it presupposes the truths logical and mathematical, of which we must make use in order to be able to calculate at all. The truth of special laws, limited in their action to a group of facts, the non-existence of which is just as conceivable as is their existence, may, as we shall presently see, be proved by means of calculation; but from what basis could one legitimately start to show the law of identity or the doctrine of disjunctive judgments to be more or less probable? The very simplest determination of any probability presupposes a disjunction of all possible cases to be given, that each of these cases is identical with itself and not the same as any other, and that each of them is exclusive of the rest. It follows that before we set about to prove an event, or a state, or a series of events to be probable, we must have presupposed the particular content in question to be part and parcel of a world, in which universal laws demarcate what is true from what is untrue, what is possible from what is impossible, what may easily occur from what may not.

But the calculus of probabilities is subject to other limitations besides these. The object, which its problems concern, must be regarded by it as not merely thinkable in the abstract; it must also presuppose the presence of conditions, which necessitate the realisation of one of the disjoined cases to the exclusion of the rest; to use the language of its formulae there must always be a certainty $= 1$, which is the sum of all the probabilities of the particular cases, which we can think of. This was noticeable in our examples throughout. *If* a die has been thrown or *if* two have been, we can determine the respective chance of any of the particular cases which may result; but unless we specify how many dice are to fall and how many times in succession, it is quite impossible to determine the scope of the disjoined possibilities and the unity by reference to which the chances of each are to be severally measured. It follows that we can only calculate such events as depend on one another within a regularly ordered world; ultimate facts, which contain an independent absolute being of their own, we cannot calculate. It would be mere senseless play of wit to reason thus: prior to all existence there is the same chance of the existence of something as of nothing; but one or the other must take place; therefore the chance of something existing is $= {}^1/_2$: but this something would necessarily be either one or many; consequently the chance of there being many elements is ${}^1/_4$, and the chance of there being one is ${}^1/_4$: lastly assuming that there are n elements, they may be all alike, or some different, or all different; the case of these being all alike would be but one of the m

cases which would thus arise, and consequently its probability is $= \frac{1}{4m}$. Prior to all existence, we must observe, there can exist no ingenious spirit to institute such a calculation of what will be; could we conceive however of such a spirit as existing outside the world and speculating as to whether it is likely to come into existence or no, still that nothing would involve no condition of any sort, which would necessitate a real settlement of the alternative presented in thought between being and not being, so that the end of the whole matter would still be nothing. But suppose the alternative to have been somehow settled in favour of being, this being could not possibly be something, which were merely thinkable *in abstracto*; it must be capable of existing and can only be some determinate being, which excludes all other thinkable being. Such determinate being would from the very beginning have a certainty of its own $= 1$, while the probability of all other kinds of being would be not exactly $= 0$, so much as an idea without any assignable meaning. It would be different if we wished to determine the probability of these ultimate facts from given data: on the assumption that all reality is bound together by law, these given data would (not as the ground of their reality, but as the source of our knowledge of them) constitute a condition which would compel us to assume the one or other form of those ultimate facts to the exclusion of the rest.

284. There is one other point we shall do well to bear in mind. In probability we have after all no more than a measure of the confidence we may legitimately place in the occurrence of an event, *before* it has occurred. *After* it has occurred however what was previously its greater or less probability does not continue to attach to it as a permanent property, from which we may regressively draw some other conclusion in regard to the causes of its being realised than just this that they have actually come about. We are victims of all sorts of illusions on this point. For example an event E occurs, which on previous calculation was very unlikely to occur as compared with a whole class of cases, which for the convenience of thought we gather into one and collectively oppose to it as a second rival case non-E; in such a case we are apt to imagine that not only a special and peculiar but a *higher* cause was necessary in order to bring about E. To take a common instance: some object is of little significance, almost unknown and seldom spoken of. We stumble on its name once, after which it meets our eye again and again in conversation, in books, in periodicals; here is a coincidence, the chance of which, if calculated beforehand, was infinitely small and we call it a very strange incident. But a moment's reflexion will convince us that there is very little strangeness about it; how infinitely more numerous are the cases in which the incident does not turn up than those in which it does! How many names just catch our ear once without ever being repeated in such a way that their repetition strikes us as odd! To put what I mean in a perfectly general way, let us suppose there to exist some condition B or some group B of different but co-operant conditions; these according to the different and in the abstract equally possible variable attitudes, which they can assume towards each other, would bring about a number n of different results E; the probability of each individual E is then $= \frac{1}{n}$ and by consequence the same as that of every other *determinate* E, but, if n be infinitely great, infinitely small as compared with the probability that *any* one indifferently of all the remaining $n-1$ events will happen, which we collectively oppose to it. But the latter, the collective probability has another significance than the former—the individual, for all the $n-1$ events cannot be realised, but only one of them to the exclusion of the rest.

A famous instance will illustrate the extent to which we may be led astray by such false comparison of things, which are essentially different. The planetary system, according to Laplace, and so far as he knew it, consists of 11 planets and 18 satellites; we are acquainted with the revolutions of the sun, of 10 of the planets, of the moons of Jupiter, of Saturn's ring and of one of his satellites; the rotations of these bodies together with their revolutions form a group of 43 movements in the same direction; would we assume that this uniformity is all a matter of chance, we find on calculating it that the probability of such an assumption being true is something less than unity divided by four billions. I have no doubt that fresh advances in astronomical science will leave this number substantially correct, but what follows? Simply that the particular cause or grouping of causes adequate to produce this state of things is or has been real. It does not follow that this grouping of causes itself requires any other cause than just that so-called chance, by which we mean no more than that the mutual relations of several realities, which we presume to form a group, may without contradiction be combined in an infinite number of ways. Out of all these four billions of possible cases never more than a single one can be realised, and no matter which of them it be, we shall feel just the same surprise at its happening in particular as we should have at any other of the four billions, had *it* happened. The case would be utterly different, if all those other dispositions of events really formed a single second case, capable of being realised as such; in that case its chance, denoted by four billions, would have at least admitted of being directly compared with the probability of the other case, i. e. with unity, although even then the sort of conclusion would not have been justified but only rendered more attractive. No doubt a plausible attempt may be made to justify the reduction of the whole number of cases to a single pair of alternatives; nothing, it may be said, but this given disposition of all masses and motions could secure the stability of the planetary system and the continuance of its movements; no one of the million other arrangements would have served to produce this state of equilibrium. We concede all this, but might it not be also pleaded in favour of each of the other arrangements, that it too had in store for the planetary system a particular destiny of its own, that on presupposition of it alone among the many millions of possible presuppositions could that peculiar destiny be fulfilled, so that if the uniqueness of the result constitutes a claim to a higher origin each of these rival dispositions may with equal right prefer such a claim? It would seem then as if it was not the unique result as such, but the unique result which was better than its fellows, which finally prevailed. But why should superiority in itself constitute improbability? And after all *would* this case which ultimately prevailed be better than others? No doubt as things are our blood is ever fresh and new, yet at the expense of for ever circulating in the same forms; but is it really and without qualification a finer thing that it should circulate as it does than that it should not? The perpetual repetition of these forms may no doubt appear very grand to us to-day, to-morrow may it not strike us as rather tedious? Would it not be a finer thing if the planetary system were not in such stable equilibrium, if all its relations were for ever changing, so that vegetation and natural beauty, animals and man should develope in ever new and interesting forms and history be really the history of a progress, of a manifest advance, instead of the chronicle of a cycle of ever recurring events? And to conclude, inasmuch as the heavens are infinite, may not all the millions of differently ordered systems be actually realised therein? With us the system of equilibrium, at unknown distances the rest? And then surely our own system would only possess such reality as its probability entitles us to claim for it, it would be but one among millions.

285. Thus far we viewed the calculation of chances as a mere means of ascertaining accurately the confidence we may repose in the occurrence of future events, there now arises a natural desire to know how far these previous calculations are as a rule confirmed by the actual course of events. The answer usually made is that the more numerous the cases, which make an event *F* possible, the more closely does the number of times it actually occurs tend to coincide with the number calculated. We can only get an answer that is at all trustworthy by means of experiments of the simplest kind, in which care is taken to restore after each m^{th} experiment the group *B* of conditions, upon which each particular case *F* depends, in such a way as to leave the composition of *B* exactly the same after as before that experiment, differenced only by the variations, whose influence on the net result of the series of experiments it is the very object of our enquiry to ascertain; taking care at the same time to prevent the entrance of any alien cause not implied in the idea of such variation, whether that cause consists in external circumstances or in a change of the object of the experiment or in unfair intervention, on the part of the person experimenting. These conditions are fulfilled in experiments made with dice. We calculate beforehand that, if two dice are thrown once, the chance of getting a particular combination of points, e.g. $5 \cdot 6 = {}^1/_{18} = 0{\cdot}056$, which for a thousand throws would be 56; if now we make trial of these 1000 throws one after another, and find, as in fact has been found to be the case, that the specified combination occurs 50 times, we see that this number already approximates pretty clearly to the number calculated; yet more when in 10,000 throws it rises to 570. Each single throw in such a case depends, if we leave out of account the uniform or changing resistance of the air, on the following conditions: on the velocity with which and the angle at which the die impinges on the receiving plate, upon the position of its sides and corners at the moment of impact, upon its own elasticity and on that of the plate. We may regard the last of these conditions as constant, for as we should expect from a calculation of its probability, the die will extremely seldom touch the same point of the plate, so that the elasticity of the point of impact will not change to any appreciable extent, if it was the same to begin with for all points of the plate. If, however, we would still regard it as variable, it may be included, just as well as the slight and gradual changes in the shape and elasticity of the die, among the variations of the conditions, the effect of which is being investigated; for since the two changes do not depend on each other, but may co-operate, they do not when taken together favour one particular throw more than another, but favour now one now another indifferently. The first-named condition, the velocity and direction of the die, depends of course on the movement of the hand that casts it; but were one even disposed to favour a particular throw by this means, one could hardly do so effectually; for *after* we have obtained a particular throw, we neither retain a clear recollection of the group of muscular feelings, which accompanied it, nor are we able to reproduce the exact movements, on which those feelings depended, so as to exactly copy the throw; and the least deviation would have the effect of favouring some other combination of points than what we wished to throw. These very changes therefore of our movements are among the legitimate variations of the conditions of the result we are investigating.

The same advantages are presented by a rotating drum, into which m white and p black balls are introduced and into which each time we have drawn we re-introduce the ball drawn before drawing again. If we then turn the drum we do not of course restore exactly the same position to the balls, which they had before we drew, but still we only produce one of the variations of this position, with the influence of which we wish to become acquainted. If we distinguish the whole condition B, upon which the event F in each separate case depends, into a constant and a variable element, we may say that in the first case the shape of the die forms the constant element as does the number of black and white balls in the second, while the variable element consists in the first case of the velocity and direction of the die, in the second case of the relative positions of the balls and the direction given to the hand in drawing. If we actually make the second experiment the result obtained is similar; the greater the number of draws the more nearly does the ratio between the numbers of the white and black balls drawn approach to the ratio between the numbers m and p, in which they were present in the drum.

286. Theoretical considerations have been based on the results of these experiments, which I cannot persuade myself are correct. A vicious circle is involved in all attempts to show that the results mentioned occur always with an *intelligible necessity.* In the first place one cannot argue from m series of experiments, in which we have really obtained it, to its being obtained in every $(m+1)^{\text{th}}$ series, so long as the unknown variations of the conditions, which have there produced the said result and would produce it here, are *individually* subject to no rule whatever. For the idea that they will at least *on the whole* continue to compensate themselves in the same way here as there—and it is only on this condition that the attempted universalisation of the observed results would be permissible—this idea, I say, has no *objective* validity, nor is it to be deduced from anything we already know to be real; it is indeed simply a way of expressing our subjective and almost tautological maxim, that that is most likely to occur in reality, which previous calculation shows to be most likely. Provided, that is to say, no uniform cause gives the preference to one of the possible cases of the kind F over another, we must ascribe the same chance of being realised to all cases, which in their idea are co-ordinate or equally possible; in which case all that is meant by saying that some particular fact or event is to be expected as most probable is just this, that in a great number of experiments the *actual* number of times a case F occurs is equal to the *calculated* number. If this expectation is verified in m series of experiments, that has *actually* occurred m times, which *before* the occurrence was the most probable: but it does not therefore become a demonstrable necessity that it should again occur in every $(m+1)^{\text{th}}$ series of experiments; that indeed remains the most likely thing we can expect, when we are brought face to face with this new series of experiments, but it may always turn out to have been a wrong expectation.

In the second place no single series of experiments can really comprise an infinite number of experiments; it must always stop short at some finite number, however large it be. Thus it can never be a real fact of observation, that the number of *actual* realisations of F approximates *without limit* to the *calculated* number as n increases: it is always an inference from the facts. Now assuming that in n experiments we reach a point at which the two numbers coincide or that they have so nearly coincided that their difference need not be considered, it would be a very arbitrary procedure to break off the series just at this point. It is obvious that the law of such equality or approximation comes true if we continue the series till it comes true and *no longer*. But what if we prolong the series? Possibly the theoretical and the real results will converge still more; possibly again each additional round of n experiments will have the same or nearly the same result as the first had, and the difference d will not be sensibly diminished by prolonging the series: and to these possibilities we may add any other less regular succession as *also* possible. Only these different suppositions have not the same amount of probability; so long as we consistently avoid presupposing any constant cause, which in a series of experiments to be made might give one case F a preponderance over others, our most probable assumption is merely, that as n goes on increasing the number of observed realisations of *F* will continually approximate to the number of them calculated beforehand. If in a large though limited number *n* of experiments this expectation is *not* verified, a constant or uniform condition *may* be chargeable with the result, though it *may* also be due to the combination on no principle of variable conditions. As often however as our expectations are verified by what really occurs, we are presented with a *fact*, at which we cannot feel any surprise just because it was beforehand not improbable, but still a fact of which we can just as little prove that it was necessary as we can prove of the verification of any mathematical chance that it was necessary. In the experiment with the drum and balls an uniform proportion gradually revealed itself between the numbers of the differently coloured balls drawn; I cannot believe that this uniformity is really *explicable*, if that means anything more than probable. The distinction made between the constant and the variable or accidental causes which jointly produce an effect is a very true and significant one, but this is not in my opinion the place to appeal to it. It is argued that however irregular the successive arrangements of the balls may be, there still remains one constant element, namely the unchanging proportion of white to black balls; this in a great number of draws *must* make itself felt by producing some constant effect: for there is no reason to suppose that just where the hand alights balls of one colour will be found oftener than in proportion to their relative number: if such were found to be the case, we must, it is argued, break with our supposition and assume some constant collateral cause which favours that colour. Against this view it may be objected that the constant causes spoken of could not make themselves felt by merely being there, but only so far as they act. In the experiments with dice the shape of the die and the position of its centre of gravity were such constant causes and both took effect in each single case. In virtue of the former the die could only fall on six sides and not on a seventh, in virtue of the latter it could not help falling on one of its sides instead of coming to rest on one of its angles or corners; on which of its sides it should fall, however, was just what these constant causes did not determine. It is the same with the experiments with balls in a drum; two of the conditions are constant; firstly the colours are only white and black, so that no blue or red ball can be drawn; secondly their numbers *m* and *p* are constant, though the relative numbers of the few balls which come within reach of the hand each time it draws are none the less to be classed as variable elements in the condition; hence it follows that this constant condition, the ratio $m:p$, does not take effect, though it actually exists. I do not therefore see the necessity of assuming—what contradicts our presupposition—a constant collateral cause to account for the apparent anomaly of a different proportion of balls being drawn to what is in the urn. On the contrary all that is needed to produce such a result is that the positions of the balls should be changed on no principle, and such irregular change is just what we presuppose and try to bring about by turning the drum. Such change of their positions renders possible every and any combination of the balls, makes it possible even that all the balls of one colour should be missed and even that this exclusion of one colour should be repeated over and over again in successive experiments, everything being designedly so arranged that each $(m+1)^{\text{th}}$ experiment is entirely independent of the m^{th}. All that can be said against this is that it is the reverse of probable; all that is probable is that the number of times a ball of a particular colour is grasped will tally with the number of balls of that colour which there are in the urn. But this too is no more than a probability; if it be nearly verified by experience we have got a fact, not inexplicable, inasmuch as we see quite well how easily the causes which contribute to that result may come together, but not explicable in the sense that one could demonstrate both that and how they *must* thus coincide in the long run of cases, whereas, when the cases are few in number, there is no must about it.

287. In the foregoing examples we were cognisant of the nature of the constant causes as well as of the extent to which those which were not constant might vary; hence we could in anticipation of experience make assumptions as to how often an event they conditioned would occur, and find our assumptions verified by experience. We now turn to events of which we know neither the constant nor the variable causes, but which we observe to occur over and over again. What conclusions can we draw from the regularity with which they occur? Here we know neither how many are the cases, which are barely possible, nor how many chances in favour of the event in question there may be among them. The only distinction we make is between the occurrence or non-occurrence of E, regarding as cases, in which its occurrence is possible, all those which realise the particular centres of relation, which being given make E intelligible, and comparing with the numbers of these the number of cases in which it is realised. The constant and variable causes on which blindness depends are hidden from us; but the number of cases in which this defect may owing to those causes occur is equal to the number of the population. If we compare with the entire number of persons belonging to one generation the total number of blind people among them and conceive of this comparison being extended over several generations, we can quite see how it would transpire, whether such a constant ratio is on the whole to be found between the two numbers, as would point to the presence of a group of constant causes favourable to blindness in the mass of men, but in the individual modified in their effects by variable causes. But this is not all; in most cases it will probably need a considerable length of time for the variable causes to realise themselves by turns in such completeness that they cancel each other's influence and allow the constant ones to assert themselves. Hence it is usual to try and discover units of time, in which the ratio of the actual to the possible cases of *E* becomes the same, due regard being had of course to the periodical changes to which the number of the latter are liable. Now the year happens to be the particular unit of time during which most of the variable conditions, which affect men at all generally, run through the cycle of their possible values, and so the first question to ask in investigations referring to human affairs is naturally this: within these units of time does the ratio of the actual to the conceivable cases of *E* remain uniform or approach to uniformity? The answer to all these questions may just as well be negative as affirmative. If an event *E* occurs at all frequently during a certain period of time, there must be within that period some constant cause of it, at least in the sense that some ratio exists, which to a definite extent promotes the combination of variable cause which favours *E*. Then as often as an unit of time recurs and the same proportion of real cases to possible ones is reproduced, so often are we warranted in the regressive inference, that that constant cause has existed. But it is not at once clear how we can argue forwards, that for the next equal interval of time the same proportion will hold good as a predetermined law. Such an assumption can only be regarded as the safest rule to go by in judging of the future, when no data are known pointing to the intervention of some change in those unknown conditions. If the rule holds good as foreseen we are justified in once more making the same regressive inference as before, and sure enough the oftener we can make it, the oftener that is the rule is confirmed by the facts, so much the stronger becomes the probability, that the group of conditions, which has remained constant over so many units of time, will remain unchanged for the future. To more than this probability however we can never attain, and so it is very unsafe to characterise the results of such observations as *laws* of what happens, or actually to speak, as we sometimes do, of laws of big numbers, as if the mere bigness of a number of compared cases must of necessity introduce a regularity in the course of a certain class of events, which has no independent foundation in the nature of those events and their conditions. A law, as we have seen, is an hypothetical judgment and enunciates the necessary validity of a consequent provided the antecedent be valid. Statistical laws must not aspire to satisfy this definition or they certainly lose their value; for they say no more than this: *if* in the next unit of time T *all* known and unknown conditions be as in the last, then will the series of all the consequences, consequently also the sum total of E, be the same. Of course it will, for if we suppose the past to take place over again, it will wear just the same aspect as it wore then. Those who talk about statistical laws no doubt do not mean to be guilty of tautology; on the contrary they mean to state their antecedent clause categorically, that is, to assert that such an identity of all conditions will take place; it is obvious however that such an assertion can never be certain, but only probable. Such propositions therefore are not laws, but *analogies*, which extend a proportion, which has held good in n cases, to the $(n + 1)^{\text{th}}$, not proving but only assuming that between n and $n + 1$ there is no change in the conditions on which their validity depends.

288. Among events, which in their frequent recurrence depend at once on constant and on variable conditions, may be properly classed our own *observations*, by which I mean the simplest kind of observation, viz. the measurement of a quantity given in an act of perception. Here the constant cause consists in the true value of this quantity, for this under entirely similar conditions would always have the same effect on our susceptibility. The variable causes are the external circumstances and the changes in our psychical state, which modify that effect in different ways on different occasions of its repetition. To elicit from the different measurements thus obtained, the true value of the thing measured, would be impossible if we ascribed to the measurements made every conceivable degree of inaccuracy: that would mean that we thought we might substitute for the values found any others we liked as more correct, and that would be stultifying the very idea of a measurement. We therefore presuppose that knowledge, aptitude, and attention have combined to make the measurements fairly trustworthy and only leave a chance of error very small compared with the magnitudes themselves which have to be measured. Now suppose we wish to determine some single unknown quantity A, at first every isolated measurement we have of it must pass muster as a true determination of A; for even if we had doubts of its accuracy we do not know how far or in what direction to rectify it and have no grounds to go upon. On the other hand though the quantity A can only be one and the same, observation may give us different values for it; we have then no absolute ground for trusting one value more than another, and as we must now suppose all our observations to be more or less erroneous, we are most probably right in fixing its true quantity at a value, whose assumption involves the least sum of errors in the measured values. The arithmetical mean M, i.e. the sum of all the measured values divided by the number of measurements, is thus to be regarded as the most probable value of A. The difference between this mean M and the true value A is the residuary error, which, so long as we have no other accessory conditions by which to determine A, cannot be got rid of, but only reduced by multiplying the number of equally careful observations.

On the other hand if we have repeatedly measured different quantities ABC, and are still furnished with other conditions, which the values thus obtained must satisfy, it may be that the different arithmetical means, which would individually give the most probable values of ABC, will not collectively satisfy these accessory conditions, and so stand in need of rectification. For example, we may have repeatedly measured the three angles of a triangle and found that the sum of the mean values thus obtained amount to $180^\circ + d^\circ$; this d° being incompatible with the nature of a triangle will point to an error in the result, which must have arisen out of errors in the measurements and can only be got rid of by altering the values found. But the reduction required may be distributed in very various ways among the three measured angles; and the question arises, what sized error may be most probably ascribed to the measurement of each angle. This suggests an enquiry based on principles, which if not demonstrable *a priori* are at any rate very probable and in harmony with experience,—an enquiry into the relative probability of the occurrence of errors in our observations generally. In the idea of a careful observation as such there is nothing to imply error at all. The chance therefore of our having hit on the truth is always greater than the chance of our having fallen into any particular error. Similarly it is involved in the presuppositions, on which the eliciting of true values from observations always depends, that the chance of large errors is less than that of small ones, and the chance of positive errors exactly equal to the chance of negative errors of the same size. This suggests one way of picturing the problem. Let a straight line be chosen as the axis of abscissae, take in it any starting-point to correspond to a total absence of error, and from this point let there be divided off in opposite directions abscissae $\pm\alpha$, $\pm\beta$, $\pm\gamma$, in ascending order of magnitude. At the point of no error or zero-point draw to the line an ordinate of any length, symbolical of the chance of there being no error: this will be the longest of all the ordinates, and all the others, drawn at the points $\pm\alpha$, $\pm\beta$ . . will diminish in length symmetrically on both sides of it according as the errors symbolised by $\alpha\beta\gamma$, the respective chances of which they denote, increase in size. But experience at the same time teaches us that the chance of errors does not simply decrease in the same proportion as their size increases. So long as the errors are trifling the chance of them decreases less rapidly than their size increases, but the greater they are the faster does their decrease in probability outstrip their increase in size. Hence the line joining the upper extremities of all the ordinates cannot be formed of two straight lines, meeting over the zero-point and symmetrically approaching the axis of abscissae on both sides, in such a way as to form a triangle with a segment of the axis. On the contrary the line in question is a curve, the vertex of which lies above the zero-point and which branches out therefrom in two symmetrical limbs, which are concave towards the axis of abscissae. The course of the curve is thus easily followed in the neighbourhood of its vertex, it is not so easy to follow as it approaches the axis. We may regard errors of any size as possible, errors of even infinite size; these too will have their degree of probability, infinitesimal though it be; in consequence each limb of the curve must ultimately become convex toward the axis of abscissae and approach it asymptotically. But we need not take into account such infinitely large errors; we may consider that in careful observation, errors which transgress by the whole amount of the value to be measured, do not occur at all; the curve will then remain concave and cut the axis at two points.

I cannot here go into the lengthy investigations which have been instituted with a view to determine more exactly the most probable form of these curves, their equation and from that the chance that an individual error will occur. Still I should like to give my reader some idea of the means to this end ultimately employed in such speculations, though I will not follow them more closely than my purpose requires. I will at once drop out of sight the tracts of the curve which approach the axis of abscissae; we are only concerned to ascertain the chance of such errors as we must expect to fall into even in careful observations, and accordingly we shall only consider a short arc of the line, which lies on either side of the apex. We have seen that this line cannot be a straight line; the next simplest thing to assume is that its equation is of the second degree. The symmetrical values of the ordinates on this side and that of the zero-point are possible on such an assumption, accordingly we make it and choose from among the sections of the cone, which however all lend themselves to the experiment,—the circle. Let the longest ordinate r, drawn at the zero-point of the abscissae, denote at once the true value of the quantity to be measured and the magnitude of the probability that this true measurement has been obtained in our observations. Let the abscissae $\pm\alpha$, $\pm\beta$, $\pm\gamma$ denote the size of the errors by which the different measurements diverge from the true value r; for the present we think of these as expressed in parts of this true value, so that $\pm\alpha\pm\beta\ldots$ are for $r=1$ proper fractions of unity, while if we take $r=r$ they must be replaced by $\pm r\alpha$, $\pm r\beta\ldots$; finally the ordinate y which corresponds to each abscissa denotes the chance there is of that particular error in measurement being made, which deviates from the true r by the size of this abscissa: if then we assume that the equation of the circle holds for the curve in question, y is $= r\sqrt{1-x^2}$, where x is the general expression for the changing values α, β, γ. Now we saw that the chance that different and mutually independent events will concur is measured by the product of their respective chances. Bearing this in mind we shall see that where, as here, we are directly compelled to assume a number of errors in our measurements, because these do not harmonise with an ulterior condition, and where, moreover, various combinations of errors may be assumed, which would all satisfy this condition, that combination of errors is the most likely and should be assumed, which allows to the product of the individual chances of error the highest value. Now this product consists of nothing but factors of the form $r\sqrt{1-x^2}$ and it clearly reaches the highest value, when all the several factors take on at the same time the highest values compatible with the conditions of the problem. This happens, when in all factors at once the subtractive elements—here the sum $\alpha^2+\beta^2+\gamma^2$—are reduced to a minimum. This minimum value presupposes, as one easily finds out, that the sum of the errors $\alpha+\beta+\gamma\ldots=0$; and this can only be the case, when these first powers of the errors have different signs, and must be the case when the arithmetical mean of the observations to which they belong, is taken to be the true value r of the quantity to be measured. We thus find that this obvious and in simple cases satisfactory principle falls within the lines of the process of determining r by means of the sum of the squares of the errors. Now suppose we have made m observations of a quantity and have derived different arithmetical means from them, by crediting them each with this or that error and correcting it accordingly; our method seeks to determine that particular mean which comes nearest to the truth, inasmuch as it rests on the most probable combination of such corrections.

In the above we have not tried to exhaust the subject, we have only just approached it in such a way as to give a general idea of what is meant by this method of the *least squares* and of how it came to be called by that name. Our brief exposition will not serve as a basis for a number of more delicate solutions, in regard to which as well as in regard to the introduction of the calculus the reader must be referred to the classical exposition of Gauss and to the text-books which found upon that exposition. It must be borne in mind that the validity of the method always depends on certain very probable though not strictly demonstrable presuppositions; for its full and adequate substantiation we must look to the results to which, especially in astronomy, it has led.

CHAPTER X.

Of Elections and Voting.

289. Elections and voting are processes of framing judgments; judgments, that is, whose validity we mean to create by our own decision, and not merely to acknowledge. The logical calculus has taken account of these processes in various ways. It has been asked what expectation of a just verdict, or of a proper decision, or of a wise election, can be based upon different forms of procedure; but, as questions like these can never be answered apart from special and arbitrary presuppositions of a psychological kind, I shall here exclude them, and confine myself to the enquiry into the means of attaining what is formally the object of all voting, namely, a decision that shall express as completely as possible the collective will of the voters, independently of the degree of wisdom that may guide the several wills which go to compose it.

In common life such a collective will takes the shape of public opinion; and the matter which it affirms or rejects has been gradually defined by the countless reciprocal influences of all who have the power of manifesting inclination or aversion. But a logical treatment presupposes that the matter in question is already put into shape as a definite proposal V or a series of proposals V, W, Z; that the expression of will takes place by simple acceptance or rejection of what is so put forward; and finally that there is a definite and limited number S of equal votes, to which and to no others it belongs to establish the collective will.

290. To take the simplest case: if there is a single proposal V put forward, and a decision absolutely must be arrived at, the only possible ground of determination is an Absolute Majority. It is the only result which cannot help occurring either for or against V; supposing the case of equality of votes to be provided for by some fixed agreement as to a casting vote, or as to giving the preference to the affirmative or to the negative conclusion. But there are great limitations on regarding an absolute majority as the true expression of what could rightly be called the collective will of the voters. The several votes are themselves no exhaustive expression of the several wills; being restricted to 'Yes' or 'No' they have no means of distinguishing a decided will for or against from mere acceptance or non-resistance. This constant defect in all voting can only be remedied by previous Discussion. This allows fitting expression to different intensities of affirmation or rejection, and gives scope for the influence of personal authority, which has to lose its power in the actual and formal voting in which the votes must be counted and not weighed. It is for the individual's sense of propriety to decide how far in the subsequent giving of the votes account is to be taken of the division of feeling for and against, which after discussion is at least known to all. Other conventional rules, such as the requirement of a two-thirds' majority, diminish this evil without removing it; the only unambiguous result would be unanimity, but neither it nor the two-thirds' majority can be required without endangering the certainty of coming to a decision. So these two regulations are only appropriate where there are other weighty reasons for giving to the conservative preference for the existing state which is known, an advantage over the impulse to innovations whose result is unknown.

291. No general reason can be found in Logic for departing from an equal value for all participating votes; but in actual life there have been both fair and unfair reasons constantly operative to attach different weights to the votes, so as to give an advantage whether to greater wisdom, or to the more important or the more specially menaced interest, or to claims to peculiar preference which had some historical origin. It is sometimes done by simply counting the single vote of the preferred person as equal to several votes; sometimes by breaking up the totality of voters into a number of groups in each of which a separate vote is taken, and substituting the majority of the majorities which arise in this voting for the absolute majority of the total number; sometimes by having recourse to indirect voting, in which each of the groups transfers its right to a delegate and leaves the decision to the majority of these nominees.

The first case requires no separate consideration; the last withdraws itself from all logical treatment in cases where the deputy so commissioned has to vote independently himself, and not to represent a decision already taken by his electors. For the certainty with which the result in that case corresponds to the collective will depends upon the doubtful reliability of the electors' judgment in estimating the agreement between their deputy's sentiments and their own. On the other hand, the second case, that of division into groups which are to vote separately, has the following determinable peculiarities.

1. If we take the total number S of the votes as $= 2m \cdot 2n$, one of these factors indicating the number of groups made, and the other the number of votes in each group, then $(m+1)(n+1)$ will be the number of votes in the absolute majority of the several absolute majorities which result within these groups. And this value remains the same if we substitute for one or both of such even factors the odd numbers next above $(2m+1)(2n+1)$. Suppose M on the other hand to be the simple absolute majority of the total number of voters S when voting without subdivision, we may easily convince ourselves that $(m+1)(n+1)$ is less than M for all uneven S which are greater than 7 and for all even S which are greater than 12, and so in all cases that need be considered with reference to voting. Thus it is always possible by a suitable subdivision of S to bring about a decision resting on the *minority* of the total number of voters; and it may be asked which modes of division are the most adapted for making this winning minority as small as possible. A precise answer to the question would be far more lengthy than the matter deserves; for in application we shall always have to be satisfied with an approximation, because our precise estimate would be made useless by any trifling accident that prevented a vote from being given on which we had reckoned. So I content myself with what follows.

2. If we consider S as the product of two even or of two uneven factors, and thus either $= 2m \cdot 2n$ or $= (2m+1)(2n+1)$, if we replace m in the formula for the winning minority by an expression in terms of n and S, and if we differentiate with respect to n, we obtain as condition of a minimum $2n$ or $2n+1 = \sqrt{S}$ which gives the other factor $s = \sqrt{S}$, and therefore $m = n$.

If we take S as product of an even and an uneven factor, $= 2m(2n+1)$, we obtain in the same way, as a condition of a minimum, that the even factor $= \sqrt{2S}$, which gives the uneven $= \sqrt{\frac{1}{2}S}$. The manner of their deduction prevents either of the formulae from applying precisely in these cases where both the number of votes and that of groups can only increase by entire units, and not continuously; in particular, their application cannot be regular for small numbers of whose amount a unit is a considerable fraction; and lastly, the advantage of odd numbers over even, as the winning minority for $(2m+1)(2n+1)$ is not larger than that for $2m \cdot 2n$, will also be detrimental to the influence of these rules. Still, for high values of S, as unity, or the difference between odd and even, forms a progressively smaller fraction of their amount, these two formulae really give the two least values of the required minorities. They are obtained by separating S into two factors either as nearly as possible equal to each other and to the square root of S, or one of them as exactly as possible double the other. Thus 225 considered as 15 . 15 and as 9 . 25 gives the two least minorities 64 and 65, but as 5 . 45 and 3 . 75 the larger ones 69 and 76; 11025 as 105 . 105 and as 147 . 75 the least minorities 2809 and 2812, while on the other hand as 175 . 63 and as 9 . 1225 it gives the larger numbers 2992 and 3065; and finally 20,000 breaks up to best advantage as 200 . 100 and as 125 . 160, with the minorities 5151 and 5103. In the case of small numbers the influences of the different conditions cross each other very markedly; 36 taken as 6 . 6 gives the minority 16, but even as 4 . 9 gives the smaller minority 15 because of the favourable influence of the odd factor; the most advantageous subdivision is 3 . 12 giving 14 as the minority; for in this the even factor 12 comes nearer to the square root of $2S = 72$ (which is greater than 8) than does the even factor 4 in the division 4 . 9. On the other hand 81, being the square of an uneven factor, has no subdivision more favourable than 9 . 9 giving 25 for the minority; the other into 3 . 27 is too remote from both conditions. For 144 one minimum 49 is obtained out of 12 . 12, the other 45 out of 9 . 16.

3. In the first of the most favourable cases, that of equal factors, the winning minority, expressed in terms of S, $= (1 + \frac{1}{2}\sqrt{S})^2$; in the second, that in which one factor is double the other, it =

$$(1 + \tfrac{1}{2}\sqrt{2S})(1 + \tfrac{1}{2}\sqrt{\tfrac{1}{2}S}).$$

Both expressions approximate to the value $\frac{1}{4}S$ (only the second does so more slowly) as S becomes greater, but remain always greater than that fraction as long as S does not become infinite. Thus the winning minority has a lower limit, and can never, by the most advantageous subdivision, be reduced to a quarter of the total number of votes.

4. Finally, S may be a prime number, which in any case can only be made divisible if increased or diminished by a single unit at least: i.e. for the present purpose, by giving to one of the groups a single vote more or less than the others have. A choice is therefore inevitable, and may be exercised as we please; beyond a doubt it is equally justifiable to consider 67 as 66 + 1 and as 68 − 1, making in the first case 5 classes of 11 votes and 1 of 12, in the second 3 classes of 17 and 1 of 16; if it is required for the sake of fairness that the majorities which make up the winning minority shall always include those of the more numerous classes, we shall obtain in the first

case $3 . 6 + 1 . 7 = 25$, in the second $3 . 9 = 27$. When once this path is open, it is followed even where there is no need; and then the inequality of groups is readily put up with as long as it remains within reasonable limits; and it further diminishes the winning minorities very considerably. Thus we obtain for $64 = 6 . 9 + 1 . 10$ the minority $3 . 5 + 1 . 6 = 21$ (even if we make a rule of requiring the majority of the larger group to be in it) while $8 . 8$ only gave the greater minority 25. We all know that this resource has been abundantly employed from Servius Tullius downwards to a very unfair extent, which must look for its justification not to Logic, but to politics.

292. When we come to choosing between different proposals V, W, Z, Logic as such would make requirements that diverge from the usages observed in practice. If a number of persons desire to unite in a collective determination such as to produce the greatest general satisfaction, they ought not to obtain the result as an inevitable consequence of a summation of declarations of will, none of which takes account of any of the others. A rational will must attach importance to not giving its decision without knowledge and consideration of the other voters' inclinations or aversions, opposed to its own; especially as the necessity of finally declaring itself in a bare Yes or No leaves no means of finding expression for the different intensities of its volition and so of securing for it neither more nor less than the just measure of effect. Previous discussion, to which I referred above, cannot entirely satisfy this requirement; for if everyone wanted to declare himself completely, the discussion would turn into voting, only without the precision of form which makes the ascertainment of the final result easy and certain. So an attempt must be made to effect by the actual mode of voting as nearly as possible what discussion aims at doing.

If we consider V, W, Z, as three persons, one of whom is to be elected, we may adopt the following procedure. A preliminary vote upon all three candidates at once would show what degree of approval each of them meets with, compared to the others. If no one of them obtained an absolute majority of the votes, the relative majority could not be taken as decisive except in elections of very small moment; but it has an importance which may be seen in everyday life. The candidate who has the most votes compared with the others attracts attention and often gains the other votes as well; but just as often his prominence arouses antagonism, and compels his opponents to combine in support of a rival. Hence it is the general rule to require an absolute majority; it gives the only security that the sum of negative votes must be less than that of affirmatives, and that therefore the will of the majority has been hit upon; and this is the *ultima ratio* which must always give the final decision when opinions remain unreconcilable and yet a collective resolution is indispensable.

But further; if an absolute majority has been obtained[1] for one of the candidates, say for *V*, still it is neither essential, nor right in itself, to accept this at once as the decision. This preliminary voting only showed the number of voters who preferred or postponed each of the candidates to the others; the degree of such preference was left undefined, and so also was the feeling of each voter to the candidate whom he has not named. To bring this to light a second and tripartite vote would be required; being a vote of Yes or No upon each of the candidates separately so as to give each elector the possibility of directly recording his vote against any particular candidate, while before he could only express it by preferring another to him. If we suppose that in the first voting *V* obtained 11 votes out of 20, *W* 5, and *Z* 4, then, excluding incomprehensible inconsistency on the electors' part, every candidate will retain in the second process the votes of those who preferred him to the other two in the first, but the remaining votes may be very variously divided. It is possible that *V* may now meet with a decided opposition of 9 votes, while *Z* who was only preferred by 4 may find no opposition at all and win 16 votes more; and *W* may get 10 of the 15 which he had not before.

To obtain a final result out of this it must be considered that the votes obtained in these different votings are of unequal value. Those of the first process showed how many voters thought a candidate the best, and though their approval may have been very different in degree still we may regard all these *preference* votes as homogeneous and attach the same weight *m* to all of them. For the best that anyone can say of a candidate with reference to the election as such, is that he is for him; whether he respects him more or less apart from this is indifferent, for every election can only aim at the best result under the given conditions, and at not the best absolutely; any one who is for *V* or *W* under the given conditions is for him altogether. The same assumption holds of the negative votes in the second voting; anyone who has the opportunity of pronouncing directly on *V* or *W* by Yes or No, and votes against both, is absolutely against them, and has accomplished his will completely, as regards this election, if the vote of rejection is carried; no matter how thoroughly he may hate or despise V or W in other ways. So all the unfavourable votes may be considered as homogeneous and assigned the same weight q. But those favourable votes which are only obtained in the second voting are obviously of less value than those which were obtained in the first; they are only permissive, while the others were preference votes; and this difference, which marks a middle grade between voting for and voting against, is of course of importance for the election in hand. Still, what weight a permissive vote ought to have compared with a vote of preference, is a question which the voter who gave it could probably not answer with precision; besides, his acquiescence would not indicate the same degree of approval for every candidate to whom he gave it, but a greater, perhaps, for W than for Z. Therefore, though it at once involves a serious failure of accuracy, yet the only possible attempt to make a general estimate even approximately of the difference of the permissive votes from the preferential is to assign to all votes of the former class a common value p, which must be a proper fraction of m, and the amount of which can only be fixed by convention. On these assumptions the votes would be calculated in the above example as follows: for V, $11m - 9p$, for W, $5m + 10p - 5q$, for Z, $4m + 16p$; and so finally, if we arbitrarily take $m = q$ (giving the preferential vote equal weight with the vote of rejection) and $p = \frac{m}{2}$ (giving the permissive half the weight of the preferential), the result would be only 2 votes for V, on the other hand 5 for W and lastly 12 for Z, in marked contrast to the result of the first voting.

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[1] [I.e. in the preliminary vote.]

Various circumstances combine to make these logical requirements unrealisable in practice. In the first place, on grounds of social propriety, we should desire to avoid voting against persons altogether. Next, even if it were admitted, there would be great reason to doubt whether the second voting would be carried out with the requisite impartiality, even supposing it to take place before the first. Those who were determined to give their votes of preference to V would probably not admit even to themselves that they could be content with W or Z, and their 11 votes would appear in the permissive voting too as so many votes against W and against Z as well. And lastly there would in every case be the same preliminary question what was best to aim at with a view to the matter in hand, whether the completest satisfaction of the majority or the greatest average satisfaction of all; and this would have to be determined before the ratio of weight between affirmative and negative votes could be fixed. It need not necessarily be equality; on the contrary, there may be cases in which a single unfavourable vote may fairly balance more than a single favourable one, and the decision would have to be obtained not so much by the greatest number of votes for a candidate, as by the lowest number of votes against him. It clearly makes a difference whether the matter in hand is the decision of some gravely responsible business, say an election to some office of political importance, or whether it is the organisation of amusements in common, perhaps the election of the president of some social gathering. In the latter case it would be absurd to make 9 members out of 20 discontented in order to give 11 others complete satisfaction; but in the former it may be reasonable to satisfy the majority of decided wills completely, rather than light upon a choice that only met with the lukewarm approval of all. But it is just in the second case, where the method described would give the most desirable result, that the inadmissibility of negative votes makes its application difficult; in the first, where its result might be less desirable, its application would be less difficult, for in this case the votes of rejection would be a less serious slight, as they might possibly be directed against the views represented by a candidate and not against him personally.

293. There is another mode, a sort of process of elimination, that may answer our wishes when an election has to be made out of a very large number of candidates; as when a constituency has to name some one out of its own number. It is usual in these cases to take a first vote by way of fixing upon some three names which are the first to attract the electors' attention before minor considerations come in to restrict it, and at present therefore appear the most desirable to each of the voters. It is possible in this process to attach distinctions to the order in which each elector names or writes his three candidates; and to put the one mentioned first above the others; but I assume for simplicity's sake, that the order of naming is quite indifferent. Then it is conceivable, though very improbable, that the same three candidates, *V*, *W*, *Z*, may receive all the votes; if this occurs, it becomes impossible for a final decision to be obtained by election, for a fresh vote could not give a majority for any of the three unless some voters retracted their previous decision without any ground in circumstances for doing so. In this and all similar cases the only remaining possibility is either the lot, or the decision of some external will, e.g. that of a higher authority.

On the other hand, if *V* and no one else obtains a vote from every voter, his election is decided beyond a doubt, whatever number of votes *W* and *Z* may have obtained; for then there are no concealed votes of rejection which nothing but want of opportunity has hindered from being recorded. But supposing *V* to have only obtained an absolute majority, *W* and *Z* considerable minorities, and the rest of the votes to have been scattered, then it becomes possible that there are such votes. So, considering our observations above, we cannot hold it quite justifiable to break off the election at this point and regard *V* as elected; it is better to take a fresh vote for *W* against *Z*, so that actually voting against *W* may be avoided by voting for *Z*, and *vice versa.* One of the two must obtain a larger or smaller absolute majority. Supposing *W* is successful, a third vote as between him and *V* will give a final decision. Of course this final voting will be wholly superfluous if the absolute majority which was in favour of *V* on the first vote maintains itself unaltered; but a reasonable motive for a change of feeling may have been furnished by the result of the second voting. If in it the votes for *W* and for *Z* are nearly equal, it would prove either that *V*'s opponents are not united, or that no other choice meets with more uniform agreement than that of *V*, and this would give the previous majority a reason for persisting in their conviction. If on the other hand *W* got all the votes, the majority in question might think this a good reason for going over in the last vote to the minority, already considerable, in favour of *W*, in order to produce a result which should have no decided opponents. Many more modifications are conceivable; I will not follow them out, for the discussion threatens to be longer than its importance warrants; moreover it is at least doubtful whether such a process of elimination is really more flattering than open rejection. And finally, *V*, who is elected, may decline the post. This alters the conditions with reference to which the votes were given so completely, that it becomes necessary to repeat the whole process of election, or perhaps to make an independent selection of *Y*, as a third candidate besides *W* and *Z*.

294. If *V*, *W*, and *Z*, are not persons, but legislative proposals, there is no reason for shunning the direct negative vote; and it might be demanded on logical grounds that a vote of Yes or No should be taken on each of the measures proposed, but that the obtaining of an absolute majority by one should be no bar to voting on the others. The decision would then depend either on the largest of the majorities or on a fresh vote which would be final. This procedure would cause those whose opinion had gained a considerable number of votes, to adhere to it in the final voting; but any who found theirs in a hopeless minority would have time to attach themselves in the final vote to the opinion which they liked next best to their own and which might have a chance of gaining the deciding majority by their accession. Still, there is here too the same psychological reason which I mentioned before, against this procedure; any one who decidedly preferred a proposal V would not declare freely that W or Z would also be tolerable to him, but would be tempted to reject them both. Therefore as it is the traditional rule that the adoption of one proposal *ipso facto* shelves all posterior ones, the order in which $V\ W\ Z$ are put to the vote acquires great importance as affecting the choice made. I concur with Trendelenburg's statement[1] of the wish that may be felt on logical grounds respecting this arrangement, the most difficult problem of parliamentary tact; viz. that the adherents of every opinion should have an opportunity of emphasising it with all the weight they can command; of negativing what they want to reject directly, not indirectly by the acceptance of something else which has only their partial approval; of affirming what they wish immediately and exclusively, not by the rejection of something else of which they disapprove only in part; and finally, that it should be possible for everyone to begin by defending and recommending what he thinks best of all, and only to retire upon his second or third best after the first has failed. But whether the universal accomplishment of this postulate for everyone entitled to vote and in respect of every proposal before a meeting is not as a whole frustrated by a fundamental contradiction; whether, therefore, it is conceivable that everyone's sentiments should be gratified by just those proposals being broken up on the parts of which he thinks differently, and just those united which he wants to see accepted or rejected together, demands no investigation. It is quite clear that in each case the solution of the problem can only be approached by an acumen developed in long and uniform practice, after entering thoroughly into the subject-matter under discussion. The procedure to be observed can only be learned or taught by help of definite instances, not by universal symbols representing possible cases, and only in practice; general rules can give very little help.

295. It may happen to begin with that the proposals in question $V\ W\ Z$ do not compose the complete disjunction between the members of which there is a choice, but that there is a fourth member consisting of the rejection of one and all of them; i. e. that speaking generally a new resolution is as such unnecessary, and it is possible to maintain the *status quo.* There are two reasons that may lead to the choice of this alternative; either the desire to protect that particular *status quo* on principle against all innovation, or the absence of an acceptable proposal among those put forward, though on grounds of principle there is no opposition to a reform. It is important to provide expression for the distinction between these two dispositions. The mere rejection of all individual proposals successively does not provide it; this only proves that the change which would have been acceptable has not been proposed; but it should be possible to reject generally and as such the invitation to change which all the proposals have in common. This is effected by the motion to pass to the order of the day, that is, therefore, that the whole of the proposals in question should be excluded from being debated or voted on, and so their common element should be negatived in a correspondingly general form. Where anyone's will is in favour of such a negative it is his parliamentary duty to contribute to a full expression of the state of opinion by making this motion, and not to content himself with throwing out all separate proposals, until it has been rejected. Even where there is only a single proposal instead of several, the motion of the order of the day may be in place; its meaning then is to reject not this particular proposal as such, but the general intention out of which it has arisen and others might arise. Thus the order of the day when voted without a statement of grounds, may act as an expression of contempt for a proposal that is legally or morally disgraceful, or as a refusal to entertain one that is extraneous, and beyond the competence of the voters, or finally as a riddance of a dangerous proposal the mere discussion of which it is the interest of the commonwealth to avoid. Or such acts of rejection may be mitigated by a motived order of the day, which recognises in its statement of grounds what there is in the proposed measure that is just in itself, but denies the propriety of introducing and debating it at that particular moment.

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[1] ['Ueber die Methode der Abstimmungen,' Berlin, 1850.]

296. If two proposals *V* and *W* are so related to each other by way of subordination that *W* is an 'ameliorative motion' or amendment that aims at modifying the purport of the substantive motion *V* by addition, omission, or alteration, then it is logically speaking a correct usage to take a provisional vote on the amendment before the final vote on the substantive motion. For no decision on this latter can rationally be solicited from the voters till its wording is completely and unequivocally settled; certainly not while its details are open to subsequent modifications the acceptance or rejection of which if it could have been foreknown might well have succeeded in totally reversing the favourable or unfavourable impressions which had been prematurely recorded. The vote on the amendment *W* serves to fix unequivocally the purport which the substantive motion *V* has when put to the vote; therefore the rejection of *V* annuls the previous adoption of the amendment, which was only provisional.

If there are several mutually exclusive amendments *W* and *Z* to a substantive motion *V*, or several accessory proposals about the special modifications necessary to applying *V* in practice (as often happens when details of quantity remain to be fixed), the safest course would be to vote separately on all such proposals and let the decision go by the greatest majority. Only if, according to the usual practice, the acceptance of one by an absolute majority is to exclude all the others from being voted on, the order of putting them to the vote becomes important, and the obvious advice is to arrange the proposals so that the two least divergent shall always be next each other. This is the practice under a rather different form in both kinds of auction, that by bidding and that by offering[1], and in these cases people actually calculate, and quite fairly, on the uncertainty in which each person who bids must be about the degree of the others' desire. For as the bid or its acceptance[1] by the buyer are alike voluntary, the customer is merely declaring what value the object has for him according to his own estimate and no right of his is attacked by the open competition of others or by his ignorance of the absence of any eager desires but his own. Offering seems, speaking generally, in favour of the seller, for it compels the buyer to take the object at the highest price which he thinks he can afford to give, though he would give less if he could foresee the absence of competition; bidding is in favour of the buyer because if there is such absence it is available to him; if not, at least he has only to advance on the last bid, and his time for decision is not excessively curtailed. This is a procedure in which an individual tries by a contest with others to secure a legitimate personal advantage; so its analogy is not in spirit very appropriate to the efforts of a multitude to bring to pass by common action a resolution advantageous to the common weal. Still, in form, they must take pattern from the procedure of 'offering,' only, as a rule, it will be rare to find proposals that can be so simply arranged in a quantitatively graduated series; most commonly *W Z*... will differ in purport so as to their anticipated degree of conformity to the general will; those that are furthest removed from the *status quo*, that demand the most extraordinary and ample measures, and therefore have little probability of success, would have a claim to be put to the vote first; in order that, if, contrary to expectation, they were destined to turn out in conformity with the general will, the expression of that will might not have been made impossible by starting from a more likely proposal which might easily unite all the votes with premature resignation based on that erroneous estimate itself. After the rejection of such extreme proposals we might pass, as in the mathematical method of limitation, to the mean terms of the series which are more likely in themselves; with the aim of procuring the final decision in favour of the proposal which involved the least possible deviation from general satisfaction. But all these rules are ultimately inadequate; for instance, where the business is to decide upon a composite whole whose different parts can only be discussed by degrees, it must always be impossible while the special deliberations are still proceeding to discover all the inconsistencies, inconsecutivenesses, and contradictions which would arise from the ultimate conjunction of all the details of the plan, perhaps variously modified. In such cases, as in the case of amendments, the special deliberations should be treated as no more than preliminary, and an assembly should reserve to itself the power, by a vote on a second reading or by some final vote, to annihilate the monster which its united efforts have brought into existence. Lastly, it is true that the formal aim of all voting, to arrive at a collective will, would involve in the first place the establishment of a decision Z such as to give all members of the society the greatest attainable average satisfaction M, considering the lesser satisfaction of one as compensated by the greater satisfaction of another. At the same time it is also to be desired that in order to the performance of the obligation arising from the acceptance of Z an equal compliance M were to be reckoned upon in all members. I have explained why the former end cannot be attained in perfection. The latter on the other hand is of course a desire unrealisable by logical means; only this may be deduced as a logical rule which the nature of ethical ends makes necessary to their realisation, that here (for Logic can require it nowhere else) we should subordinate our personal conviction to the general opinion when different from our own.

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[1] [I. e. in a 'Dutch auction,' where the auctioneer offers successively lower prices, and the first customer who accepts is the buyer.]

BOOK III.

ON KNOWLEDGE (METHODOLOGY).

◆

INTRODUCTION.

In the enquiry instituted in the last book as to the means by which we are enabled to arrange the manifold content presented to our minds under those ideal forms of conception and association with which we became acquainted in the previous book upon Pure Logic, nothing was said as to the general question of logical methods with which a theory of the nature of thought ordinarily concludes. In passing over the subject in that place I conceive that I was guilty of no unpardonable omission, nor was it mere caprice which led me to reserve that and other questions akin to it for this concluding section of my work.

297. Since the time of Aristotle philosophers have distinguished between the analytic and synthetic methods, from points of view essentially the same in all cases, regarding them as the two ultimate forms to which all methods of scientific procedure which the movement of thought follows may be reduced. In the view of antiquity any subject-matter presented for scientific investigation was to be submitted to a process of dissection which should trace it to its simplest elements or to its most universal conditions. Thus the analytic method was a retrogressive operation proceeding *a principiatis ad principia*, while the principles when discovered formed as it were the blocks which the synthetic or progressive method proceeded to build up into the individual objects of experience.

taught us on the contrary that for the human intelligence general principles have largely to be created out of a combination and comparison of the manifold facts of experience, and they appear to us therefore when we have arrived at them, as the final outcome of a synthetic operation of thought. In the same way we are no longer disposed to limit ourselves to a point of view which regards general principles as atoms of truth, from the mere piecing together of which particular truths are derived; to us, rightly or wrongly, general principles appear rather as containing within themselves a capacity of development, and we regard the derivation of the conditioned from its conditions as consisting at least as often in an analysis of the content of the conditions as in the combination of them.

But the question of language is not worth debating further, for it is plain to begin with that neither of the two methods can be carried out, at all events in any general application, without the other. No method of analysis can arrive by the mere dissection of the particular object presented to it, at a principle or a general truth, unless at every stage it compares the result a of the last step with some general proposition T, and by endeavouring to bring it under the latter—that is to say at this point by an act of synthesis—makes sure whether a is an ultimate principle, or whether it may not involve some contradiction for the removal of which it may be necessary to continue the analysis further in one direction or another. Nor need the proposition T which has to be recognised in such a process by any means always belong to those formal logical laws whose supremacy over the *modus procedendi* of every conceivable method is acknowledged as a matter of course. On the contrary it must often, if any real advance is to be made, be a concrete proposition which Logic cannot furnish, but which it has to accept on purely extralogical grounds, and to which the results arrived at by the analysis have to be subordinated.

As little can a method of synthesis make any way without the help of analysis. Even supposing it to start with a number of elementary truths A, B, C in its hand, it could never get beyond the tautological proposition that these several truths are all true at once, unless it can go on to show how by their possessing simultaneous validity in respect of one and the same object this or that fresh consequence x or y is necessarily developed. But whether x or y will follow can only be decided after an examination of the nature of the object in question, that is to say by a return for the time being to analysis. Such an analysis can alone furnish us with the determinate minor premiss which we require to combine with the general truths with which we began, and which supply our major premiss, in order to proceed once more synthetically to a determinate conclusion.

It must be allowed that in certain departments the synthetic method has the appearance of a greater independence. Thus geometry is able to create the objects to which it desires to apply its general truths, as it goes along, and the analytic statement of the *data* which are accepted for the purpose of deducing each new proposition occupies only a small place in its demonstrations. Still in reality it cannot be wholly absent. And in the wider provinces of scientific activity which are concerned with the synthetic construction of real things, the progressive movement from principles to facts is always preceded by a retrogressive operation, consisting in a comprehensive analysis of the *data*, which is essential to determining for the synthetic procedure itself the directions in which it has to look for those minor premises with which its general principles cannot possibly dispense.

298. Thus in fact the distinction between the two methods runs up practically into the following antithesis, which has long attracted attention :—the Analytic method is essentially the method of investigation, having the discovery of truth for its object; the Synthetic that of exposition, of which the object is to exhibit a body of truths, whether obtained in one way or in another, by direct or indirect processes, in their natural and objective connexion. And by exposition I do not merely understand communication to others, a purpose for which an exhibition of the subjective process of discovery is no less necessary and no less instructive. I mean rather the framing of the results arrived at into a logical whole, in which form alone they meet the ideal requirements which the human intellect makes of any body of truth that is to be fixed and independent.

Such being the case it appeared to me hardly desirable to introduce the question of the analytic and synthetic methods among the questions of applied logic, inasmuch as neither the one nor the other affords any practical contribution towards the solution of any definite problems,—the analytic method as little as the synthetic, notwithstanding that we regard it as the type of all methods of investigation and discovery. To instruct a person to employ the analytic method is to give him no very helpful counsel; the customary general definitions of the method really contain nothing more than an indication of the direction in which the required road still has to be looked for; to find it we must turn to the special expedients of applied logic, in using under the method of synthesis or under that of analysis. In the same manner an enquirer who has the synthetic method prescribed to him has merely got his problem stated: to the question how to solve it, in which certainly the rules of a method ought by rights to be of service to him, no very sufficient answer is afforded by the general direction to work downwards from principles to results.

299. All this is changed if we surrender a certain privilege which in the sphere of applied logic we allowed ourselves to claim, and endeavour by this surrender to give completeness to our treatment of the subject. Thus far in speaking of the forms of proof, of the search for grounds of proof, of the discovery of laws, we have left our work throughout in a certain sense incomplete. Every attempt to establish a proposition went only a few steps back, and came to a stand as soon as some other proposition was reached, which served for its foundation, and the validity of which was not proved but assumed. This procedure answers to the actual course of thought in life as well as in the sphere of the special sciences. In ordinary life our judgment of things and the conclusions we draw concerning them rest not on a single proposition *T*, nor yet upon a clearly defined group of homogeneous elementary truths, but on a large number of truths of a quite heterogeneous character, yet possessed of an equal certainty; here a proposition *A*, which once apprehended forces itself upon us as a necessity of thought, there a proposition *B* expressing an immediate fact of perception, and presenting itself not exactly as necessary but as incontrovertible; a third, again, *C*, may be a principle of unknown origin, but one which at every moment is being put to the proof and confirmed afresh; finally we have many a proposition *D* springing from equally unknown sources, but admitting of no such guarantee of its truth, and yet seeming to bear within it an indefeasible claim, a claim that is which we believe ourselves bound to satisfy if the conceptions by which we bind together the *data* of experience are to answer to the truth.

Any one of these various points of certainty—and in each of them we may suppose a number of elementary convictions to be contained and compressed—is used indifferently in the living movement of time we set out from some law which is evident to us, and determine its effects; at another by repeated observation of the effects we strengthen our belief in the law; consequences which appear to flow by an internal necessity from some acknowledged principle, are rejected for their improbability as seen from a different standpoint; now we start from *A* to prove a doubtful *B*, now *B* appears the more evident of the two, and we use it to establish *A*; the truth being that whatever possesses for us at the moment the strongest psychological certainty passes for the point of vantage from which the other more wavering beliefs are to be secured.

300. So entirely unconstrained as this in its operation scientific thought certainly is not; still the actual science which we possess, as distinguished from the ideal science, which we might wish to possess, has resemblance enough to the natural processes of ordinary reflexion. Here also we hardly meet with any enquiry into a matter of fact, which does not depend for its determination upon certain presuppositions which we adopt now as undemonstrable but certain, now again as undemonstrable and only probable, and which are regarded either as ultimate principles of the particular science before us, or as vouched for by some other science. Even within a single province of enquiry the direction in which the required proof is looked for varies. Without exactly questioning the certainty of a proposition which we have begun by regarding as the source from which others are derived, we come nevertheless to believe that there is some other proposition which we may place at the head with still greater security, and from which we may derive our former first principle with all its consequences.

And if we review our knowledge as a whole, distributed as it is among the various sciences, we shall find no one of these complete and rounded off in itself. In each one of them we come upon formal or material principles, whose validity is admitted because they are self-evident, or because they explain certain facts, while as to their origin and connexion with each other no enquiry is made if the question appears to have no immediate bearing upon the prosecution of the science itself.

This point of view we kept before us while we were dealing with the subject of applied logic, and we saw no reason to depart from it so long as our subject was the nature of investigation. For what we commonly call Applied Logic, or more properly the exhibition of the possible modes in which Logic can be applied, presupposes the existence of a variety of cases adapted for its application, and this is only possible if the work of investigation consists in taking some given fixed point to start from, and then connecting this by rule and law with other fixed points which are also not proved, but assumed. Such is the character of all investigations in which we are accustomed actually to engage, and in this our knowledge bears resemblance to our life. What was the origin of our race in the first beginnings of history we know not, and as little can thought carry us to its far-off future; for most of us the memory of those who have gone before fades away in no very distant past, and for all alike the prospect granted of the fortunes of those who are to come after us is still more confined; yet in the midst of this darkness on either hand, there lies before us a certain space of life comparatively clear, with plain needs, pressing duties, and attainable goals; our joy in existence, and our confidence in acting upon the present, are but little impaired by the uncertainty of the beginning and the end. And so it is with our knowledge. That there is an eternal truth, or a perfect and self-closed circle of truths, we do indeed assume, but ordinary reflexion has for such a system of truth no form of expression adequate to it, nor has it any clear conception how the members of the system are related; single portions only become plain and evident to us in a manner which we are incapable of analysing to ourselves, in the course of our mind's operation as it comes in contact with reality. In the process of investigation we resemble men engaged in a narrow inland traffic, and we endeavour to connect the uncertain and changing scenes about us, with the isolated peaks which rise upon our view out of a coherent world of truth which we cannot see.

301. But just as there arrive moments in life when the present only seems endurable or intelligible if we can catch some glimpse of its connexion with the past and with the future, so also in knowledge there are occasions when we are tempted to pass out of the petty business of ordinary scientific investigation, and reflect upon the points it starts from, and the points it aims at; to ask where they are situated, how they are connected with each other, and whether they are secure. For the principles on which the several sciences repose do not restrict themselves to an unaggressive sway each within its own separate province. We need only point to the very different consequences in respect of the powers that shape human life which are deduced from the principles of mechanical science on the one hand, and the deliverances of conscience on the other, to see in a single emphatic example how the claims of different sources of truth may clash and conflict in dealing with a common subject-matter. But even within the field of purely theoretical science we may find inducements enough to make that which in the living processes of thought, and in the special sciences, figures as the first principle from which all enquiry starts, itself a subject of enquiry.

This important problem philosophy in all ages has kept in view and pursued, not indeed with entire success, yet not altogether without result; and assuredly its complete solution would be also the completion of philosophy; for such solution could only consist in the establishment of a perfected system of connected truths at once ultimate and concrete, from which all the principles which direct the researches of the sciences would be derivable, which would supply the key to their precise and real significance, and define the limits of their validity. It is no such comprehensive undertaking as this, but only a modest portion of it, which will form the subject of the concluding chapters of this work. Our object is not to enquire into the content of the principles in question, but into the grounds upon which in a subjective sense their certainty for us reposes; to ask not what is the truth, but what are the marks by which we recognise it and distinguish it from error; or, if we are to keep the old terminology, it is our purpose by following a method of analysis to obtain clear ideas as to the path by which we may hope to arrive at the principles of a synthetic development.

My reason for treating this part of Logic under the head of Knowledge, the further discussion upon which we are entering will elucidate—an elucidation which the above preliminary designation of our undertaking itself certainly requires. But in giving it the name of Methodology I confess myself to be employing the term to some extent in a sense of my own. Every science develops its characteristic methods, methods fruitful in their results, which it employs in dealing with a given class of problems. But Logic regards all such methods as special artifices with which it has no concern, but which it belongs to the special sciences themselves to furnish. General methods again, such as the synthetic and analytic procedure of which I have been speaking, do indeed find a mention in Logic; but by formulating them we should only be making a somewhat barren postulate, until we are clear as to the grounds of our belief that the one has actually led us to the discovery of the truth, and that in the other we have an instrument which enables us to develope and exhibit it in detail.

It is this last-named undertaking which I desire to indicate here by the term 'method,' using it to denote not a general type of procedure which has to be applied over and over again in a thousand instances, but rather in the sense of a definite operation which thought has to go through once and for all, and of which the object is to mediate between the various sources from which various kinds of certainty appear to find their way to us, and to arrive at a knowledge of the connexion between them, and of the limits of their validity.

CHAPTER I.

On Scepticism.

302. THE human mind only becomes aware of the laws of its own activity after it has already exerted it in a great variety of ways, when it turns back by an act of reflexion and comparison upon the various forms which this activity has assumed, and makes the rules, which it has been following all the while unconsciously, an object of separate attention. The question why those laws are binding and within what limits their observance carries with it the promise of true knowledge, comes still later. It can only arise after we have had experience of errors into which we appear to have been drawn not by the neglect of those laws, but by their observance in dealing with the different subject-matters presented to our intelligence.

If then this has been the case, and if further no success has attended our scattered and occasional attempts to remove the difficulties and contradictions which have arisen, by giving a better interpretation either to that which seemed to us to be truth, or to that which we regarded as the immediate deliverance of direct perception,—then arises that mood of wide and general doubt which constitutes Scepticism.

As a transient phase of longer or shorter duration this sceptical mood has its place in the development of every serious mind; several times in the history of Philosophy it has been emphatically insisted on as the normal and necessary condition of the mind, which is called upon at the outset of the scientific life to regard all traditional knowledge as so much doubtful prejudice, which has to be submitted to test and trial. Finally it has established itself as a permanent result in the sceptical schools of Philosophy, which have believed themselves to have attained to the conviction of the impossibility of certain knowledge. In this final form in which alone

Scepticism pretends to have arrived at a definite outcome, we shall not find it so entirely free as it flatters itself to be from traditional prejudices. One thing above all however is clear; an unconditional denial of all truth this final outcome of Scepticism cannot by any possibility include, inasmuch as not the solution of doubt merely but doubt itself is only possible on the presupposition of some sort of acknowledged truths.

Whoever entertains a hope of finding a path out of the labyrinth of Scepticism to any form of certain knowledge grants this already: for he can only find that path by an investigation, and any form of investigation is possible only on the assumption at all events of formal principles of judgment by which one combination of ideas can be distinguished as true from a second as false, or from a third which is doubtful. And again he who denies that such a way out is to be found, in the very act of denial acknowledges that which he denies. When the old Sophistic taught that there was no truth, and that if there were it could not be known, and that even if it could be known still it could not be communicated, in so doing it contradicted each of the propositions enunciated. For after all it gives out its three propositions for truth, and could not therefore deny all truth; it endeavoured further to prove the soundness of its contentions, and was bound therefore in its own interest to presuppose the validity of that particular form of the apprehension of truth—mediate apprehension—the impossibility of which it would have been most especially pleasant to point out; finally it denied the possibility of communicating truth, at the very moment when on the strength of its being communicable, it was setting itself to convince men of the truth of its own tenets.

Nor can these contradictions be escaped by avoiding the form of positive assertion in the expression of the results of the doctrine, and instead of denying the validity of any asserted truth simply returning a *non liquet* to the general question equally with all particular ones. Certainly those who adopt this course, and we along with them, are at liberty to give this answer where the question concerns the proof of particular contentions from truths whose validity is acknowledged; but to maintain that the validity of all truths whatsoever is doubtful is a proposition which may indeed be expressed in words, but the words have no longer any real idea to answer to them; we could not possibly explain the meaning of that *liquet* which we are negativing, if we had not in our mind certain conditions under which we should be prepared to affirm it, that is to say if we did not presuppose some unconditionally valid truth, from which is derived our right to doubt whatever cannot be proved to be in agreement with it.

But not only is any sceptical conclusion, in whatever form maintained, impossible without this assumption, but the very fact of doubt itself is impossible also—impossible at least in the only sense in which we are here concerned with it. Uncertainty indeed there would be, not sometimes, but at least as concerns the future, always and invariably, if there were no truth to teach us to distinguish between what is necessary and what is not; but on the other hand we should never in that case have occasion to raise the doubt whether a given proposition holds as tried by this or that standard, inasmuch as it would be a matter of indifference whether it did so or not, unless the standard in question be recognised as really such, as a veritable criterion, in a word as truth.

However thorough-going then the claims of Scepticism may be, still it can never get rid, not only of the recognition of some absolutely valid truth, but of this presupposition also, that the human mind is in possession of certain fundamental principles which enable it to affirm at all events the impossibility of proving this or that given conception to agree with the truth which the sceptic recognises.

303. To this admission the sceptical mind readily allows itself to be driven; it has it will acknowledge a profound belief that there is some absolutely valid truth; and again it will grant that necessary laws of thought rule all our enquiries and all our doubts: the question which troubles it is whether the two—the truth and the laws of thought—coincide. Just because we know that there must be truth, and therefore that there may be error, how are we to be sure that those necessary laws which exist in our mind may not belong to the side of error, and everything therefore be quite different in itself from that which by the laws of thought it necessarily appears to us?

It is clear that a scepticism such as this, which is not driven to doubt through any special cause residing in the nature of its subject-matter, but which simply looks upon the possibility of raising a doubt as ground sufficient for actually raising it, can never admit of being refuted by demonstration. For every argument which can be brought into the field against it can only rest upon the self-evidence and necessity with which it is thought, and must belong therefore to that sphere of necessities of thought as to which the old barren question can always be renewed to infinity, whether after all things may not be in reality quite otherwise than thought makes them.

hilosophy; at the beginning of the modern era by Des-
after convincing himself as he thought that the soul is
ith an equipment of innate necessary Ideas, presented the
the following vivid form:—might not an evil Demon
stituted our nature, that all our thoughts should be ne-
se, and yet appear to ourselves clear and necessary truths?
pothesis he considered he could only refute by pointing
that among these innate Ideas is to be found the con-
an absolutely holy and perfect God, but, he argued a
could not have constructed out of itself that which is
itself, the thought of the Infinite; only an actually
God could have implanted this in us, and it would con-
ature of this holy God to practise a deception upon us.
e feature in this demonstration which is worthy of atten-
derlying thought that in the immediate assurance which
e significance of the moral Idea lies the security also for
our knowledge: but certainly the off-hand way in which
thrown together here in Descartes' conclusion will con-
e. For after all what exception can justly be taken to
us views, which also set out from the belief in a holy
it perfectly compatible with the purposes of His wisdom
tion of mankind, that He should have wholly withheld a
of the truth from our human knowledge? And sup-
He had denied to us not a portion of truth only but all
place of it had furnished our soul with imaginations
re necessities of thought, what right should we have had
withholding of truth and bestowal of error by the hard
eception, until we had first proved that our soul pos-
ht to the grant of truth which God could not disregard
udice to His own holiness, and that the apprehension of
as it really is, was the necessary prerequisite for the
those purposes which we believe that in His holiness He
ccomplish? Such proof Descartes has neither furnished
d to furnish; he abandons himself in the above line of
all confidence to the guidance of certain assumptions
heir limited place in determining questions within the
intercourse of human beings with each other, but be-

would not hinder us in fact from assuming, not indeed that a malicious Demon, but that some creative power had so fashioned us that all things should actually appear to us by a necessity of our thought otherwise than they are.

Two alternatives are open to us. First we can if we please leave any person who is disposed to assent to such a hypothesis, to himself, on the ground that we acknowledge the impossibility of refuting him, so long as his doubts are suggested not by any positive difficulty which renders them irresistible, but merely by the possibility of continually renewing them without any positive ground whatever. In the presence of this sceptical disposition we should fall back for the purposes of science upon a principle from which in the ordinary affairs of life our opponent himself cannot escape and does not shrink,—faith in reason. We should continue to regard a necessity of thought as true until through the conclusions which it itself produces it proves itself to be no such thing, and compels us to declare it a 'show of being' only, which in such case would be not entirely a vain show but an appearance standing in a definable relation to the truth with which it can no longer be identified. This attitude towards the sceptic is that which we find observed in life, for through the world's history this groundless scepticism has always reappeared from time to time, but as often as it has made its appearance men have simply turned their backs upon it.

But in science such a treatment of the question is not altogether becoming. The second alternative appears to me the more helpful one, to lay bare the essential groundlessness of this curious solicitude, which asks whether after all things may not be quite other in themselves than that which by the laws of our thought they necessarily appear. What after all is the meaning of this addition 'in themselves,' or this being in itself of something which we oppose to our necessary conceptions of the very same something and which is supposed to be different from them? We are here in fact, as we now propose to show, in presence of a prejudice springing up from the accumulated effects of experience and education, which has crept into the heart of that very Scepticism which conceives itself to have got rid of all prejudices.

304. He who begins to reflect upon the foundations and the sources of his knowledge finds himself at starting entangled in all the prejudices which have grown up in him unconsciously as his mind has developed, whether arising out of his individual experience, or accepted from others. For the first attitude of the mind can never be doubt;

it begins always with entire confidence in all its perceptions. Now no one of these prejudices is more universal than the conception of an independent world of things with which we habitually contrast our own world of thoughts. Errors which meet us within this latter world we regard as trifling blemishes easily cured, in comparison with the great and dread delusion in which it may be the entire system of the world of thought is involved as judged by that other world of actual things.

The doubting question, therefore, whether things may not be in fact quite different from what they necessarily appear to us, has *prima facie* an intelligible sense only upon the assumption that human knowledge is intended to be a copy of a world of things, and in fact that truth regarding the possibility of which for man uncertainty is felt, has been most commonly defined as the agreement of our ideas with the real condition of the things which they profess to copy. The ordinary consciousness in practical life never departs from this standpoint; philosophy, on the other hand, in the course of its speculations has abandoned it not unfrequently on the strength of knowledge of which it believed itself to be already in possession. But a scepticism which in an enquiry into the possibility of knowledge professed to renounce all prejudices, was bound above all things not to retain unquestioned a definition of the truth of which it was in search, founded upon the uncriticised prejudice that there is such an external world of things. To dispute that this assumption is a prejudice is possible only for one who never raises a doubt at all, but who feels so complete a satisfaction in the direct deliverances of simple perception as to find in it at once a convincing evidence of the existence of the external world, and an infallible revelation of its nature. But he who once entertains a doubt of the truth of any perception, and at the same time holds fast as if it were a matter of course to the assumption of the existence of the fact, to which the perception ought by rights to correspond, can, to begin with, only be raising such a doubt at all, on the strength of definite convictions as to the nature of the 'fact' in question, convictions which appear to him to be necessities of Thought, and which forbid him to take the given perception as its true representation. But further, as he can no longer regard the thing itself as given him by direct perception, it follows that the obligation to retain the belief in its existence at all can in its turn only rest upon an innate necessity of thought compelling him to supplement and complete the manifold world of perception by this thought of a world not perceived, in order to bring his ideas in their totality into an inward harmony in agreement with the laws of his own thought.

A philosophical review of these questions is necessary, not indeed to establish our immediate faith in this world of actual things, but to give us scientific justification for holding to the assumption of its reality, and on this point the systems of Idealism and Realism have arrived at opposite results. To bring so comprehensive a problem to an issue is not in the least our business here; on the contrary it is our purpose to show that as a matter of arrangement the question ought not to be imported into these introductory discussions on the theory of knowledge. With this view we have to consider a single thought in two aspects; we have to remind ourselves in the first place that any decision of the question postulates the recognition of the competence of thought, secondly we have to show that nothing else but the connexion of our ideas with each other can ever be made the object of our investigations.

305. A few words will suffice for the first point. Every criticism of the entire apparatus of our faculties of knowledge *P*, undertaken with a view of enquiring into its agreement with the nature of things, must presuppose in order to its decision a second source of truth *Q*, which gives us a knowledge, free from all alloy of error, of what that nature is: for we can only compare known with known, not known with unknown. Supposing now that this *Q* were given us, it matters not whether in the form of a comprehensive revelation imparted originally to our soul, or as a certainty coming suddenly upon us as an answer to particular questions one by one as they present themselves, in either case how are we to compare it with the claims of *P* which requires us to connect our single ideas according to determinate laws?

If *P* and *Q* agree, how could we distinguish the one from the other, in order to convince ourselves that it is not only our subjective cognition *P* which is speaking to us, but that it has the additional confirmation of the higher objective truth *Q*, evidencing its agreement with the things themselves? We could not do it at all; the united utterance of the two together would be liable to precisely the same doubts and questionings as that of *P* by itself. If on the other hand *Q* told us something different from *P*, how should we decide between them? Even supposing that as a matter of fact *Q* gave us truth and *P* error, how else could our faith in the superior credibility of *Q* be arrived at, except through the greater immediate certainty which attended its utterance as compared with that of *P*? But this certainty is inconceivable, except on the condition of *Q* coinciding with that very truth which constitutes the universal law of our subjective faculty of cognition *P*; what contradicts this would, even if it were given us in immediate perception, be always a riddle to us, and not a revelation. If then *Q* and *P* remain opposed, what we arrive at is not a refutation of *P* on the strength of the higher claims of *Q*, but we experience an inner conflict between two utterances of that faculty of knowledge which is peculiar to our minds, a conflict which either can find no higher court to appeal to, and in that case can never be resolved, or must be settled by the discovery on the part of that faculty itself of a higher point of view within its own province from which one or other of the conflicting utterances may be corrected, and the apparent contradiction removed. We see then that to thought and its necessary laws we are as a matter of fact limited in every resort; the faith which reason entertains that truth whatever it may be is discoverable by thought, is the unavoidable postulate of all enquiry; what that truth is can be discovered only by the reflective operation of thought, continually trying and testing its single results by the standard of the universal laws of its activity.

306. It is in vain to shrink from acknowledging the circle which is here involved, for that there is no escape from it everyone after all must see. It is also superfluous, because there can never come a moment in our experience—and this is the second point we have to urge—in which the supposed mischief which our vague suspicions apprehend could possibly become known to us. All we know of the external world depends upon the ideas of it which are within us; it is so far entirely indifferent whether with Idealism we deny the existence of that world, and regard our ideas of it as alone reality, or whether we maintain with Realism the existence of things outside us which act upon our minds. On the latter hypothesis as little as the former do the things themselves pass into our knowledge; they only awaken in us Ideas, which are not Things. It is then this varied world of ideas within us, it matters not where they may have come from, which forms the sole material directly given to us, from which alone our knowledge can start. In them, and in the course which they follow as they change and connect themselves, we endeavour to discover a regular and orderly arrangement, guided in our search by those universal principles of our thought which determine for us what we are to account as order and truth, and what as involving a contradiction or a problem to the reason.

Every discovery of such a law, regulating universally and without exception any two determinate ideas[1] *B* and *F* in their conjunction with each other in our minds, is the attainment of a fragment of that which we call knowledge of Fact[2]. If we fail in our effort to discover such a constant connexion between the two, then we have a problem before us, which we always set to work to solve in the same way. First we endeavour to find some universal relation between *B* and what is contained in a third idea *M*, and between *F* and what is contained in a fourth idea *N*, and then to show that by reason of a variable relation which obtains between *M* and *N*, that between *B* and *F* cannot be expressed in the form of a simple law such as we were looking for, but only through a law of a different kind which takes account of *M* and *N* as well.

If finally we are in doubt whether a relation which we have discovered to exist between two ideas *B* and *F* in our minds, corresponds to the reality of things, this can only mean that we doubt whether whenever *B* and *F* reappear as ideas in our consciousness the relation between their contents which we have collected from only a limited number of instances, will continue to obtain universally and without exception. But if the question be once more repeated: is a relation between *B* and *F* as established for consciousness even by invariable experience also true in itself, such a question is only intelligible at all on one supposition, namely if the relation existing as a matter of fact in consciousness does not accord with the universal postulates of thought,—those postulates which thought constrains us to make in the case of any relation between any two objects whatever, and therefore also of those which we are endeavouring to conceive as obtaining between real existences independent of ourselves. It is not this assumed external world of the Real which comes in here between our ideas as the standard by which their truth is to be measured; the standard is always the conception of which we cannot get rid, of what such a world must be if it *does* exist, the conception that is to say of a thought in our own minds; this it is by which we measure the truth of other thoughts, whether they contain the evidence in themselves, or are such as to require elucidation from without.

307. It is perhaps superfluous, but it may not be without its use, to repeat this simple argument, starting from the opposite side, and to ask what it is that must happen if we are to discover a supposed piece of knowledge *Z* to be a delusion. Suppose we knew from our own observation that between two ideas *B* and *F* frequently recurring in our experience the relation *Z* does not uniformly obtain, but on the contrary varies according to the varying relations in which *B* is found conjoined with *M*, and *F* with *N*. Suppose on the other hand that another human being lived within a sphere of experience where those conditions exclusively obtain under which the relation *Z* between our *B* and *F* does become a necessity. It will follow that he will never have occasion to doubt the universality of that relation *Z*, nor will his faith in it prejudice the coherence of the rest of the world of his ideas, provided only that *Z* does not conflict with the universal laws of his thought. Unquestionably the assumption that *Z* is an absolute relation between *B* and *F* independently of further conditions, may make it much more difficult for him to find a simple law for the relations between other constituent elements of his experience, as *C* and *E*, which he would at once discover if he was aware of the dependence of *Z* upon conditions which do also in fact determine the relation between *C* and *E*. But so long as he does not extend his faith in *Z* beyond the objects contained in the world of his ideas, he will still be in a position to systematise the objects connected in that world, however awkwardly he may have to express their connexion. We indeed who possess the experience which he lacks are aware of his error, but we can only convince him of it by taking him out of his more limited circle of experience and transplanting him into a wider. Then when he himself finds fresh conjunctions of ideas arising in his mind distinct from those which he formerly experienced, he will allow that he has been in error; still all he will have to concede will be that he was mistaken in supposing the relation *Z* between *B* and *F* to obtain universally; that relation still holds true when the conditions are added upon which, though unawares to him, its validity all the while depended.

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[1] ['Vorstellungen.']

[2] [Sache.]

And now how will it be if we place human reason as such in the position of this unfavourably situated observer, and imagine it confined to a mode of mental representation, coherent indeed in itself, but not corresponding to the real relations which obtain in a world of things external to it? How is the standing delusion, in which in that case we are all involved, to become known to us, or how will our knowledge suffer supposing it to continue? Setting aside for the moment the instruction which an angel from heaven might impart to us, what we find is this: it is certainly not the things themselves which are here making their way all of a sudden into the midst of our thoughts, and laying bare their falsity; even if the world of things running its independent course were to enter some day upon a new arrangement which diametrically contradicted the conceptions which we had previously formed of it, such contradiction could only come within our observation through the new influences awakening in us a set of ideas which we find no longer to observe those laws of combination which we had hitherto assumed to be their laws. Then we have fallen into one of those errors of the understanding to which we of course allow that we are liable; we have wrongly interpreted this variable world of ideas, that world which is the sole material that lies open to our intelligence; we recognise now that we have learnt a new lesson, and that the proposition Z does not possess the universal validity with which we had credited it; but we have learnt also that it does continue to be valid still when the conditions of that validity with which we have now become acquainted are reckoned in. And now the universal validity of Z being erroneous, so also its limited validity is true, and we come to see that inasmuch as error can only be observable by us through an inner contradiction in our world of ideas, it follows that the recognition of truth itself consists only in the discovery of laws of connexion which this ideal world is destined always to observe, to however infinite a distance we may imagine its varying course to be prolonged. Undoubtedly the discovery of these laws is an undertaking which must remain incomplete; we are not in possession of the whole truth, we are in search of it; still so often as we correct a previous belief Z at the instance of fresh experiences in our world of thought, we have not indeed reached as yet the full truth, but we have removed the errors which without such correction would have lasted on.

308. This argument, unless I am much deceived, will satisfy no one. We are left after all, it will be urged, even if all inner contradictions are removed, walled in within the all-embracing delusions of those ideas which have grown up into a solid mass within us, and never see the truth in itself, but only as it necessarily appears to us. Now then let us call in our angel from heaven, who beholds from his purer atmosphere things as they are. What a shock we fancy it would be to us if all at once he withdrew the veil from our eyes, and we saw how entirely different things really are from what we had imagined them.

And indeed we should experience a very agreeable shock if in that moment it were revealed to us how our old ideas, with all their old

But also on this condition only. If it were an entirely new world which rose upon our view, bearing no resemblance and no relation to that in which we had lived before, we simply should not perceive that everything was different from what we thought; for what that meant to us was that everything we thought was different from what we thought it; the wholly new spectacle admitting, as it would, of no comparison with the former one, could on this ground at all events give us no shock at all, pleasurable or otherwise; it could not so much as occasion us surprise, except through a sense of contrast, that is to say by being brought into relation with our previous world of illusion. But again we who now see ought to be the same persons who before were blind. If that moment of revelation had at the same time transformed the laws of our thought, and altered the conditions under which hitherto we had distinguished truth and error, we should indeed, if our newly discovered world completely answered to these new conditions of truth, have no occasion to doubt about any particular fact in it; but what could preserve us from the grand fundamental doubt, whether this new world of ideas with all its self-consistency may not in its turn distort the true nature of reality, and things be once more quite different in themselves from that which in this new world they appear to be? Do you hope to exclude these doubts on the ground that on our own assumption it is the truth of things themselves, and nothing else, which makes up the content of our new perceptions? But to exclude the possibility of doubt, the fact that our representation of things *is* the true one would not of itself suffice. We must also possess means to arrive at a certain *knowledge* that it is the true one. Now such means we do possess in regard to particular parts of our experience; we can measure their truth by asking, are they as judged by the universal laws of our thought in harmony with the rest of that same experience? But it is impossible to test the truth of the entire world of our ideas as such by comparing it with a reality which so long as it is not an object of knowledge is for us non-existent, and if once it becomes so must be subject to the same doubts and uncertainties to which all ideas simply as such are liable.

And finally the supposed case is in itself impossible and absurd. What can be the meaning of saying that this higher intuition, perception, cognition, gives the thing in itself, as it really is? We may exalt the intelligence of more perfect beings above our own as high as we please; but so long as we desire to attach any rational meaning to it, it must always fall under some category of knowledge or direct perception, or cognition, that is to say it will never *be* the thing itself but only an aggregate of ideas *about* the thing. Nothing is simpler than to convince ourselves that every apprehending intelligence can only see things as they look to it when it perceives them, not as they look when no one perceives them; he who demands a knowledge which should be more than a perfectly connected and consistent system of ideas about the thing, a knowledge which should actually exhaust the thing itself, is no longer asking for knowledge at all, but for something entirely unintelligible. One cannot even say that he is desiring not to know but to *be* the things themselves; for in fact he would not even so reach his goal. Could he arrive at *being* in some way or another that very metal in itself, the knowledge of which in the way of ideas does not content him; well, he would *be* metal it is true, but he would be further off than ever from apprehending himself as the metal which he had become. Or supposing that a higher power gave him back his intelligence while he still remained metal, even then in his new character of intelligent metal he would still only apprehend himself in such wise as he would be represented to himself in his own ideas, not as he would be apart from such representation.

309. In dealing with these fundamental questions I ought not to be blamed for the lengthened discussion which I have permitted myself. It is true the outcome is small. We have convinced ourselves that this changing world of our ideas is the sole material given us to work upon; that truth and the knowledge of truth consist only in the laws of interconnexion which are found to obtain universally within a given set of ideas, and are confirmed as often as those ideas recur in our consciousness; that as the thoughts which lead us towards this order of truths make way, the antithesis between our ideas and the objects to which we conceive them to be directed, itself a part of that same world of ideas, necessarily arises; that the question as to the truth of this antithesis, and the value which according as we answer it will belong to our ideas, is a question of metaphysics which has no business to be mixed up with an introduction to the theory of knowledge such as the present; that in regard to this or that among our thoughts we may doubt as to the possibility of bringing it into harmony with the rest of the content of our consciousness, and that such doubts resting on definite grounds are compatible with the endeavour gradually to remove them; that on the other hand a scepticism which indulges the apprehension that everything may be in reality quite different from what it necessarily appears, sets out with a self-contradiction, because it silently takes for granted the possibility of an apprehension which does not apprehend things but is itself things, and then goes on to question whether this impossible perfection is allotted to our intelligence. Finally we see that if we set aside this inadmissible relation of the world of ideas to a foreign world of objects, there still remains a further line of enquiry open to us,—the endeavour to discover within the world of ideas itself what are the fixed points, the primary certainties, starting from which we may be enabled to bring the rest of the shifting multitude of its ideas into something like orderly connexion. I shall find and shall avail myself of various opportunities hereafter for elucidating this point of view; I go on at present to glance at the different methods of procedure which the sceptical philosophy has followed, and which have been pursued in the various departments of enquiry to which they have been applied, upon the whole with greater completeness in antiquity than in modern times, when many of its questions are no longer able to excite an active interest.

310. Sextus Empiricus has left us a *collectanea* of the tenets of Scepticism down to his own time. The Sceptic does not any more than other men deny the sensuous perceptions, the feelings of pleasure and pain, which we experience. They force themselves irresistibly upon him, and are independent of his opinion. On the other hand everything that is contrasted with these phenomena, as a noumenon, or as a thought, which itself not given in the phenomenon, seeks to bring the content of the perceptions into some inner connexion,—all this is made open to doubt, and any statement we venture from this point of view may be met with equal propriety by another which contradicts it. Nothing therefore remains for the wise man but to refrain from either affirming or denying either the one proposition or the other, and to find in this suspension of judgment that peace of mind which so long as he considers it his duty to decide between two conflicting hypotheses, he must necessarily seek in vain.

But when Scepticism, not content with representing an abstention from any affirmation as the condition of mind actually found in its adherents, undertakes to prove it on logical grounds to be the only legitimate attitude of the mind, it becomes at the very outset false to itself, presupposing as it does at all events at this point the truth of those logical laws of thought by which alone it can establish the cogency of its own reasonings. And not only so but in its efforts to expose the impossibility of dogmatic statements it is compelled to assume a variety of dogmas which can never be directly given in phenomenal experience, but can only be derived from them by those very processes of reasoning whose legitimacy is contested. The ten τρόποι or logical grounds of doubt, which Sextus begins by rehearsing, all come to this, that sensations by themselves cannot discover to us what is the nature of the object which excites them. The first τρόπος calls attention to the different organisations of different animals; when it goes on further to the proposition that by reason of this any object must appear different to the senses of one animal from what it does to those of another, it appeals to the Dogma that unlikes cannot be affected alike by likes. Nothing short of this argument would have justified his conclusion, for as we cannot place ourselves inside an animal's consciousness, this supposed difference between the sensation of one animal and that of another is a conclusion given by reasoning, which can never be established by immediate perception. More than this, the argument affirms too much; there is nothing to prove that visible differences in bodily organisation are an invariable indication of corresponding differences of feeling; no one will easily believe that a cat, by reason of its elliptically shaped pupil, must necessarily perceive the world of space differently to a man with his circular one.

The second τρόπος applies the same argument within the circle of human beings. They too in their turn are variously organised; if then, the τρόπος argues, it were proposed—though we have no grounds for so doing—to give the human sensations a preference over the animal, and to regard these as true and adequate to the thing itself, we are again defeated by reason of the individual differences which exist between man and man. So that all we can say is, that to one man the thing appears in one way, to his neighbour in another way; how it is in itself remains unsettled.

The two next τρόποι lead to the same result; the third appeals to the differences among the senses themselves; to the eye honey is yellow, to the tongue sweet; it may be that there are other forms of sensation, lacking to us, to which it appears something different again: what it is in itself must therefore be relegated to uncertainty, as we have no reason for accounting the deliverance of one sense truer than that of another. Even supposing however that we keep to a single sense, the fourth τρόπος points out that here too there are variations of feeling, according to age and state of health, according as we are hungry or satisfied, asleep or awake, so that still we can only say how a thing appears to our sense under each of these varying con-ɔf one and the same thing: the sixth points out that no obje duces in us the impression of itself unmixed with those of other seventh that the composition of various elements in single causes qualities to appear in them which are entirely wanting simple elements themselves, and effaces others which belon them; so that we can never do more than state how eac appears in its several combinations with other things, nor is like in itself and by itself apart from the various phases by reason of those relations it passes through. It is impossi read the examples to these last *τρόποι* without a feeling of ast ment that the scepticism of antiquity should have seen in throughout only impediments to scientific knowledge. In n science they have become one and all starting-points of er Modern science has not been content with raising a general l tation over the changeableness of phenomena under changing ditions; it has questioned experience; it has enquired what a special connexions which obtain between any one of these con and this or that particular change in the phenomenon, and it this way arrived at a knowledge of the general laws which this endlessly changing play of events. We have not indeed what a thing is like in itself when it stands wholly apart from conditions of its manifestation to intelligence; but that the pr so presented is absurd the ancient scepticism was itself awa we find it expressed in the eighth *τρόπος*: Everything sta relations of one kind or another, if not to other things, yet alw least if it is to become an object of apprehension, to the s apprehending it; what it is like in itself, apart from all rel remains therefore beyond our power to say.

The last two *τρόποι* are of less interest for us: the ninth re here pass over. It will here be evident that so far the Scepticism we have been considering does not deny the reality of truth, for it is the impossibility of attaining to truth which it laments over, and one can only make that an object of quest in the reality of which one believes. Nor does it doubt that conformity to the laws of thought is the necessary condition of any thought being true. It is incessantly enumerating, in disjunctions alleged to be complete, sets of cases which are inferred on the basis of these laws to be possible and to exclude one another; and it is by this same logic of thought that it undertakes to bring us to acknowledge the necessity of withholding judgment altogether. It is true that this procedure has to be subsequently corrected. The sceptical argument is at pains to include itself in the uncertainty to which, by one of those very affirmations which it seeks to get rid of, it condemns the whole of our pretended knowledge. The forms of argument which are employed for this purpose are many and curious. If the Sceptic arrives at his negations by a process of demonstration, he is not, it is said, in so doing, here any more than elsewhere, laying down any positive doctrine; he is simply stating that to him, here and now, at this particular moment of his life, and in the particular state of mind in which he happens to be, the opinion which he has announced appears to be the true one. He does not guarantee its continuing so to appear even to himself at every future moment; if he is driven to acknowledge some one else's argument to be convincing, he can always answer, the truth *Z* which this man teaches, has up to this moment been unrecognised, yet all the same if it is truth it has been so always and been always valid; and where is our security that some third person may not hereafter discover and demonstrate a new truth to upset *Z* in its turn, which at the present moment, though it already holds good, is neither recognised nor capable of being either apprehended or proved.

These questions are independent of the relation of our knowledge to an object outside itself; they concern the ground of certainty generally, and our right to the confidence we repose in the truth of any thought in our minds; in this view we reserve them for consideration later. But apart from this the arguments of Sextus involve at once a prejudice and a fallacy; the prejudice of the existence of that World in itself with which knowledge was contrasted, a prejudice which may be just or the contrary—that cannot be decided here—the fallacy that the conception of a knowledge which apprehends things not as they are known but as they are, means anything intelligible at all, as to the possession or non-possession of which it is possible to raise a controversy; whereas the truth is that upon this at least thought is perfectly clear and at one with itself, that knowledge under whatever form can never *be* things in themselves but only represent them.

312. There will be a disposition to express this contention in the form that we only know phenomena, and not the essence of things in themselves, and so stated to recognise it as the primary truth of every theory of cognition. I avoid that particular form of statement because it still contains a prejudice which I should wish to see abandoned. The actual assumption indeed of the existence of this world of things which is given by the categorical form of the proposition might be avoided by transforming it into the hypothetical: *If* things exist knowledge apprehends only their appearance, not their essence. But even then the proposition plainly carries the idea of a thwarted purpose. That 'only' implies that our knowledge which was intended by rights for the apprehension of the higher, the essence of things, has to be content with the lower, the phenomenon. Such a valuation is once more a prejudice, it may be legitimate, it may be not legitimate, as the further progress of Science may decide which we are not here in a position to anticipate. But we can see at once that it is an arbitrary proceeding to place knowledge in the position of a means which is *not* adequate to its supposed end of apprehending things as they are. And we may at once pronounce an opposite point of view to be conceivable, which should regard things as mere means to produce in us in all its details the spectacle of the ideal world. If this were so we should not indeed know things as they are but we should not therefore fail of any end or aim; in the appearances which things present to us would reside then that element of higher dignity and value which we sought to indicate by the name of essence; and in the discovery of the purport, the connexion and the laws which govern this inner world of phenomena, the knowledge of truth would lie not indeed exclusively but pre-eminently, and at least as truly as in that which we are now so painfully anxious to arrive at, the apprehension of that which must always remain outside our own and every other intelligence, the system of means through which the series of phenomena is called into existence within us. But to continue this discussion further would be to overstep the limits of my undertaking. I repeat once more what I desire the reader to understand this to be; let us leave entirely out of the question the opposition between our world of ideas and a world of things; let us look upon the former alone as the material we have to deal with; and let us endeavour to ascertain where within this world the primary fixed points of certainty are to be found, and how it may be possible to communicate a like certainty through the medium of these to other ideas which do not in themselves equally possess it. By following certain circuitous paths which will be found to be no deviation from our proper route, we may perhaps arrive at clearness on this subject.

CHAPTER II.

The World of Ideas.

313. THE problem which we have set before us is one which ancient philosophy long ago declared again and again to be insoluble. That all is in flux was the familiar doctrine of Heraclitus, a doctrine however of which it is difficult to determine the precise significance. That it was understood in the half pathetic sense of a lamentation over the rapidity of change appears in the heightened form subsequently given to his saying that it is impossible to cross the same river twice—'it is impossible,' it was added, 'even once.' But against the testimony of observation to the transitoriness of things the most ordinary experience might have set counter examples of duration through incalculable periods of time; philosophical reflexion could only have universalised the former set of experiences into the doctrine cited by establishing in opposition to superficial appearances, that the latter also do but veil a slow process of change to which in fact they are always subject. We do not know how far this actually took place and whether these speculations passed over without notice the circumstance that the differences in the speed of one set of changes and another at once introduce into the play of phenomena a contrast between the relatively fixed and the more transitory which might be turned to fruitful account. Once more, that nothing can wholly withstand agencies of change operating from without, that everything therefore must be *susceptible* of change, is a conviction too easily derived from the experience of every-day life to have needed a philosophy to discover it. But it remains doubtful how far Heraclitus passed beyond this, and taught that there are changes in all things springing from causes in their own nature and not merely occasioned by outside influences, and whether he taught this simply as a fact of experience, or whether he held continual movement to be the condition of the possibility of all natural existence, and that stable,

There is much to lend probability to a view which should credit him with this more advanced conception, but the question can as little be certainly decided as the more important one what precisely is to be understood by the 'all' to which he ascribed this ceaseless mutability. The expression included beyond question the things of sense; in fact the very starting-point of the doctrine could have been found nowhere else but in the changing combinations of sensible qualities and relations. But did it include at the same time the content of the ideas by means of which we think this world of sense? Was it intended that not only all that is real but all that can be thought as well is subject to this eternal flux? I doubt if Heraclitus held this latter opinion; the universal instability of all determinations of thought would of course render all enquiry and all affirmation impossible. We may however assume from the lively picture which Plato draws in the Theaetetus of the later activity of the school, that they at all events had no hesitation in giving this extension to their master's doctrine.

At this point it is taken up by the Sophists. I do not mean that section which under the leadership of Protagoras acknowledged only the subjective validity of every perception for the person who experiences it, I mean those who, disciplined in the Eleatic dialectic, set themselves to demonstrate that every conception signifies at once what it does mean and what it does not mean. This contention was met, principally in the field of Ethics, where it produced its most pernicious effects, by the sound instinct and sense of truth of Socrates, who called attention to the fact that the conceptions of good and bad, just and unjust, are fixed and unchanging, and cannot be determined now one way and now another at the pleasure of individuals, but that they have to be accepted as permanent and self-identical conceptions to which everyone has simply to subordinate his own ideas on these subjects. Plato followed, at one with these aims of his master, but impelled by more many-sided motives, and expanded the convictions received from Socrates into his own doctrine of Ideas, a first and most characteristic attempt to turn to account the truth which belongs to the world of our ideas in itself, without regard to its agreement with an assumed reality of things outside its borders. The philosophical efforts of antiquity have the attraction of exhibiting in full detail the movements, the struggles, and the errors of thought, into which every individual still falls in the course of his development, and which notwithstanding the culture of our own day has no longer the patience to follow up and investigate. I shall permit myself to enter

points which seem pertinent to our present enquiry.
The Platonic expression Idea[1] is usually rendered Universal
on[2], and so far approximately that there are Ideas, according
, of everything which can be thought in a universal form,
›m the particular perceptions in which it is presented. At
: time it is only for the purposes of a later set of conceptions
e shall meet with presently, that it becomes important to be
hink of the ideally apprehended content[3] as something com-
many individual contents, that is as a universal. What is
here at the outset is not so much that it can be separated
'erent particular instances which contain it, as that it has been
shed as a content with a meaning of its own which we *pre-*
urselves, from a mere affection which we *experience.* In the
ıse it might have been involved by the Heraclitic or pseudo-
c doctrine in its ceaseless flux of events, of which each one only
: moment in which it occurs, and no one has an abiding
n or significance in the world, because there is no reason
ng once occurred it need ever recur again in identically the
m. The former conception on the contrary turns the mere
of our sensibility into an independent objective[4] content
gnificance once is its significance once for all, and whose
to other contents have an eternal and self-identical validity even
it nor they should ever be repeated in actual perception.
had occasion to explain my meaning here in an earlier part
›rk (p. 11 ff.). Perception shows us the things of sense under-
ınges in their qualities. But while black becomes white and
ır, it is not blackness itself which passes into whiteness, nor
ɜtness become sourness; what happens is that these several
each remaining eternally identical with itself, succeed each
he thing, and the conceptions through which we think the
ve themselves no part in the mutability which we attribute
nt of their changes to the things of which the qualities are

supposed to have changed. It is a very simple and unpretending, but yet a very important thought to which Plato here gives expression for the first time. The continual change which goes on in the external world may affect us like a restless whirling eddy, bewildering our intelligence, yet it is not without a pervading truth. Whatever mutability the things may display, that which they are at each moment they are by a transient participation in conceptions which are not transient but for ever identical and constant, and which taken together constitute an unchangeable system of thought, and form the first adequate and solid beginnings of a permanent knowledge.

For it was one of the conclusions at which we arrived before[1], that to the making of this earliest immediate stock of knowledge there contribute not merely the separate unity of each conception in itself, nor again simply the fact of a mere uniform contrast between this and all other conceptions, but also those graduated relations of resemblance and affinity in which different conceptions stand to each other. If the white becomes black and the sweet sour they do not merely become different in the abstract, but pass over from the domain of the one conception in which they participated before into that of another which is separated from the first by a fixed and determinate degree of contrast, a contrast stronger for example than that which obtains between white and yellow, and altogether incommensurable with that absolute gulf of separation which exists between white and sour.

315. I refer to these simple examples once more in order to make it clear how a knowledge may be possible, the truth of which is wholly independent of the question of Scepticism as to its agreement with a world of things outside it. If the current of the outer world had brought before us only once in a transient appearance the perception of two colours or two sounds, our thought would immediately separate them from the moment of time at which they appeared, and fix them and their affinities and their contrasts as an abiding object of inner contemplation, no matter whether they were ever presented to us again in actual experience or not. Again supposing we could never learn how these ideas are able to appear as predicates in things, and in what that which we have called the participation of things in them exactly consists, a question would indeed be left unanswered which might in the course of our reflexions prove important, but still the certain knowledge would remain to us undisturbed that the series of colours and the scale of musical tones themselves are each a connected whole with fixed laws, and that in regard to the relation of the members to each other, eternally valid true propositions are vitally opposed to eternally invalid false ones. And finally the question whether after all colours in themselves and tones in themselves are not different from what they appear to us, is one which no one will care to raise again. Or rather we do meet with it again in the confused notion that sounds are in fact merely vibrations of the air, colours merely quiverings of the ether, and it is only to us that they appear in the form of the subjective feelings which we know. It is unnecessary to enlarge over again on the consideration that these feelings do not cease to be real, and are not got rid of and banished out of existence as intruders, because we have discovered certain external causes not resembling them, which are the occasions of their making their appearance to us. Even if these vibrations of external media appeared to differently constituted beings in the form of modes of sensation entirely unknown to us, still the colouring and tones which we see and hear, would constitute for us, when once we have experienced them, a secure treasure of knowledge with a validity and an orderly connexion of its own. The feelings of such other beings would remain unknown to us and ours to them, but this would only mean that we have not *all* truths for our portion, but that what we do possess we possess *as* truths in virtue of the identity of every such content of perception with itself, and of the constancy of identical relations which obtain between different contents. Thus we readily understand the significance of Plato's endeavour to bind together the predicates which are found in the things of the external world in continual change, into a determinate and articulated whole, and how he saw in this world of Ideas the true beginnings of certain knowledge; for the eternal relations which subsist between different Ideas, and through which some are capable of association with each other and others exclude each other, form at all events the limits within which what is to be *possible* in experience falls; the further question what is real in it, and how things manage to have Ideas for *their* predicates, after appeared to Plato not to be the primary question, and was for the time reserved.

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[1] [Cp. p. 24 ff.]

316. There is one wide-reaching difficulty connected with the first-named aspect of this question. How precisely are we to conceive colours when they are not seen, or tones and their differences when the former are not heard and the latter not apprehended by comparison? Are we to say that they are nothing or that they do not exist, or are we still to attribute to them some predicate which we can hardly define, some kind of being or reality? We shall not be disposed at first to consider them to be nothing at all; for as long as we fix them in our thoughts, as at present in searching for an answer to this very question, every tone and every colour is a determinate content distinguishable from every other, and so a something and not a nothing. Still this decision becomes doubtful when we consider the answer which we feel ourselves compelled to give to the second part of the question. In regard to things we do imagine ourselves, dimly enough, to know wherein their being consists even when they are objects for no intelligence, but exist purely for themselves; but what is meant by a tone when it is heard by no ear and when even the silent idea of its sound is not called up by any mind, we can no more say than what a pain is when no one is hurt by it. But how can that which is not either in itself or in our consciousness, be any longer anything at all or be distinguished from anything else? Still this conclusion again we hesitate to affirm. There is clearly in our first conclusion, speaking quite generally, a certain element of affirmation, which is not entirely to be cancelled by the denial contained in the second. Perhaps it may appear to us a way out of the difficulty to turn the categorical form of our judgment into a hypothetical; two sounds which are neither heard nor imagined are not indeed actually anything, and stand in no actual relations, but they *will* always be something and the one will be different from the other and stand in a definite relation of contrast to it, *if* they are heard or imagined. Yet even this does not at once satisfy us, for in order even to imagine how the notes a and b can be subject to this varied fortune of being presented to imagination at one time and not so presented at another, and then how it happens that when they are presented in experience the relation Z is necessarily thought along with them, whereas whenever certain other sounds are presented, they are no less necessarily accompanied by a different relation Z^1,—in order to imagine this we are constrained to ascribe to them existence and definite existence, at a time when according to this view they did not in fact exist at all; for so alone can we explain their subsequent existence and the definite form which their relations then assumed.

I will not pursue these refinements further, but will conclude with the following remarks. We have undoubtedly a conception of affirmation or 'position' in an extremely general sense, which meets us in various fields of enquiry, and for which languages, dealing as they do in their early stages with highly complex and concrete notions, and not with the simplest elements of thought, have commonly no abstract term which expresses it with the requisite purity. But it would not be which always are what they are, no matter whether or no there are things which by participation in them make them manifest in this external world, or whether there are spirits which by thinking them, give them the reality of a mental event. But the Greek language then as afterwards was wanting in an expression for this conception of Validity as a form of Reality not including Being or Existence; and this very expression Being came, often indeed quite harmlessly, but in this instance with momentous consequences, to fill the place.

Every possible content of thought, regarded as an individual unity, distinct and separate from others, all that class of things for which the language of the School philosophy in later times invented the not inappropriate name of *Res rationis*[1] was to the Greek a Being (ὂν or οὐσία); and if the distinction between a really valid truth and a pretended truth came in question the former was distinguished as ὄντως ὄν. The language of ancient Greece never found any term to express the reality of simple Validity as distinguished from the reality of Being, and this constant confusion has prejudiced the clearness of the Platonic phraseology. ✓

318. We may easily see that everything Plato says of the Ideas presents itself when understood in the manner so explained as natural and necessary, and that the various devices to which he resorts in setting forth their nature have this purpose and no other, to exhaust the conception for which no adequate term could be found, by the help of a variety of expressions limiting and supplementing each other. Eternal, without beginning, and imperishable (αΐδια, ἀγέννητα, ἀνώλεθρα) the Ideas could not but be named in the presence of the flux of Heraclitus, which seemed in danger of sweeping them away along with the sense-world in its stream. The reality of Being indeed they have or have not, according as transient things of sense are clothed that which has no validity and could have no validity in itself by the fact of its never occurring at any moment of time.

Again, we understand the ideas being called separable or separate from things (χωρὶς τῶν ὄντων), first because the image (εἶδος) of their content can be still called up to memory after the things which originally occasioned its appearance in us have vanished from real existence, and next, because the content is taken to include what can be apprehended in a universal form, and remains the same in different external manifestations, so as to be independent of the mode in which it is realised to sense in any particular instance.

But it was not Plato's intention to represent the ideas as independent merely of things while still depending for their special mode of reality upon the mind which thinks them. Reality of Existence it is true they enjoy only in the moment in which they become, in the character of objects or creations of an act of presentation now actually occurring, members of this changing world of Being and Becoming; but on the other hand we all feel certain in the moment in which we think any truth, that we have not created it for the first time but merely recognised it; it was valid before we thought about it and will continue so without regard to any existence of whatever kind, of things or of us, whether or not it ever finds manifestation in the reality of Existence, or a place as an object of knowledge in the reality of a Thought. This is what we all believe with regard to truth when we set out to search for it, and it may be lament over its inaccessibility at least to any form of human knowledge; the truth which is never apprehended by us is valid no whit less than that small fraction of it which finds its way into our intelligence.

The independent validity of the Ideas Plato emphasises again in a somewhat different form, in answer to the doctrine of Protagoras, rescuing them, in their character of being in themselves that which they are, (αὐτὰ καθ' αὐτὰ ὄντα) from the relativity in which the famous dictum of that Sophist was in danger of involving them. Even granting that his doctrine has its truth so long as it is confined to the impressions of sense, and that viewed in this relation Plato's opposition to it rests upon a misunderstanding, granting that is to say that my sensation is as true for me as yours which differs from it, is for you, Plato would still be right in insisting that for neither of us could the sensation be possible at all, unless that which we felt in the sensation whatever it be, red or blue, sweet or bitter, had a definite and constant significance of its own, as a member of a world of Ideas. This world of Ideas is the permanent and inexhaustible treasure-house from which things exist, we can only say that it possesses Validity.

And finally we must not ask what in its turn is meant by Validity, with any idea that the meaning which the word conveys clearly to us can be deduced from some different conception; as if, for example, it were possible to find certain conditions by the operation of which either the Being which belongs to things could be so modified and attenuated, or the momentary act of Becoming or occurring, in which the transient reality of ideas regarded as excitations of our consciousness consists, could receive such fixity and independent existence, as that both the one and the other in different ways might pass into this conception of Validity, which at once excludes the substance of the valid assertion from the reality of actual beings and implies its independence of human thought. As little as we can say how it happens that anything *is* or *occurs*, so little can we explain how it comes about that a truth has Validity; the latter conception has to be regarded as much as the former as ultimate and underivable, a conception of which everyone may know what he means by it, but which cannot be constructed out of any constituent elements which do not already contain it.

317. From this point of view some light I think is thrown on a surprising statement which is handed down to us in the history of Philosophy. Plato, we are told, ascribed to the Ideas of which he had achieved the conception an existence apart from things, and yet, as these same critics tell us, of like kind with the existence of things. It is strange how peacefully the traditional admiration of the profundity of Plato acquiesces in the ascription to him of so absurd an opinion; we should have to abandon our admiration of him if this really was the doctrine that he taught, and not rather a serious misunderstanding to which in a quite intelligible and pardonable way it has laid itself open. The expression of philosophical ideas is dependent upon the capabilities of each language, and it is hardly possible, in giving utterance to our meaning, to avoid using words which language has coined to express a cognate thought only which is not our real meaning at all. And this is pre-eminently the case when a new field is being opened out, and the necessity of distinguishing the precise meaning intended from the ordinary meaning of the word is as yet little felt. This is I think the explanation of the misunderwise to invent a technical term to represent it, the meaning of which would always be doubtful, because it could never come naturally to the lips or to the thoughts of any one; the very term 'position' which is frequently used for it suggests by its etymological form the entirely alien sense of an act, or operation of establishing[1], to the execution of which that state of affirmation which we wish to express then seems to owe its being. It is best however to keep to ordinary speech, and select a word which can be shown to express in common usage, approximately at all events and unmistakeably, the thought with which we are concerned. We may express it in our own language by the term Reality[2]. For[3] we call a thing Real[4] which is, in contradistinction to another which is not; an event Real which occurs or has occurred, in contradistinction to that which does not occur; a relation Real which obtains, as opposed to one which does not obtain; lastly we call a proposition Really true which holds or is valid as opposed to one of which the validity is still doubtful. This use of language is intelligible; it shows that when we call anything Real, we mean always to *affirm* it, though in different senses according to the different forms which it assumes, but one or other of which it must necessarily assume, and of which no one is reducible to or contained in the other. For we never can get an Event out of simple Being, the reality which belongs to Things, namely Being or Existence, never belongs to Events—they do not exist but *occur*; again a Proposition neither exists like things nor occurs like events; that its meaning even obtains like a relation, can only be said if the things *exist* of which it predicates a relation; in itself, apart from all applications which may be made of it, the reality of a proposition means that it holds or is valid and that its opposite does not hold.

Now misunderstandings must always arise, when under the persuasion that the object which we are considering must have some sort of reality or affirmation proper to it, we endeavour to attribute to it not that kind of reality which is appropriate to it, but a different kind which is alien to it. Then arises the conflict just noticed between the conviction on the one hand that we are right in ascribing to it some sort of reality, and on the other that the particular form of reality to which our misconception has brought us is inadmissible.

Now Ideas, in so far as they are present in our minds, possess reality in the sense of an Event,—they *occur* in us: for as utterances minate place, no fixed relations of affinity or difference in the universal world of thought, but stands in complete isolation, bare of all relations to anything in that world, the possession of a single individual mind alone, is in fact an impossibility. ”

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[1] ['Setzung.']

[2] ['Wirklichkeit.']

[3] [Cp. 'Metaphysic,' p. 1, and for 'Objectivitat' contrasted with different forms of 'Wirklichkeit' see above, p. 11 ff.]

[4] ['Wirklich.']

While Plato by thus describing the Ideas, takes security for their independent validity, he has at the same time abundantly provided against the confusion of the validity thus implied with that wholly distinct reality of Existence which could only be ascribed to a durable thing. When he places the home of the Ideas in a super-celestial world, a world of pure intelligence (νοητὸς, ὑπερουράνιος τόπος), when again more than this he expressly describes them as having no local habitation, such language makes it abundantly clear to any one who understands the mind of Greek Antiquity, that they do *not* belong to what we call the real world. To the Greek that which is not in Space is not at all, and when Plato relegates the Ideas to a home which is not in space, he is not trying to hypostasize that which we call their mere validity into any kind of real existence, but on the contrary he is plainly seeking to guard by anticipation against any such attempt being made. Nor is it any objection that the Ideas are called unities (ἑνάδες, μονάδες), for there is no occasion to interpret these titles from an atomistic standpoint, whether in the sense of material indivisibility or of a self-identity resembling that of a self-conscious subject. For in fact what constitutes the *meaning* of an Idea, and of a complex no less than of a simple Idea, is that it manifests itself as a unity, unifying the elements which cohere in it and rejecting that which is alien to it. Nevertheless although these various expressions point one and all to the fact that Plato never asserted the existence of the Ideas but only their external validity, he had still no better answer to make to the question, what then are they, than to bring them again under the general denomination of οὐσία. Then the door was opened to the misunderstanding which has since widely spread, though no one has ever been able to say what the nature of that existence, into which he

Aristotle. For it is really the very definite language of Aristotle which has established the doctrine of the reality of the ideas as a dogma of Plato, whereas Plato's own statements are in no way inconsistent with the other interpretation which we have preferred. It seems incredible that the most acute of Plato's disciples, informed by personal intercourse with the master, should have misunderstood him in a point of such serious moment as this. At the same time we are justified by the nature of his polemic not against particular statements of Plato but against the doctrine of Ideas altogether, as well as by many details in his criticisms, in assuming that his attack is in part directed against certain misunderstandings of the Platonic doctrine which had gained hold in the Academy at an early period. For he could not well have challenged Plato himself to show *where* the Ideas are, when Plato had said in plain terms that they were nowhere. He could not have directed against Plato the criticism that there must logically be Ideas of products of art, for one passage at least is to be found in the Republic which is entirely in agreement with that criticism, and how far Plato was from having overlooked the difficulty there involved, is evidenced by the opening of the Parmenides. Finally as to Aristotle's objections to the Ideas that they are superfluous, being mere copies of individual objects, and the assumption from which his elaborate analysis frequently starts, that there are as many examples of every Idea as there are instances of its application in reality, these are criticisms which do not really apply to the doctrines of Plato himself. That every Idea is what it is once for all, that what we are to understand by it is not an individual thing but a universal comprehending many things, and that all its manifestations are only copies of this one essential reality, is the doctrine which he never abandons, whatever obscurity may still attach to that operation on the part of the individual things, described as imitation or participation, by which they provide the one Idea with a countless number of realisations in the world of actual existence.

The discussion therefore which fills the XIIth (XIIIth) book of Aristotle's Metaphysics and of which the purport is to exhibit the absurdity of attributing to the Idea a reality identical with the reality of actually existing things, I cannot regard as a refutation of the pure Platonic doctrine, and the less so inasmuch as at the end Aristotle himself equally fails to find a decisive and unambiguous expression for that more appropriate form of reality which he desires, in contradistinction to this, to ascribe to them. To him the only genuine *οὐσία* is the individual thing, and there we must certainly agree with him; to the individual thing alone belongs the reality of Existence;

still for Aristotle as much as for Plato the object of knowledge is always the universal; not only in the sense that we are incapable of exhausting the meaning of the individual thing, but that so far as we investigate it in its nature and its workings with any prospect of a result, we invariably proceed according to universal principles. But Aristotle is entirely at one with his predecessors, that that which is not, or has no reality in any sense, cannot be an object of knowledge either, and so in regard to the universal we cannot say that it simply is not, but that in a sense it is and in a sense it is not.

I do not propose to enter into Aristotle's further treatment of this question in detail. I must however remark that by placing the universal and the Idea within the Individual things and not outside them he does not explain the possibility of knowledge; for the mere fact of the presence of the Idea in one individual does not entitle us to transfer all the consequences which flow from it to a second individual in which it happens also to be found; it can only justify us in concluding from the doings of one real thing to those of another, if it includes within itself a number of characteristics so related that the appearance of any one necessarily implies the presence of the rest. Such considerations would at once conduct Aristotle back to the admission that the Idea is certainly in a sense *χωρὶς τῶν οὔτων*; but in what sense it is so was impossible for him to define, since he no more possessed than his master did a technical equivalent for our term validity; and thus eventually the universal conception or Idea came to be for him also an *οὐσία*, not indeed a true or *πρώτη οὐσία* but still a *δευτέρα οὐσία*.

320. It may appear to us a strange spectacle to see two of the greatest philosophers of antiquity struggling with imperfect success to arrive at clearness upon so simple a distinction as that which we have been considering. But such a view would do both of them injustice. The apprehension of the simplest relations of thought is not the simplest act of the faculty of thought, and the whole long history of philosophy teaches how ready we all are at any moment to be guilty of a degree of obscurity in the application of ideas which if reduced to its simplest terms would appear to us incredible. Whenever men have believed themselves to have discovered a principle which appears to represent the universal element in the constitution and development of the real world, they invariably go on to exalt it into the position of an independent reality and to represent it as a pure form of being, in comparison with which the individual things retire into a position of subordinate and even unreal existence. I need not even refer to the latest phase of German philosophy which aspired to set on the throne of the Platonic Ideas the one absolute Idea, for the same tendency is apparent enough in spheres of thought outside the circle of philosophy. How often do we hear in our own day of eternal and unchangeable laws of nature to which all phenomena and their changes are subjected; laws which would indeed cease to manifest themselves if there were no longer any things for them to control, but which would even then themselves continue in their eternal validity and would revive with their old effective power the moment a new object presented itself from any quarter for them to apply to; nay there is not even wanting on occasion, the enthronement of these laws above all existing realities in that very super-celestial habitation which with Plato is the home of the Ideas. Nevertheless those who hold this language would indignantly repel the imputation of ascribing to those laws an existence whether as things or as persons outside the things which are governed by them, and Plato may resist with equal justice a similar misinterpretation of his doctrines.

Finally it must be added that we ourselves, in drawing a distinction between the reality which belongs to the Ideas and laws and that which belongs to things, and calling the one Being or Existence[1] and the other Validity[2], have so far merely discovered, thanks to the resources of our language a convenient expression which may keep us on our guard against interchanging the two notions. The fact which the term validity expresses has lost none of that strangeness which has led to its being confounded, as we have seen, with existence. It is merely that we have been so long accustomed to it; we use our thought as we do any other natural faculty without troubling ourselves about it, and take it as a matter of course that the content of manifold perceptions and phenomena does invariably adapt itself to general conceptions and can be read by us in the light of general laws, in such wise that the consequences which those laws lead us to predict are found to coincide with the actual phenomenal order which supervenes. But that this should be the case, that there should be universal laws, which have not themselves existence like things and which nevertheless rule the operation of things,—remains for a mind which realises its meaning, a profoundly mysterious fact which might well inspire rapture and wonder in its discoverer; and that he should have made the discovery will always remain a great philosophical achievement of Plato, whatever the problems it may have left still unsolved.

321. One of these problems is that of the exact nature of the relation of things to the ideas which Plato describes by the terms participation or imitation. I do not propose at present to discuss this question at large; but there is one defect in the doctrine of the Ideas which a criticism of Aristotle's—in itself not well-founded—may suggest to us. Among the reasons which led him to regard the Ideas as both superfluous and useless, he especially emphasises the fact that they supply no beginning of motion. However true this objection may be in itself, the fact that they do not perform this task proves little against the doctrine of the Ideas; the real objection is that they do not, as we shall see, adequately perform the task for which Plato intended them. As concerns Aristotle's criticism let us turn to the sciences of our own day. What shall we say of our Laws of Nature? Do they contain in themselves a beginning of motion? On the contrary, they all presuppose a series of data which they cannot themselves establish, but from which, *once given*, the necessary connexion one with another of the phenomena which ensue is deducible. No natural law ordains that the different bodies in our planetary system should move, or that their course should be directed towards one and not another quarter of the heavens, or that the acceleration which they impose on each other by the force of attraction should have the particular amount which it has and not a different one. But is the whole system of mechanical truths useless and mere empty babble (κενολογεῖν) because it leaves these first beginnings of motion to be explained from some other source, and starting from the fact of motion as it actually finds it, is satisfied with explaining its different phases in their necessary connexion with each other? There may be obscurity enough—though after all not more than in our own mode of representing the matter—in Plato's relegation of the primary motive impulses upon which the succession of phenomena depends, to that dim world of ὕλη which represents to him the material which is given for the Ideas to be applied to. But for all that to see in the world of Ideas the patterns to which all that is, *if* anything is, must conform, was a thought of which the importance is unfairly ignored by Aristotle. For he was himself on a later occasion to have recourse to that very same thought, for the explanation of individual phenomena: he too found himself unable to allow the cause of motion which communicates the actualising impulse also to control its issue; this had been decided from all eternity by those universal laws, which in their turn take no part in the communication of the impulse.

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[1] ['Sein.']

[2] ['Geltung.']

On the other hand it must undoubtedly be admitted to be a deficiency in the Platonic doctrine that this, which was its actual undertaking, it only half accomplishes. An account of the necessary connexion of two contents of thought must always assume the logical form of a judgment; it cannot be expressed in the form of a mere notion which does not in itself contain a proposition at all. Thus we have always employed laws, that is to say propositions, which express a relation between different elements, as examples to explain the meaning of Validity in contradistinction to Existence. The term cannot be transferred to single concepts without some degree of obscurity: we can only say of concepts that they *mean* something, and they mean something because certain propositions are valid *of* them, as for example the proposition that the content of any given concept is identical with itself and stands in unchangeable relations of affinity or contrast to others. Now Plato apprehended the elements of the world of thought which he discovered almost exclusively under the form of the isolated concept or the Idea. We need not look beyond the general impression which his Dialogues leave with us to be aware how rarely by comparison we meet with general propositions; they are by no means entirely absent, on the contrary they are made on occasions the subject of important disquisitions, but that it is propositions as such and nothing else which must necessarily form the most essential constituents of the ideal world, is a truth which never forced itself upon Plato's mind. His peculiar point of view is not without modern parallels. Kant himself in his search for the *a priori* forms which were to give the unity of an inner coherence to the empirical content of our perceptions, made the mistake at starting of developing them in the form of single concepts, the Categories, and that in spite of the fact that he derived them from the forms of the judgment itself. And now having got them, as he thought, in his Categories, it became the more evident that there was nothing to be made of them, and thereupon followed the attempt to derive judgments out of them again, and so he arrived at the 'Principles of the Understanding' which it was now possible to apply as major premises to the minor premises furnished by experience. It seems therefore that this disposition to bring into the inadequate form of a single concept truths which can only be adequately expressed through the proposition, is natural to the imagination at all times, and is not peculiar to the plastic mind of ancient Greece. It may however be remarked in passing how dangerous a tendency it is, leading the mind as it does away from the full concrete reality which is the true aim of its enquiries to a barren playing with empty ideas which have become separated from their natural foundations.

Thus we find our present requirements hardly at all satisfied in ato, and even the need of satisfying them not clearly or adequately cognised. It is true the abstract thought that the Ideas are not ıly a multitude of individuals but that they make up all together ı organic and articulated whole—this thought is the soul of all his aching, and he describes with enthusiasm the delight which he finds the dialectical exercise of resolving the complex structure of the eal world into its elements, following the natural joinings, and then ıtting them together again; even the different degrees of agreement of contrast between individual ideas and the possible modes of ımbining them are mentioned as subjects worthy of investigation. ıt in the examples which he gives of the application of his method, e art of Dialectic ends almost invariably in a mere classification of eas, by which we are shown the place which belongs to any one ea in a system of division in virtue of the elements which it comnes, but which furnishes us with no single proposition, adds no jot our knowledge concerning the nature of any one of the Ideas which ıuld not have been arrived at equally without this circuitous route of ıssification. If we want to know what can be said or cannot be said any Idea we have still to learn it, after the classification as much before, from other sources. The joinings and articulations of truth ıich Plato's sole aim was not to mutilate he ought to have investited with a firmer hand; instead of making a systematic collection the flora of the Ideas, he ought to have turned his thoughts to the neral physiological conditions which in each single plant bind limb limb according to a law of growth. Or, dropping the figure, the istence of a world of Ideas possessing a definite meaning and unchangeable validity being once clearly and emphatically establed, the next task was to investigate the universal laws which vern its structure, through which alone, in an Ideal world as ewhere, the individual elements can be bound together into a whole. (?) ıus the question to be dealt with at this point was what are those *'t principles* of our knowledge under which the manifold world of 'as has itself to be arranged. This is the more precisely defined n which the systematic enquiry into Truth and the source of Truth v assumes for us.

CHAPTER III.

The a priori and the Empirical Methods.

322. WHEN we feel in doubt about any particular point of belief within the sphere of our knowledge we endeavour to clear it up by analysing the conditions which have led us to entertain the belief; we expect to learn from the history of its origin whether it is true, or if it is false how it must have grown up. And whenever in the history of philosophy the question has arisen as to the capacity of the human mind for the attainment of truth in general, mankind have thought that the same path would lead there also to the goal. It has been supposed that the claims of our ideas and our judgments to the name of truths could be decided by considering the process by which they have been formed. This belief, which is worth considering inasmuch as it lies to a great extent at the root of certain tendencies of philosophical enquiry even in our own day, leads me to quit for the moment the subject upon which I have entered. It is necessary to attempt to point out, that this method of criticising our Ideas by tracing their genesis does not present the advantages as applied to the subject of human knowledge generally which it does undoubtedly possess in the case of particular beliefs or ideas.

For the two cases are not alike. If we desire to test the accuracy of any particular opinion, we have a basis for our decision in other truths of which we are in acknowledged possession, on the one hand general principles with which all other propositions if they are to have validity for us must be in agreement, on the other hand established facts, which must not be contradicted by those other facts which are either affirmed or assumed by the view under question; finally we have certain laws of thought by which, given certain valid premises, logical conclusions derived from them are distinguished from illogical. Throughout we start from some truth which operates upon the mixture of our thoughts which is submitted to the test like a fermenting matter, assimilating that which is akin to it, and rejecting ition of them, viz. to our supposed insight into the *origin* of our edge. The theory is that the mode in which knowledge ori- s is to decide its claims to truth, that truth moreover, as is sed by this view, having regard to a reality which is foreign to anscends knowledge. But if this is our aim we cannot move a step without making certain more definite assumptions; first he position of the knowing subject as regards those objects of owledge, next as to the nature of that relation, *between* it and objects, by which the process of knowledge is carried on. For ily by understanding these circumstances that we can learn to te the dangers which stand in the way of the formation of true otions.

pretence therefore of setting to work to ascertain the process ich knowledge comes to us, by a simple act of observation, ling all prejudice and eschewing all admixture of principles validity can be called in question, is in fact a groundless illusion. attempt to carry out such an undertaking is necessarily full of ıysical assumptions, but assumptions disconnected and un- ed, because they are merely taken up at the moment as they to be wanted to clear up a difficulty. The circle is inevitable, ıad better perpetrate it with our eyes open; the first thing we do is to endeavour to establish what meaning it is possible for ttach to knowledge in its widest sense, and what sort of relation conceive to subsist between the subject which knows and the of its knowledge, consistently with those yet more general which determine the mode in which we have to conceive eration of anything whatever upon anything else. What we do is to obtain the last-mentioned conception, which amounts etaphysical doctrine, and to treat the relation of subject and object as subordinate to it; we are not to begin by setting up some chance theory more or less probable as to that one relation, and then to use this as a test of the capacity of the human mind for apprehending truth at all. I say nothing of the question how far it is really in our power even to establish the facts regarding the gradual development of our world of thought; certainly the process of that development cannot be directly observed, for every observer has left it long ago behind him. And even though in many cases the developed consciousness may still retain the recollection of the road by which it has come to its present set of ideas, it will be admitted on the other hand that in many other cases these pretended observations of the development of our ideas are merely somewhat fanciful theories of the mode in which we think we may conceive it to have taken place.

323. If we follow the attempts which have been made to arrive in the first instance at some fact beyond the reach of doubt, from which we may proceed with security to test the origin and the truth of human knowledge, we are met at the outset of modern philosophy by the maxim of Descartes, '*cogito ergo sum*,' the one certain truth which the doubt of all received opinions seemed to him to leave standing. This proposition has been frequently taken as a point of departure, and it has always approved itself, from as far back as Augustine, in whose writings we first find it, for a truth as unquestionable as it is absolutely barren. Not the smallest step towards the establishment of any theory of knowledge whatever has it been possible to take from this proposition by itself, without calling in other and wholly independent principles to help. The very criterion which follows next in order, that all ideas are true which are equally clear and evident with this, Descartes himself did not venture to derive from that primary principle, without securing himself by the roundabout argument, alluded to in a previous chapter, against the objection that we may be all the while deluded by entirely false ideas possessing an equal degree of evidence with the true.

In point of fact it is easy to see that from this beginning we never can get to anything further. If we take the proposition in its negative sense, that is to say that nothing is certain for us except the fact of our own thought alone, and there is no such certainty in regard to the real existence of an external world, then I recall an observation already made: even if such external world be really existent, still it is only an ideal picture of it and not that world itself which can be present in us: the fact therefore that nothing possesses immediate certainty for us excepting our own world of thought, can never settle the question whether it alone exists, or whether there is a world of existence outside it to which it enters into relation. And even if the idea of this external world could be proved to be a necessary product of our thinking activity which we are compelled to form through the organisation of our mind and the laws of interconnexion to which our thoughts necessarily conform, if that is to say we could deduce from the fact of the *cogito* that our assumption of an external world of existence must necessarily have a *subjective* origin in the laws of our own minds: even then the truth of the assumption would be neither proved nor disproved; for even if that external world does really exist, it would be impossible for us to arrive at the idea of it unless the nature of our mind and the workings of our thoughts were such as to render it indispensable for the avoidance of contradiction within the world of thought itself.

On the other hand if we turn our attention to the affirmative aspect of the proposition, we find that it is not formulated in a way adapted to its purpose. It is no longer the expression of an immediate fact but of an abstraction. I do not complain of Descartes for keeping to the first person of the verbs '*cogito*' and '*sum*,' for obscure as the idea of the '*ego*' may be which they contain, and provocative as it is of further enquiry, it does unquestionably belong to the original form of this simplest of all experiences, and a theory which seeks to supplant the '*cogito*' by the '*cogitare*' and the '*sum*' by the '*esse*,' as the primary and most certain fact of experience, has no claim whatever to the credit of resting on a basis free from all presupposition and prejudice, which it is its ambition to share with the exact methods of the natural sciences. There never meets us as the simplest of facts an idea which merely exists and which no one has; we never meet with a consciousness which presents itself simply as consciousness and not as the consciousness of an '*ego*,' which in it is conscious *to itself* either of itself or of something else. Science may attempt afterwards to separate by one means or another the occurrences of thought and knowledge from this their constant condition of reference to a subject whose nature remains impenetrable: but they are originally given and their certainty along with them only in the form '*cogito*' 'I think,' not in that of the infinitive '*cogitare*.' But while Descartes is entirely correct in employing the personal form of the verb, it must be acknowledged that its significance was overlooked by him, and the interpretations which it received at the hands of Kant we cannot enter into here.

We may add that Descartes' principle was expressed in an unserviceably abstract form, emphasising as it does in the various mental states which carry with them this immediate certainty of personal experience, exclusively their universal quality—that is to say it emphasises exclusively the fact of cogitation or consciousness in the widest sense, which is an element entering equally into very various mental states, sensations and ideas, emotions and the will, distinguishing them all alike from that which we suppose ourselves to conceive as the condition of a being without a self and without a soul. No doubt this element of consciousness enters into everyone of our mental states which we observe, but what can be the use of noticing this common quality alone to the exclusion of those concrete elements apart from which it cannot really exist or become an object of direct observation at all?

The really fruitful starting-point of enquiry would have been, not the fact that the '*cogito*' is found in every form which consciousness can assume, but the question, what are the forms in which it is found? Not the bare fact *that* we are conscious or struck teaches us the truth we know; it is *what* we think, the matter or content of our cogitation, which supplies not only the original datum from which we start, but the sole source from which that which we ought to think or that which we cannot but think can be derived. Descartes himself points out that even the Sceptic in his doubt or in his denial of all knowledge, by that very act confirms the fact of cogitation, and just because it is associated indifferently with all true knowledge, and with every act of doubt, and with every kind of error, it cannot possibly serve to distinguish the true from the false.

324. Thus a fresh starting-point for the enquiry into human knowledge was unavoidable. It was given by the belief in the truth of innate Ideas. We must not allow the expression 'innate Ideas,' which has introduced a long controversy into the history of the theory of knowledge, to excite prejudices in our minds which a little care and consideration may certainly allay. Even the ancients in speaking of that *quod a natura nobis insitum est*, and all philosophers who have used the like expressions, were certainly very far from assuming that a truth, in itself foreign to the mind, was stamped in upon it at some particular moment when its life was beginning, and became thenceforth a permanent object of its conscious thought. What they meant was no more than this—the mind is of its own nature so constituted, that under certain operative conditions it necessarily develops certain habitual modes of combining its ideas. These constitute, to begin with, a method which the mind follows unawares, but finally as it comes to reflect upon innumerable acts of thought performed in accordance with them, the rules of its procedure hitherto unconsciously followed become themselves the objects of its conscious reflexion. These Ideas were called innate from the impression that it was not sufficient to represent the mind in which they were supposed to grow up as merely possessing a certain formal character, or general capacity for ideas, in such a way that given the same conditions the same set of Ideas would necessarily grow up in every being so endowed; it was held essential that every mind should have its determinate natural capacity, such as might conceivably distinguish it from other thinking beings, dictating the form which its thinking activity should take, and in which its particular acts of thought should be combined. It is true there was no occasion to take this assumption of a *possible* distinction between different beings endowed with a like capacity of thought for anything more than a fiction, which served to illustrate the truth that no adequate basis of human knowledge is to be found in the mere abstract fact of consciousness (*cogitatio*), but only in definite and concrete forms of it which at the same time are in fact shared by all minds in common. Nevertheless when once the conceivability of such a distinction had been admitted, it was no longer possible to resist the question what would be the result if it were accepted as real? And then the two sides of the Cartesian conception, the *a priori* character of the Ideas and their truth, parted asunder. To each individual that must necessarily appear to be truth which follows from the laws of its own nature; and so if each is furnished at birth with a stock of Ideas in the way supposed, then it is a mere act of faith, a faith quite irrational however firmly held, to imagine that the Ideas which are allotted to mankind contain a higher measure of truth than those which it may be force themselves with a no less convincing evidence and with a divergent message upon beings of a different constitution. It will be seen that such doubts are justifiable not only when we contrast the general sum of our knowledge with an objective world of existence of which it is supposed to be the copy, but even when—a thing which seems still more unavoidable—we insist on counting that only as truth which appears to all minds equally necessary, as distinguished from that which presents itself differently to different minds. This is the point from which the modern polemic against the Ideas takes its start, insisting that if our Ideas are innate they have no claim to truth, and that such a claim can only be allowed if they are regarded as independent of the possible differences between one mind and another and dependent only on the nature of a world of objects common to them all.

325. Before we enter upon the arguments on either side in regard to the questions which are here raised, it is incumbent on us to realise that we have now arrived at a point at which, instead of the unavowed assumptions to which we are in the habit of surrendering ourselves, it becomes necessary to make one express assumption which we admit in plain terms to be such. No enquiry of this nature can establish its conclusions, whatever they may be, without making some kind of assumption by the way as to the mode in which the object of knowledge may be conceived as operating upon the subject which apprehends it. Let us, instead of thus assuming our postulate by the way, place it at the head of our enquiry, in the shape in which the varied experience of the human mind has taught us to formulate it. Wherever between two elements A and B of whatever kind any event which we call the influence of A upon B occurs, such influence never consists in a constituent element, or predicate, or state a separating itself from A to which it belonged, and just as it is, and without undergoing any change, passing over to B, to attach itself thenceforth to this new object, or be adopted by it, or become one of its states (however we like to phrase it); what happens is, that a, the property residing, or change arising in A, becomes the cause by reason of which, given a relation C already established or coming for the first time into play between A and B, B also is necessitated in its turn to evolve out of its own nature and as a part of itself its new state b.

How this necessary connexion between the states of A and B is brought about, how it happens that B is necessitated to follow the changes of A, what again the relation C, which may be constant or may vary in different cases, but which is essential to the production of the effect in question, consists in;—all these questions, as well as the preliminary one whether they admit of an answer at all, may be left outside our present enquiry; for us the abstract principle enunciated is sufficient, no matter what the mode in which it is realised in fact. That principle however gives us this result, that the form of the effect b can never be independent of the nature of the object B which experiences it; it changes with that object; and the same relation C which obtained between A and B, will as between A and B^1, produce in B^1 a new effect b^1 quite distinct from b. As little is the effect b independent of the nature of the active agency A or of the relation C; it changes with both; if A^1 instead of A enters with B into the relation C, it will become β, and β^1 if B and A enter into the relation

C^1. But all these different results b, b^1, β, β^1 will make up in themselves a complete series of events which are only possible in B, and A and C are only to be regarded as exciting causes, determining which of the many effects of which the nature of B is susceptible are to be realised at a given moment, and in what order they are to come about. If we like to apply here the favourite designations, receptivity and spontaneity, we may say that every object is receptive of various kinds of stimuli to its spontaneity, and never operates spontaneously without such stimulus.

326. The operation of objects of knowledge upon a subject apprehending them comes under this general principle. Every assumption, to begin with, is wholly inadmissible which places the origin of our knowledge exclusively in the object: a very little attention will discover to us that even in the '*tabula rasa*' to which the receptive soul has been compared, or in the wax, which it has been supposed to resemble in being a mere recipient of impressions, a spontaneous reaction of the recipient subject is indispensable. Only because the tablet by virtue of certain modes of operation peculiar to its nature and consistence retains the coloured points and prevents them running into each other, only because the wax with its cohesive elements presents the properties of an unelastic body readily receptive of the stamp and capable of retaining it—only by virtue of this peculiar nature of theirs are the tablet and the wax adapted to receive the colours or the stamp impressed upon them; an object which presented no such qualities of its own to meet the stimulus from without would not possess so much as the character of pure receptivity ascribed to it.

Further it is necessary clearly to understand, that in an act of knowledge the direct contribution from the side of the object may be absent, but never that which is furnished by the subject's own nature. For it is conceivable that two ideas α and β, having once arisen in the soul through a stimulus from without, should then combine in obedience to laws having their source in the constitution of the mind alone, and without any renewal of the external stimulus, in a new result γ; but it is quite inconceivable that we could receive an impression from the world outside with the shaping of which our own nature had nothing to do. And therefore we cannot assent to the distinction between the matter and form of knowledge as it is drawn by Kant. The idea is indeed perfectly just, but he formulates it inaccurately when he ascribes the entire content to experience and the form alone to the innate activity of the mind. Kant was well aware of the fact which we are here emphasising, that even the simplest sensations, which in the strictest sense furnish the original content of all our perceptions, do not come to us ready made from outside, but on the contrary (if we are to hold to the conception of an external world) can only be considered as reactions of our own nature of combined sense and intellect in response to the stimuli coming from that world. They are the *a priori* capacities of experiencing sensation having their seat in ourselves which the external forces do indeed summon into actual existence in a definite order, but never transmit simply to us ready made. And when we pass to the composite result of these simple elements, the image of a particular form presented in space, the succession in time of the notes in a melody, or of a series of events, these too, in every particular and detail of the picture, are no whit less the product of the thinking subject, no whit less therefore *a priori*. For even if we assumed that things exist in a real extended space or occur in a real order of time in the same positions or in the same order in which we thereupon apprehend them, even then our temporal and spatial idea of them would be something quite different from their temporal and spatial *existence*; we could not manage to bring our ideas α, β, γ, into the same order as obtains among their objective causes a, b, c, unless our own nature and the laws of our mind enabled and obliged us to do so.

327. Or do we wish to delude ourselves with words and to reply that this trifling business of *copying* may be taken as a matter of course and requires no such labour of re-creation as we have attributed to the mind? But what do we mean by this word copy, and how is an image or a picture produced? We will say nothing at present of the eye, *for* which alone after all a picture is a picture, and we will ask only what are the conditions which make it possible for a mirror to present to the eye the image of any object? It can only so present it by reflecting the rays of light which it receives from the object in a fresh direction, while maintaining their original arrangement relatively to one another, and for this office it is absolutely dependent on the smoothness and the shape of its surface. It depends on these qualities of its own whether it scatters the rays in such disorder that no eye can combine them into a picture, or whether it so reflects them that although they diverge they can still be collected by the eye, or so that by converging they compose a real image which becomes visible to the eye as a new object.

But even when all this is done the mirror only supplies the stimulus which acts upon the organ of sight similarly to the object itself, and can be taken therefore to represent it; but if we ask how it is brought to pass as a result of this stimulus that the picture reflected can be seen, we are at once sensible how inapt the comparison of knowledge with a copy is. The apprehending consciousness is no resisting surface, curved or plane, smooth or rough, nor would it gain anything by reflecting rays of light no matter in what direction; it is in itself and its own co-ordinating unity, which is not a space, and not a surface, but an activity, that it has to combine the separate ideas excited in it into the perception of a spatial arrangement, which perception again is not itself an order in space but only the idea of that order. For even if, as some persons may perhaps imagine, the idea of a point to the left were actually placed to the left in our consciousness side by side with that of another point to the right, and the idea of an upper point above that of a lower, still this fact would not by itself give us the perception of this fact; all that this by itself would do would be to place us this time *really* in no better condition than that of a mirror in which some other mind might discover the disposition of the points, but again only on the supposition that *it* succeeded in accomplishing that which our own mind had not done, that is to say that it not merely received and retained the impression of the rays with their order of arrangement as reflected from our mind, but also turned those impressions to account, by producing, on occasion of them, a co-ordinating perception of that order.

Nothing is left therefore of this inexact comparison except the conviction that even the mere perception of a given state of things as it really is, is only possible on the assumption that the perceiving subject is at once enabled and compelled by its own nature to combine the excitations which reach it from objects into those forms which it is to perceive in the objects and which it supposes itself simply to *receive* from them.

That the case is the same with all the ideas which we form as to the inward connexion between one perception and another, is a fact to which I need only briefly advert, for it is here that the criticism has been most generally admitted. It is allowed on all hands that we do not see the causal connexion between two events, but that on the contrary the idea of such a connexion has to be superadded by ourselves to that mere succession of events in time which is alone directly perceived; and the admission of the *a priori* origin of the causal nexus has been used by one school of philosophy to establish for it the superior dignity of a necessary idea of universal validity, and by another to deny it all validity whatever in relation to the world of things in our perception of which its origin is not to be found. Both the one deduction and the other is unsound. In regard to the second I recall once more this simple consideration; even if a causal connexion does exist between the events of the world outside us, it still could not possibly be presented to us as the direct object of a purely receptive faculty of perception; the mode in which individual impressions are connected can never do more than afford a stimulus to thought to introduce the conception through its own activity, nor can such stimulus actually operate unless our intellectual nature is itself necessitated, in order to complete and account for the observed combination of impressions, to supplement it by that conception.

328. The *a priori* character however, which we thus claim in so broad a sense for our knowledge, is only one side of the matter. If we regard all forms of sensible perception, our intuition of space, our conceptions of thing and quality, of cause and effect, lastly the ethical ideas of good and evil, as modes of manifestation innate in the mind, then and for that reason the ground for this and that particular application of them, one necessarily excluding another, cannot possibly be found in the mind. In our perception of space there are innumerable figures possible, but at a given moment we only observe certain definite ones; we are capable of seeing many different colours and hearing very various successions of sounds, but we cannot alter the red which we have before us here and now, though blue or yellow in the same place would be equally perceptible to us, nor can we substitute for the melody to which we are now listening any other of the countless melodies which we have heard at other moments; events follow one another independently of us, now forcing us to recognise a causal connexion between them, now making such an assumption impossible; finally this grouping of the incentives which are offered us to the exercise of our *a priori* faculties varies as between one individual and another, and cannot therefore have its foundation in the common nature of the mind.

To what it is that we are to attribute them is here indifferent. It may be that the ordinary opinion in which we all acquiesce in practical life, and from which the present discussion started, is the true one; that there does exist a world of things outside us, in which we have ourselves our assigned places, and which affects us in varying ways according to the changes which take place in itself and to the different and varying positions which we occupy in it. In that case the complex web of ideas which forms itself within us, cannot indeed claim the name of truth in the sense of presenting to us a real likeness of that which exists or occurs in the world of things; still each several conjunction or separation or transformation of the phenomena which float before our consciousness, will in its character of a *consequence* bear witness to a definite process of change, though it may be of a different order, in the relations of that world of things which operates upon us. And we should be led to the same conclusion by the rival doctrine of Idealism which never becomes natural to us in ordinary life, and is recommended solely by arguments which lie purely within the field of philosophy. It may be, as this belief supposes, that there is no world of things or events outside us, but *only* the appearance of such a world brought about in all minds alike by a single unknown power which penetrates them all, and that in such a manner that the pictures of the world which different minds seem to themselves to see all round them, fit in one with another, and all find themselves members, each in its own place, of one and the same universe. This theory, like the other, has necessarily to admit that the stimulus which excites any individual mind to create *its* particular picture of the world, is a stimulus foreign to itself, and at the same time not explicable from the universal spiritual nature which it shares with all other minds. Wherever it may come from, it remains an empirical or *a posteriori* element in our knowledge. And again: every conjunction or separation or diversification of the phenomena which so arise in us, will point to a distinct occurrence elsewhere, to changes taking place, not indeed any longer in the relations of manifold external objects, but in the action of that one power which creates within us this dream of an external world. Here finally as on the former hypothesis it would be well worth while to establish by observation and comparison of the phenomena those unchanging laws which they follow through all the play of change; and the accomplishment of that task will still give a knowledge of truth, even though there were no means of deciding what is the nature of that distinct set of laws obtaining in an unknown outer world which are the source of the orderly government of our own world within. The view I am here representing is in essentials that of Kant, and is one which German philosophy ought never to have deserted. But in so doing I expressly decline to give any answer to the question last alluded to. Let a man believe himself ever so much to possess an immediate certainty of the existence or the non-existence of an external world of things; the nature and the manner of that existence can still only be unriddled by conclusions drawn from phenomena. Here therefore our footing must be secured to begin with; we must first establish those certain principles which are to determine the judgments we form in regard to the system of this inner world, before we can talk of applying the conclusions so obtained to the further metaphysical question.

329. But now supposing that we assume certain truths as innate, in the previously accepted sense of the word, whence do we arrive at the knowledge of them, unless it be by discovering them within us, that is to say, by inward experience? So that after all experience will be the sole source of all our knowledge? This criticism has been made, and it will be felt *prima facie* to be as barren as it is unanswerable. For certainly to know a truth we must be conscious of it, and if we were not conscious of it before, then the passage to the knowledge of it is an event which we must necessarily live through or experience; in this sense of the word our whole existence is a fact which only experience discovers to us. This objection therefore to the *a priori* nature of innate ideas cannot hold; on the contrary, supposing there to be innate ideas, supposing them to exist even in the sense of being unceasingly present to consciousness, still the mind reflecting on them could, to begin with, only be aware of their presence as a fact given in its experience or its conscious life. Taken then in this broad acceptation the conception of experience no longer offers occasion for a difference of opinion; the only point of importance is *as what* do we experience the thoughts in question? Do we experience them as innate truths, or as matter of experience in that narrower sense, in which they indicate in contradistinction to such truths that their origin is foreign to the mind itself? With this distinction the question about experience seems at first sight to take a more urgent form; if, that is, we go on to ask for marks which may distinguish the one of these cases from the other. We then find that the impressions which come to us from outside are forced upon us and we cannot alter them; but the *a priori* truths also present themselves as unavoidable and unalterable; that the compulsion in the first case comes from without, and in the second is that of our own nature, we may indeed conjecture, but how are we to prove it? The truth however is that if we take the unsophisticated intelligence we find that *this* which to us in the course of our methodological investigation was the most important fact, is not *to it* the primary one at all; the truths in question are not matter of experience in respect of their alleged quality of being innate in us; what first strikes us is that as a matter of fact what they assert is self-evident, so that when once we have had occasion to think of them in any particular instance, we see them to be independent of any further confirmation through fresh instances, and thus independent of experience which might supply such instances. And hence *universality* and *necessity* have always been the two characteristics which have been ascribed to *a priori* knowledge. We understand by the term *universality* that invariably as soon as the subject is thought of the predicate which belongs to it appears in self-evident conjunction with it; and again it is in this self-evidence and in nothing else that necessity or necessary validity in this sense consists, for clearly necessity attaches to universal truths in quite a different sense from that in which it belongs to those conjunctions of various objects which our changing experience brings before us. These objects, it is true, are also presented to us in such a way that at the moment in which they occur we cannot dissolve the conjunction at our pleasure; but though the content of experience possesses necessity in the sense in which every fact which cannot be denied does so, still it lacks that perfect self-evidence which consists in an inherent connexion of elements which are unthinkable apart from each other.

But after all, what gives us the right to affirm that that which may appear to us self-evident at this particular moment will appear so equally at every other, that is to say, to ascribe to it a *universality* which can make it a fixed principle of judgment in face of a perpetually changing experience? This question was raised by the early Sceptics and led them to declare *all* general propositions inadmissible. And in point of fact, whatever principle we may choose to devise to justify us in concluding from the certainty of a proposition at the present moment to its certainty for all future time, must itself be subject, *as* a universal principle, to the precise suspicion which it was intended to remove. Thus we should have no means of assuring ourselves of the universal validity of any proposition if we cannot be satisfied with the self-evidence with which its content, once thought, claims for itself eternal validity in anticipation of experience. And it would have to be a matter of consideration that this incapacity for attaining to universal truth could not be deplored as an infirmity peculiar to the human intelligence; it would be shared by all minds whose experience as being developed in time at all resembles ours; the very truest truth which might be innate in such a mind could only come into its consciousness at a definite moment, and all the self-evidence it might possess for it at that moment would not remove the uncertainty whether it would remain a necessity of thought in the next.

330. This result will perhaps be eagerly admitted, and it will be urged that it proves the futility of our defence of *a priori* truths;

even when the mind has got them it has no means of distinguishing them from the results of experience. Or in other words it is only experience which teaches us that they have universal validity; that is to say, when we find their self-evidence confirmed by each successive attempt to think them, we have not indeed strict proof but we have the strongest probability that they are valid without exception, and it is to this gradually increasing empirical probability that the whole of our knowledge is in fact restricted.

In this there is an element of truth which I shall consider presently; but taken as a whole it is a false position. If we assume, as this view admits, that the certainty of a given proposition as experienced at one moment does not guarantee the experience of its certainty in the next, then just because this is so a thousand repetitions will not make it a whit more probable in the thousand and first case than it was in the second or third. If after a series of cases of the connexion of two events *a* and *b* unbroken by any instance to the contrary, we look for fresh instances with constantly increasing confidence, we do so on the strength of very definite assumptions. If the connexion of *a* and *b* is not of such a kind as to make it self-evident the moment it is presented to the mind, if its eternal validity is not at once apparent, then we explain its constant occurrence by the fact that the conditions which might have produced a different result have not so far come into operation; that they are not likely to do so at any future time we conclude after numerous instances of similar experiences on the strength of one special assumption and not otherwise, the assumption that the course of the universe in general and of this part of it to which the events in question belong in particular, proceeds in a fixed order, which by examination of a sufficient number of instances, becomes discoverable. Then, starting from this assumption that a particular set of conditions whenever they recur in the future will be equivalent to what they were when observed in the past, we draw our conclusion: given like conditions a like result must present itself. If we are wrong in that assumption this will mean that we have set up as universal a false generalisation concerning a matter of fact, which will be refuted by future experience. On the other hand, if our universal principle, that under like conditions like consequences follow, is no longer to be regarded as really universal, then the entire method of logical procedure by which we expect to pass from particular experiences to propositions of even probable universality, is absolutely baseless and vain. For every time we argue from m to $m+1$, whether we are undertaking to establish a strictly universal or a merely probable conclusion, in either case we assume the strict universality of that logical principle.

It is clear therefore that the attempt to derive the entire body of general knowledge from experience, that is to say from a mere summing up of particular perceptions, breaks down. We have invariably to help ourselves out by assuming at one point or another some one of those self-evident principles, some principle to which when once its content has been thought we at once concede with intuitive confidence that universal validity to which it makes claim.

331. Now in practice as a matter of fact there has never been any dispute on this point. Mathematical demonstrations have often been subjected to fresh examination, but never with any other object than to establish whether each one of the several propositions which made up the chain of reasoning was either itself self-evident or was logically derived from others which were so. We never set to work merely to prove over again the self-evident propositions themselves, to see whether some moment may not arrive in which their direct contraries, the equality of unequals for instance, or that the part is greater than the whole, would be equally self-evident; and even supposing so unexpected an event had on some occasion occurred, no one would have doubted that there was an error somewhere, which could only be attributed to an oversight in the calculation. On the other hand much difference of opinion does exist as to the extent of these universal and self-evident truths, and here we are brought in view of that element of truth which I could not help allowing above, in the theory just combated. I by no means intend however to imply that experience as such could help us to establish what holds universally not merely as a universal fact, but as a self-evident and necessary truth; on the contrary it is precisely experience with its repeatedly recurring uniformities which at last deludes us into taking for necessary and self-evident truth, that which is merely matter of fact, or not even that.

I have spoken before of the delusive certainty which many principles assume, merely because our limited experience has constantly presented them to us without any instance to the contrary. The psychological association which establishes itself under such conditions between the ideas *a* and *b*, representing two events which have constantly followed each other, very soon assumes the appearance of a self-evident connexion in fact between the contents of the ideas so presented. I observed then that the attempt to think the direct contradictory of a proposition which has come to be thus self-evident may serve sometimes to dispel the illusion, and we then find to our astonishment that a hypothesis which contradicts our apparently self-evident proposition presents no difficulty to thought, that it is just as much thinkable as the other, and that accordingly the certainty which we ascribed to our belief cannot depend upon any universal self-evident connexion in its content. I was obliged however even then to add that this attempt to think the contradictory will not always be a decisive test; the influences of previous experience which nullify its value are in fact very various. If we could be certain, in applying it to any proposition, that we have not only determined with perfect exactness, with nothing lacking and nothing over, the meaning of the subject *a*, and the predicate *b*, and also of the copula *c* or whatever the connexion may be which we wish to establish between them, but also that in the final decision as to whether that relation *c* which we have established is self-evident or not, we have been guided by no sort of consideration save the fixed meaning of the three conceptions; then undoubtedly we should all agree in our conclusions, positive and negative alike. And wherever these conditions are susceptible of fulfilment, as is the case in mathematics, such agreement is in fact found. The complex notions on the contrary of real objects are very far from admitting the same exactness of analysis, and every reasonable man looks for results in this sphere only from experience or rather from the accurate manipulation of our experiences. Finally those simplest and most universal conceptions and principles to which we should desire to subordinate that manipulation, would unquestionably admit of the highest degree of such accuracy, did not the influence of past experiences come in the way. We certainly intend something very simple and definite when we use the words, being, thing, cause, force, effect, matter; but in our use of any one of them we are commonly determined by our limited circle of experience or our favourite study or pursuit. Thus we are led on the one hand to apply them only to a fraction of the subject-matter which we in fact hold that they ought entirely to dominate, and yet on the other hand to bring them into a variety of connexions which are not indeed impossible to them but still do not essentially belong to them. Thus we might perhaps if we were required to define one of these conceptions agree in our definitions, yet the ways in which we actually look at its meaning might be different enough, as different at all events as in the case of the same objects seen in different lights. Now all these unanalysed side-thoughts, the emotional suggestions and the wishes which thus attach themselves unawares to the object of thought, and give it its characteristic colouring, dispose us to find the certainty of self-evidence in predicates which we should not be warranted from the nature of the object alone in applying to it at all. This is at once the value and the danger of experience; except as suggested by experience the universal principles of our judgment cannot be presented to consciousness at all; but as thus occasioned they are at the same time subject to one-sidedness, deficiencies in one direction, superfluities in another, from which later reflexion has much ado to purify them. Here begins a work of criticism which has to be unremittingly pursued; the useful labour of investigating the psychological origin of the particular form which these conceptions have come to assume in our consciousness; the object being not so much to show how all certainty and truth arises little by little out of the deliverances of experience, as, on the contrary, to make it clear how much foreign matter due merely to the peculiarities of the instances observed, has incrusted itself upon the substance of those original truths, truths which, if once they were seen in their simplicity and purity, would be not only recognised as necessary and self-evident but would prove so in all their applications.

332. Such a criticism of prejudices, as I may shortly call it, cannot I conceive be conducted otherwise than piece by piece in connexion with definite problems which offer themselves for solution; for it is only difficulties which rise upon us in working out individual problems, which lead us to suspect the soundness of our principles and to cast about for the sources of the errors we have fallen into. I refrain therefore from entering into the subject here in detail; on the other hand it is necessary that I should vindicate the method I have thus far pursued as against the opposite theory, which not content with freeing the primary truths by this process of psychological analysis from the erroneous side-thoughts which have grown up about them, aims further at giving a systematic explanation of the nature of thought and demonstrating the validity of its first principles. I have maintained the opinion throughout my work that Logic cannot derive any serious advantage from a discussion of the conditions under which thought as a psychical process comes about. The significance of logical forms is to be found in the meaning and purport of the connexions into which the content of our world of ideas ought to be brought; that is to say in the utterances of thought or the laws which it imposes, after or during the act of thinking, not in those productive conditions of thought itself which lie behind. Conditions of this kind there must certainly be, not only those conditions of a psychical mechanism which determine at every single moment every single one of its motions, just as every feature in an event of external nature is determined by the physical conditions which are given at the moment of its occurrence,—but more than this, the necessity with which, speaking generally, thought follows unawares those logical rules of its procedure which later reflexion formulates into consciously apprehended principles, must be an unavoidable consequence of the nature of the mind itself, which it belongs to Psychology to investigate. But if we knew all that we could desire to know on the subject, it would still be a delusion to suppose that we should be thereby any the better able to judge of the truth of our logical principles; on the contrary the validity of those principles themselves would still be the necessary postulate without which the successful enquiry into their psychological history could not have been undertaken at all.

To touch here for the last time upon this logical circle which has wearied us so often already; it must be clear enough that no sensational or empirical theory of the origin of thought and knowledge can possibly either prove or disprove the principle of identity or excluded middle; in every step of the argument it needs them both. As little can it be left to such a theory either to establish or to destroy the validity of the law of causation. For every attempt to reduce our application of it in the field of experience to the association and reproduction of ideas presupposes its validity in another form in relation to the interaction of psychical states; so that it can neither be accepted nor rejected unless its validity be established to begin with—a premiss from which certainly the rejection of it could only be arrived at by a very curious sort of logical suicide. Nothing then remains but to restrict this psychological analysis to the task of showing how truths which have their own validity in themselves find realisation in thought and for thought, regarded as a psychical process, as rules of its procedure which it follows unawares.

333. And now I should like to make clear that of all that we might wish to know in this direction we in fact know nothing at all, and that Logic would have to renounce for a long time yet any profounder understanding of the operations of thought if she had to look for it in the psychological analysis of their origin. In the works of the sensational school, which have been produced in such numbers and such variety on the model of Locke's Essay—which is here unrivalled—and of Condillac's bold venture, I can find nothing that answers in a general sense to this requirement. Regarded as a criticism of the prejudices of human thought, Locke's work has enjoyed the full measure of influence in the development of modern philosophy to which the wide horizon which it opened and the keenness of its analysis entitled it. But in dealing with all the variety of those inner processes of the mind, which he undertakes to criticise, Locke has no other instrument to apply but 'common sense,' a faculty which, versed in the criticism of the course of events in the outward world, imagines that the very respectable and probable but quite unsystematic maxims there acquired are sufficient to meet all emergencies. It is more to my purpose at present to consider the attempts which have been made in this direction in German philosophy. When we speak of explaining any set of processes, and regret its non-accomplishment, we think, as the type of the wished for ideal, of the body of the natural sciences. By the strict observance of the laws of thought and the careful application of them to the results of exact observation, natural science has succeeded in arriving at a small number of original facts from the interaction of which exceedingly various phenomena can be shown to follow with logical necessity. A series of happy inspirations[1] have within quite recent times added to this domain a portion of the inner life of the soul, at least in regard to the dependence of sensations upon external stimuli. And this result was due not to attempts to construct the entirely peculiar set of events which we call psychical out of physical processes, which can never be brought into any comparison with them; but of investigations of which the aim has been simply to apply exact quantitative determinations to the members of the two series which the order of nature does actually unite together, though in a manner unknown to us, and from the pairs of correlated values thus ascertained to develop the laws of their correspondence. And previously to these enquiries a valuable attempt had been already made[2], not indeed resting on the exact observation of special facts, but upon hypotheses suggested by experience generally, to bring the purely inward phenomena of mental life under a mechanical theory of their origin. At the same time all these achievements which have given the psychology of the present day a very great superiority over the views of earlier times, do not reach those obscure regions of enquiry, the illumination of which might open new paths to Logic. They merely instruct us concerning the interaction of different psychical states to which measurement has been applied, in regard to the changes they severally undergo when brought into connexion with each other, and thus in regard also to the total state of the soul at any moment, considered simply as the mechanical result of all these reciprocal influences. But they do not equally explain the fresh reactions to which the soul is stimulated by each one of these states of itself as they thus arise, and which are not calculable consequences of certain quantitative relations in the co-operating conditions, but depend, in obedience to a necessity of a wholly different order, shall we say a dialectical or teleological necessity, upon the meaning or the idea which the soul is destined to realise.

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[1] [For an account of these investigations, see 'Metaphysic,' § 258.]

[2] [An allusion to Herbart, see 'Metaphysic,' §§ 269, 270.]

The investigation of external nature leaves questions of this sort behind, but for its purposes it does not need to answer them. In what way it happens, by what means it is brought about, or to what purpose it tends, that particles of matter attract each other with a force determined by their distance, are questions which may be left undecided. When once the law of this reciprocal influence is ascertained, it can be reckoned as a constant element in the course of nature, that is to say in the present case as an element into the determination of whose variations in each several instance the given circumstances enter. The more we succeed in reducing all natural processes to homogeneous motive forces of this kind, the more possible will it become to construct even the form of every single natural event out of the conditions which occasion it. This would all be altered if the natural sciences had cause to suppose that the material elements which had hitherto been regarded as unchangeable, experienced under the operation of forces of this kind certain inner changes which had the effect of stimulating them to wholly new modes of reaction, giving them a new influence in the play of events. No doubt those new influences so far as they operated to bring about changes in the physical surroundings could still be directly connected with the ascertainable outward conditions under which they arise, or, to express it in general terms, they could be regarded as functions of the conditions; and thus there would be apparently no interruption in the continuity of the scientific construction, only an increased difficulty in carrying it out. But in point of fact a breach of continuity would certainly have taken place. For the simple fact that given a certain set of physical conditions m a new mode of operation μ will make its appearance and given another set n a second new result ν, would remain after all a new *datum*, a fact known indeed from experience, but not to be derived analytically as a necessary and self-evident consequence from the physical conditions given.

Now the case in which we find ourselves in regard to the present question is analogous to this. All the mental processes which psychology teaches us are necessary presuppositions for the realisation of any act of thought, are merely the conditions m or n which give occasion to the logical reactions μ and ν to present themselves. They cannot explain the fact that μ and ν do thus appear upon the scene, nor again do we find in this fact in itself the least explanation of the further relations of constantly increasing complexity which thought establishes between its μ and ν or other of the elementary products of its activity.

I should dwell upon this point further were it not that the subject of the following section will oblige me in any case to call attention later on in detail to the deep gulf which remains unfilled between the psychical mechanism and thought; I content myself here with the expression of my conviction that all logical reactions of the mind have to be conceived as a connected whole, as expressions of a single tendency whose separate utterances can in so far as their *meaning* is concerned be apprehended and arranged in an intelligible series, but in their origin as psychical processes remain wholly incomprehensible. It is an illusion in psychology and a corruption of logic to take the conditions which occasion the logical operations of thought for the operations themselves. There is only one delusion more desperate still,—to imagine that a complete physical theory of the nervous system will explain that which is itself the condition of any theory being possible at all.

CHAPTER IV.

Real and Formal Significance of Logical Acts.

334. FACTS of perception we acknowledge without question; our misgivings begin with the interpretations of those facts by discursive thought, more especially when we consider the protracted and intricate web of ideas which thought spins in abstraction from the facts of sense, yet always with the expectation of reaching a final result which perception will confirm. Thought as an activity or movement of the soul follows laws of the soul's own nature; will these laws which it necessarily follows in the connexion of its ideas, lead to the same result as that which the real chain of events brings round? Will the outcome of the process of thought, when at the close of it we turn once more to the facts, be found in agreement with the actual results which the course of nature has produced? And if on the whole we consider it improbable that thought and being, which it is natural for us to regard as made for one another, should be entirely divorced, are we also to suppose that every single step taken by thought answers to some aspect of that which actually takes place in the development of the things thought about? Such are the doubts which give rise to the theory of the purely formal or subjective validity of thought. That theory is perfectly clear in what it affirms; the logical forms and the laws of their application are the conditions through the fulfilment of which thought satisfies its own requirements, and brings the connexion of its ideas with one another into that form, which for it, for thought itself, is truth; but it is not at all clear what is the relation—though some such relation cannot be dispensed with—in which these forms and laws stand to the content which they do not create but find, and from the manipulation of which alone after all that which is truth for thought draws its material.

Can an object, we ask, be brought into forms to which it is not adapted? Or even supposing that we are able to force our material into a form which it does not naturally assume, still must there not be some quality in the material which at all events makes such an operation possible? Must not every given subject-matter therefore, which thought casts into its own forms, possess some relation and affinity to those forms, of which the most we can say is that it may be misused? Finally, must not this assumption hold as regards every single logical operation? Not one of these could be carried out even as a mere subjective process of thought, unless the object upon which it is exercised contained in itself some characteristic which invited or at least allowed it. Now we know that the distrust of thought spoken of above, does not find confirmation in experience in the universal sense we dreaded. However wrong we may go in protracted chains of reasoning, daily life shows how well our conclusions taken in the average agree with the actual course of events. Why should we not hold fast to that confidence in the veracity of thought which is the natural attitude of our minds before scepticism disturbs them? Why not mount a step higher still, and regard the objective[1] content of our world of ideas as bound by no other laws than those which thought imposes on it? Then we should need nothing more than careful attention to the subtle and intricate logical processes of the mind, to find reflected there as in a mirror the real or objective forms in which all existence appropriately developes.

In this way the belief grows up in a Real significance of thought, a belief which in its more general features appears in the history of the human mind earlier than its opponent, but which stated in these explicit terms and in this thorough-going form, is a product of recent times. Between this and the opposite theory the history of philosophy has a long controversy to recount. We cannot decide it by placing the logical forms and laws side by side with those of real existences and events and comparing the one with the other, for we have no knowledge of the latter in which thought is not already present and operative. But we can ask what is the judgment of thought itself on its own operations, and how far it pronounces the forms which as a psychical movement of the thinking subject it is constrained to assume, to be a determination belonging to the object-matter upon which it operates.

335. To whatever act of thought we direct our attention we never find that it consists in the mere presence of two ideas *a* and *b* in the same consciousness but always in what we call a Relation of one idea to the other. After this relation has been established, it can in its turn be conceived as a third idea *C*, but in such case *C* is neither on the one hand homogeneous with *a* and *b*, nor is it a mere mechanical effect of interactions which in accordance with some definite law have taken place between the two as psychical processes with definite magnitudes and definitely various natures. We may take as the simplest examples of what I mean the identification and the distinction of two ideal contents. If we assume *a* and *a* identical[1] with each other, then unquestionably the idea *a* is present twice over in our mind, but the only result to which this circumstance can lead us on mechanical analogies will be either that the two ideas must count as one because they exactly cover each other, or that as similar affections of the soul they will become fused into a third idea of greater strength, or that they simply remain apart without any result at all. But that which we call the comparison[2] of them, which leads to the idea of their identity *C*, consists neither in the mere fact of their co-existence, nor in their fusion; it is a new and essentially single act of the soul, in which the soul holds the two ideas side by side, passes from one to the other, and is conscious of experiencing no change in its condition or in the mode of its action during or by reason of that passage from the one idea to the other.

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[1] ['Der sachliche Inhalt des Vorstellens.']

Again: let us compare two different ideas *a* and *b*, red and yellow. Two external *stimuli*, which acting by themselves would have awakened severally one of the two sensations, might acting simultaneously coalesce in the nerve, through which they propagate themselves still as physical states, into a third excitation intermediate between the two so as to occasion in the soul only a third simple sensation. But two ideas which have once arisen as ideas in the soul, never experience this sort of fusion. If it were to occur, if the distinctive existence of the two ideas were to vanish, all opportunity and possibility of comparison, and therewith as a remoter consequence, all possibility of thought and knowledge, would vanish also. For clearly all relation depends upon the preservation in consciousness of the different contents unfalsified by any interactions of one upon the other; the single undivided energy of thought which is to comprehend them must find them as they are in themselves, so that passing to and fro between them it may be conscious of the change which arises in its own condition in the transition.

In using this language I am fully aware that it may be fairly objected that my designation of the energy in question contains mere descriptions which cannot be embodied in a construction. But this is exactly the point upon which a clear understanding is essential,—that the intellectual processes upon which all thought depends do bear no sort of resemblance to those physical events on the analogy of which such an objection would like to see them modelled. An activity which cannot be said simply to *be* a movement but which executes a movement, which relates itself to two objects without introducing any change into them, which finally becomes conscious of the direction and the length of the path it has travelled by the differences which it experiences in its own states,—such an activity cannot be brought under the ordinary category of unchanging elements with changing relations, or of the equality of action and reaction; and yet at the same time it is something whose reality we all feel; it in fact and nothing else is the instrument by means of which we accomplish those much admired constructions which we would fain apply to it. These characteristic peculiarities we have simply to acknowledge, and to look for a new set of conceptions which may enable us to formulate them without falsifying their nature, an order of conceptions which are still a desideratum in philosophy, and which I by no means consider my own very incomplete formulae to have supplied.

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[1] ['Gleich,' i.e. the same both in quantity and quality. Neither 'equal' nor 'like' fully render this meaning. Cp. 'Metaphysic,' p. 47, note.]

[2] ['Vergleichung.' The emphasis on the connexion of 'Vergleichung' with 'gleich' cannot be rendered.]

336. In the instances taken above, *a* and *b*, red and red, or red and yellow, were objects directly given in perception. The ideas of identity or difference *C* which we obtained as the result of the act of relation introduced by the mind, are no longer of this character. As a relation of one *to* the other, the identity of *a with a*, or the difference *between a* and *b*, they cannot be really thought without at the same time recalling on the one hand the ideas of *a* and *b*, which form the terms in the relation, and on the other that movement of thought which carried us over from the one to the other Thus every time we use the term identity or difference we are called on to renew once more all those operations of thought through which alone it is possible to use them with a meaning; but when we express the final result which we wish to produce by the process of thought, by saying that *a* is the same as *a*, or *a* is different from *b*, we are implying that the objective knowledge which it was our object to arrive at lies entirely and exclusively in this final step of the completed comparison. It is not to *a* and *b* that we ascribe the movement backwards and forwards between them through which we discovered their relation to each other; this movement is merely a psychical process, without which indeed our result could neither be obtained in the first instance nor repeated afterwards in memory, but which has nevertheless to be abstracted from the real significance of the act of thought to which it ministered, as a scaffolding is withdrawn when the building is completed. Thus we see at once in an example of the simplest possible kind the antithesis between the merely formal significance of an act of thought and the real significance of its product. Before I follow up this line of thought further I wish to advert to two sets of processes which add a confirmation on a large scale to the conclusions which we have seen suggested by a particular instance.

In the first place we receive the sensible perceptions from which thought starts almost without exception under the form of space,—in spatial shape, arrangement or relations; hence we come to apply terms of space symbolically to every sort of complex relation in order to give it that vividness to the imagination in which it would otherwise be deficient. We represent ideas of difference by terms of distance, distance long and short, in this direction and that; the multiplicity of what is the same by distribution at different points of space; the self-identity of unity[1] by the notion of an unchanging place which we assign to the idea in question whenever we think it; lastly we find it difficult to make our conceptions clear, wherever the manifold orders of relation which present themselves to thought are such as the formulae derived from space are inadequate to express. And yet for all this we are conscious that these formulae do not reach the heart of the matter; all these symbols are, we are aware, mere subjective aids to the understanding, convenient paths for thought which has to travel up and down to reach its goal *C*, which is in itself wholly distinct from them; what we *mean* is independent of the mode in which we *figure* it.

Secondly we are accustomed to clothe our thoughts in speech, and even in the silent processes of thought it has long become habitual to us to call up the appropriate words before the mind; perception, recollection, expectation, hardly reach perfect clearness until we have found adequate expressions for them in spoken propositions. The advantage thus gained is not in its own nature dependent on speech and its sounds, but rather on an inward act of analysis and combination which would remain the same if it employed other forms of communication; still in point of fact, now that speech is there for the purpose, it is undoubtedly the case that the forms which the processes of thought assume and the facility with which they are conducted are dependent upon the means which speech provides, and thus present even national differences, when many and various causes have combined to render the formation and syntax of different languages dissimilar. Thus the logical meaning of a given proposition is indeed in itself independent of the form in which language expresses it; but in practice all human thought is compelled to represent its meaning by separations, combinations, and readjustments of those ideas which the growth of language has attached to single words. It is only in this its discursive character, in contradistinction to Perception, that thought is a psychical fact. It is in this character also that it has been the subject of our logical treatise. Logic has never concerned itself with a thought which did not make its various ideas, one after another, the object of its attention, which did not move amongst them comparing and relating them to each other, which did not symbolise abstract ideas by spatial images, which finally did not express its thoughts in the forms and constructions of a language. We must expect therefore to find in what we call logical operations, logical forms and laws, a considerable amount of purely formal apparatus which although indispensable to the exercise of thought, yet lacks that Real significance which for the ultimate results of its activity thought does undoubtedly claim.

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[1] ['Identität des Einen mit sich selbst.']

337. Let us now return to consider this result. When in comparing *a* and *b* we are conscious of a change C which we experience in passing from one to the other, there is no doubt that C must depend upon the nature of the two terms of the relation, for it would alter and become C^1 if they were replaced by *c* and *d*. At the same time the connexion of C with that objective relation seems to be one of dependence merely, and not to consist in being an identical copy of it; as a subjective excitation in us it falls short of the objective reality towards which knowledge is directed. I should not advert to so subtle a refinement of criticism were it not that it gives me an opportunity to return once more to the difficult subject of the nature of the act which presents ideas. The act of presentation is not that which it presents, the idea is not that which it means. And this not merely in the obvious sense that neither the one nor the other is the fact presented: but I mean that even the very simplest ideas, the content of which can only exist in thought and is not a thing, have not their content as their own predicate; the idea of yellow is not yellow, the idea of triangularity is not itself triangular, or the idea of timidity timid, or the idea of a half half as large as that of the whole. At the same time the act of presentation is not so completely separable from its content, that it could be, or occur, or experience change by itself; it *is* only in as far as it presents that which, itself, it is not; it changes only in exchanging one of these contents for another. Thus even the change of which it becomes conscious in its own condition can only consist in a change in the contents presented, which with its single activity it comprehends and compares; it cannot be sought in an affection of a wholly different character which the mind experiences merely as an after *result* of the stimulus given it by those contents, and which becomes observable to consciousness apart from those contents as an idea *C* having no resemblance to their own relation. He who finds red and yellow to a certain extent different yet akin, becomes conscious no doubt of those two relations only by help of the changes which he himself as a subject of ideas experiences in the transition from the idea of the one to that of the other, but at the same time he never entertains the apprehension that the relation of red and yellow may be something quite different in itself from that of the affections which they occasion in him, that red for instance may be in itself exactly like yellow and only appear to us different from it, or again that in reality there is a greater difference between them than we know and that their apparent affinity is an appearance only. Such scepticism might not be groundless if the question was one concerning the relation of our world of thought to a world of things assumed to be external to it, but so long as we are considering not this external world, but our own ideas, we never doubt that the relations of likeness and difference which we experience in the comparison of them, on the part of our presentative susceptibility, signify at the same time an objective relation on the part of those contents which our ideas present to us.

338. But now after all how is this in strictness possible? How can the propositions '*a* is the same as *a*,' and '*a* is different from *b*,' express an objective relation, which, as objective, would subsist independently of our thought, and which thought could only discover or recognise? We may suppose ourselves to know what we mean by a self-existent identity of *a* with *a*, but what are we to make of a self-existent distinction *between a* and *b*? And what objective relation can correspond to this 'between,' to which we only attach a meaning so long as it suggests to us the distance in space which *we*, in comparing *a* with *b*, interpolated by way of metaphor for the purpose of holding the two apart, and at the same time as a connecting path on which our mind might be able to travel from one to the other? Or otherwise expressed: difference being neither the predicate of *a* taken by itself nor of *b* taken by itself, of what is it the predicate? And if it has a meaning only so far as *a* and *b* have been brought into relation to each other, what objective connexion, we must then ask, obtains between them, if we consider the relating activity through which we have conjoined them in our consciousness as not being exercised? Many errors in ancient Dialectic were occasioned by the fact that these questions were ignored. Attributes which can only belong to things in the reciprocal relation which our combining thought establishes between them, were predicated of them, not without violence to the logical imagination, singly and by themselves. In order that *a* and *b* might be represented as different, without thought being required to establish the difference, the attribute was ascribed to each separately of being in itself a ἕτερον, and the act of comparison with a second thing, which alone gives any meaning to the term, was to be left wholly out of account. The negation which thought, comparing and distinguishing, expresses in the proposition '*a* is not *b*' was then treated as a positive predicate of *a* as such, the negatived term *b* being dropped out. That is to say it was treated as a not-being which yet is, and became thus credited with a reality of its own; and this confusion was reckoned an important and profound discovery. If *b* is less than *a* and greater than *c*, it was a riddle which much vexed philosophers, how the two predicates, less and greater, which, once separated from the terms of the relation to which they belonged, stood in direct opposition to each other, could be associated in the same *b*.

It would be a task not without interest from many points of view to follow up these erroneous conceptions in detail, but it would lead us too far afield for the purposes of the present discussion, which I may be content to close with the following remarks. If *a* and *b* are as we have thus far been regarding them, not things belonging to a reality outside and independent of our thought, but simply contents of possible ideas, like red and yellow, straight and curved, it will follow that a relation between them can exist only so far as we think it, and by the act of our thinking it. Only such is the constitution of our soul and such do we assume that of every other soul to be which inwardly resembles ours, that whenever and by whomsoever they may be thought, they must always produce for thought the same relation, a relation which has its being only in thought and by means of thought. This relation therefore is independent of the individual thinking subject, and independent of the several phases of his thought; this is all that we mean when we regard it as subsisting in itself as between *a* and *b*, as an object having a permanent existence of its own, which our thought discovers. It has in fact this permanent and assured character, but only in the sense of being an occurrence which will always repeat itself in our thinking in the same way under the same conditions. And this holds not only of difference but of every relation whatever which we may discover between *a* and *b*. Every time that any mind forms the idea of a perfect circle it will be found, in this case, it is true, only through a chain of intermediate ideas, that the ratio $1 : \pi$ obtains between the diameter and the circumference; this proportion therefore is valid in itself; but although thus possessing objective Validity, it possesses Being only in the form of the thought which apprehends it.

The case is different if *a* and *b* are taken expressly to signify realities, things, beings, which we do not create by thought but recognise as objects outside thought. In that case the name Relation expresses *less* than we have to suppose as really obtaining between the related things. Only so long as we are merely placing the thinkable contents of this *a* and *b* by a voluntary act side by side for comparison, would a proposition affirming a relation between *a* and *b*, or more properly in this case between the ideas or thought-pictures of them, adequately express our meaning. If on the other hand we are led in order to explain some connexion between these ideas which perception has thrust upon us, to have recourse to a relation *C* such as to subsist not between the ideas but between the things *a* and *b* themselves, of which the ideas are the thought-pictures, then we must recognise that this *C* which we have invoked cannot be a relation *between* *a* and *b*, cannot any longer therefore be a relation in the ordinary sense of the word at all. For it is thought and thought only which, passing from the idea *a* to the idea *b*, and becoming conscious of the transition, creates that which we call here a 'between,' and presents it as a mental picture which thought finds intelligible; accordingly it must always be a vain endeavour to attempt to ascribe to this relation, which at once separates *a* and *b* and brings them together, and which is nothing more than the recollection of an act of thought performable only by the unity of our consciousness,—to ascribe, I say, to this relation a real validity in the sense of being something in itself apart from the consciousness which thinks it. This supposed 'relation' can only subsist independently of our consciousness, or objectively, if it is something *more* than relation, and then it subsists not *between* *a* and *b* (for this 'between' has no existence except in us), but rather *in* them, as an influence which they reciprocally exert upon and receive from each other. It is merely for us when we think it that such influence takes logical shape in the weakened form of a relation, which no longer expresses its full significance. I must leave it to the Metaphysic[1] to show what are the conclusions to which this observation leads; to certain questions nearly connected with it I shall return directly.

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[1] [Cp. 'Metaphysic,' p. 141 ff. and Book III. ch. 3.]

339. The comparison of *a* and *b* does not lead merely to the affirmation of identity or difference; we also try to present identity *in* difference under the form of a universal as the content of a separate idea *C*. It is a criticism frequently made in Logic that our general conceptions do not possess the fixity with which ordinary thought credits them; their content is formed and their structure developed little by little, and the same conception means different things at different stages in the evolution of our growing knowledge, as fresh experiences continually enrich it. This is very evidently true of those conceptions whose content is drawn purely from experience, and therefore can only become gradually known to us; on the other hand the conceptions of an integer or a fraction, a line or a figure, will not be found necessarily imperfect in the same way. The conception of a triangle as such contains in it no more to the geometrician than it does to the scholar who follows him; the difference is that to the geometrician it *suggests* numerous relations which the scholar is as yet unacquainted with, and in this way the conception of the triangle as such appears to be richer in content to the one than it is to the other, whereas the truth is that it is only his knowledge about it which is more extensive. But leaving this, the point I wish to emphasise is that a general conception, even if we consider merely its content at any one moment, indicates a task which no actual idea, that can be presented to the mind, can fulfil. A specific red or blue colour we can see, colour in general can neither be seen nor yet presented in the same sort of imaginative embodiment as the images of red and green recalled in memory. He who speaks of colour in general reckons on his hearer first of all summoning up the mental picture of some definite colour, red perhaps, which however is accompanied at the same time by a negation by which it is made to stand not for itself, but as an example of colour in general. To this negation however, if it is not to deprive the idea of all content whatsoever, he can only give effect by calling up at the same time the ideas of other definite colours to his imagination, and becoming aware in passing from one to the other of the common element which remains constant throughout the changes of his conscious states.

It is a series of psychical operations of this nature which is the task prescribed to us, when we hear the name of any universal; but that towards which those operations are directed can never be presented as an actual idea; we can never separate that which makes red and green colours from that which makes red red and green green. It is commonly admitted as a self-evident truth, that the class to which a real object belongs is not itself real; this individual horse we see, horse in general is nowhere to be found; but it has to be understood that in thought too the universal is never more than an idea strained after but always unrealised, floating over the forms of the individual instances of it which are imaged in the mind. To these purely inward operations of thought no objective significance can attach; they remain subjective efforts of our mind, and the very form in which we express the result to which they lead us, 'in intension the universal is included in the particular, in extension the particular in the universal,' merely indicates in the symbolism of space those operations of thought through which the mind endeavours to represent as an idea the objective relation between them. And now inasmuch as, more than this, we never find the object of our search, our universal, in actual presentation at all, we are led to ask whether it really has any objective significance? Or are we to approve an opinion widely current, that it is merely the mechanism of the mind which misleads us into grouping similar impressions under general names by blurring the real distinctions between them to the prejudice of accurate thought? This theory however in fact acknowledges that which it sets itself to deny. In order to make the fact intelligible that not all but only similar ideas are thus drawn together under a common name it presupposes the fact of that similarity, and clearly, with it, only in another form, the objective validity of our assumption of a universal, which, however inseparably, is contained in them. On the other hand, if we could merely point to an innate tendency f thought to *search* for a universal, such tendency might very well be without any objective significance, but the fact that the object of the search is *found* gives it such a significance at once.

This is only an apparent contradiction to what I was saying just now, for although the universal cannot be held before the imagination, the effort to think it is still not without result. We could not so much as bring red and blue under the general name of colour, did not that common element exist in them, to our consciousness of which we testify in framing the name; we could form no class notions of animals and plants if the marks of individual plants or animals, and the modes in which those marks are conjoined, did not really possess such points of comparison as allow us to arrange them under general marks and forms, and thus by setting these in the place of the merely individual, to construct the thought-form of the class, however impossible it may be to picture it to the mind. Thus in the fact that we are *able* to think a universal, there is undoubtedly contained a truth of real and objective validity; the contents of the world of ideas which thought does not create but finds do not fall into mere individual and atomic elements, each one admitting of no comparison with the other, but on the contrary resemblances, affinities, and relations exist between them, in such wise that thought as it constructs its universals and subordinates and co-ordinates the particulars under them, comes through these purely formal and subjective operations, to coincide with the nature of that objective world.

340. If we pass from these more simple instances to the main forms of logical thought, and enquire into the significance of universal notions, we are met by the controversy between Nominalism and Realism, which excited such passionate agitation in the middle ages. To both parties the question at issue had other than a purely logical importance; the metaphysical interest predominated, leading them to think of the world of ideas mainly in its relation to the world of things. Thus Realism first misunderstanding and then exaggerating the independence of the Platonic Ideas, came to look upon the general notion as the only real existence in things, all distinctively individual characteristics being relegated to the position of merely transient and subordinate though mysterious appendages to the eternal substance of the universal. Nominalism starting from the sound Aristotelian doctrine, that reality of *Existence* belongs only to the individual thing, found no way of reconciling this with the *Validity* of the universal. Thus the Nominalist came to regard general notions as at the best mere aids to the mind in the arrangement of its ideas, possessing no significance whatever in relation to the things which the ideas represent. They even erred so far as to deny them so much value as this, and to declare them to be mere sounds which may be uttered and heard, but are wholly devoid of content or meaning.

I am desirous in the first place to avoid dealing with the subject thus exclusively in relation to the question of existence, which involves an undue limitation of the issue. In Mathematics where we find ourselves dealing not with existing things and their essence at all, in Moral Philosophy and Jurisprudence where we speak of virtues and crimes, which *ought* or ought not to exist, more than this, when in actual life we endeavour to arrive at a decision in a matter of importance by bringing the given case under a general notion:—in all these instances we meet with the universal and its laws, in dealing with objects which are given us as matter of knowledge although they are not things[1].

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[1] ['Sachlich, aber doch nicht dinghaft.']

341. If we can get out of the habit of always thinking exclusively of class notions in natural history as examples of the universal, if we recollect that we also frame general notions of figures and numbers, events and relations, truths and errors, the wild ambition to ascribe to general notions as such a reality like that of things, or at any rate of some actual existences, vanishes at once. To the original forms of substantial existences, of the plant, the animal, the human being, our imagination may if it pleases attribute an independent and eternal existence in a hypostasized world of ideas, as objects of intuition to souls which are yet unfettered by the limitations of an earthly existence. But the general notions of rest and motion, resemblance and contrariety, activity and passivity, could not possibly *exist* side by side with the former even in a world of ideas, they could only possess *validity* as predicates of the ideas. This fact, from which it is easy to see that there is no escape, we do indeed sometimes forget. We are tempted to treat qualities, relations, or occurrences, to which some prominent interest attaches, objects of our reverence or of our dread, as universals with a reality like that of an actual existence, misapprehending their purely predicative nature. We speak for instance of 'the beautiful' as of a being which is merely to us unapproachable, but in itself an object of possible intuition; we speak of 'sin' not merely as of an act which becomes real when we commit it, but as if it were a substantial force which operated upon us with an independent reality of its own. We confound the importance which belongs to the content of such conceptions in the entire system of the world with a form of reality which it cannot possess, and in attributing which to them we are merely expressing in the most emphatic terms at our command their independence of our recognition of them. This mistaken habit of thought, which is not altogether harmless, is however here easily renounced; it is only from that class of general conceptions the nature of whose content necessitates its being apprehended from the first in a substantive form, that this hypostasizing tendency continues to draw support. Here too however it has to give way before a very simple consideration. We are not content to frame, starting from the particular of perception, a single universal Q, but we go on to combine this with others like it in a higher universal P, and as we proceed with this operation, it rests within wide limits with our own logical good pleasure to determine through how many such links in the chain of universals we may choose to connect our Q with the highest universal A, at which the process of abstraction will be arrested. Each one of these universals would have an equal right to such substantial existence; side by side with animal in general would appear vertebrate in general, mammal in itself, one-toed animal in general, horse in itself, black horse in general, all equally real. I say deliberately side by side with each other, for in fact our imagination is totally unable to transfer that relation of subordination through which in our thought one such general notion includes another, to beings such as these, which are conceived as possessing all alike actual existence. Placed thus however side by side with one another they could no longer have the meaning which they purport to have. Thus we find ourselves confirmed in our conviction that this Reality which we desire to recognise in the general notions which are created by our thought is a reality which is wholly dissimilar to Existence, and which can only consist in what we have called Validity or in being *predicable of* the Existent. But how much of the full meaning of a general notion possesses this validity, and what is the meaning of possessing 'validity' at all, are questions which need some further discussion.

342. I remind my readers to begin with that we are not concerned with the question of the objective value which may attach to one or another of the general notions evolved by thought in virtue of its content being correctly constructed; the question relates to the general significance of the logical *form* as such. That this like every other of the forms which logic prescribes as ideals may be given a content which is not adapted to it, needs no special mention, but a critical review of the countless modes in which the form of the notion may be applied is not our business here. To proceed then: we saw that any content of thought S is conceived under the form of the notion, when we do not merely grasp its manifold constituents as some sort of whole, but present to ourselves at the same time a universal M whose general characteristics P, Q . . . standing in determinate relations to one another, become severally modified and defined in S in the specific forms p^s, q^s. This constitution of the logical notion does not correspond to anything which takes place in things or external objects[1] themselves; and neither does it answer to the actual nature of a content which is presented to us as matter of knowledge but not as a substantial thing. There is no moment in the life of a plant in which it is merely plant in general or conifer in itself, awaiting some subsequent influences answering to the subsequent logical determinations in our thought, to settle the question what particular tree it is to grow up into. It is true that the plant is not

[1] ['Eines Dinges oder Gegenstandes,' contrasted with 'Inhalt' (content) and 'sachlich' (matter of knowledge) in the following clause.]

while still in the germ its future self in perfect miniature; still its manner of development is not that certain conditions superadded from without produce a special determination of characteristics which were present in a general and indeterminate shape; on the contrary its characteristics are already fully determined when the conditions enter in. From the two in conjunction new results are produced of which it is misleading to say that they were contained in the earlier and more general properties as mere potential species and are now for the first time actualised to the exclusion of all other alternatives. An ellipse has no natural existence and development like a plant; still here too it is not the only way of arriving at a true apprehension of its nature to think of it first as a curve possessing the general properties of all curved lines, and then to define these properties further till we reach the particular form of them which belongs to this particular curve. We *may* indeed arrive at the conception of it in that way—supposing for instance an unpractised memory only allows us at first to recall the general outlines of the figure required, and we need subsequent reflexion to draw it exactly; but in the mathematical equations, whether they refer the shape of the line to arbitrary points of origin, or take account of some graphical method by which it may be generated, the curvature itself is not directly expressed at all; it only appears as a consequence which may be deduced from the definite ratios of the co-ordinates. These considerations hold equally as concerns the subordination of notions to one another in classification; it has no real significance in relation to the actual structure and development of things themselves. This horse was not to begin with animal in general, then vertebrate in general, later on mammal, and only at the last stage of all horse; nor can we by any means at any moment of its life separate off as an independent set of qualities the more fully defined group of properties which make it a horse, from the more general and less determinate which would make it a vertebrate, or from those most indeterminate of all which would merely constitute it an animal as such. Add to this that not only do different classifications of the same objects conflict owing to imperfect knowledge and observation, and thus introduce various and diverse ladders of universals between the highest universal and the objects, but the logical right of thought is incontestable to start from any point of view it pleases, and so to subsume the same object *S* under different general notions, or to construct its conception of the object by means of several widely divergent series of successive determinations. In such a case we are at liberty to ask with a view to the particular purpose as would contain within them the key to all problems which arise, then we might go on to select out of the various possible ptions of an object that highest or best conception, which would te its place in this supreme classification, and in which all the conceptions of it would be contained as logical consequences. reatly as the *value* of this conception for knowledge would be ced if this ideal were attained, from the importance which would attach to its content and to the mode in which this content be internally connected, for all this the Logical structure which gs to it as a conception would still represent no Real structure ponding to it in the object itself.

s value for knowledge however, which we do not dispute, gives e other side of the question, that which we mean when we all in spite of everything, that the general notion and that classifica- o at all events contain something which has to do with the thing We shall perhaps be disposed to express it by saying that the series of intersubordinated universals are contained not *actu* but *â* in the essence of the thing itself; and this proposition will be led to other and different ways in which a given content is con- ed or conceived: not really but potentially is every mark of n contained in the continuous magnitude which we break up by of it; potentially all simple motion in a straight line contains he two component motions into which we may choose to resolve *s* not 4 + 3, but certainly it admits of those figures being substi- for it for purposes of calculation. We may interpret these s into more definite language; all the processes which we go h in the framing of conceptions, in classification, in our logical uctions, are subjective movements of our thought and not sses which take place in things; but at the same time the nature se things, of the given thinkable contents, is so constituted, that nt by surrendering itself to the logical laws of these movements own, finds itself at the end of its journey if pursued in obedience se laws, coinciding with the actual course of the things them-extends and is knit together, a diversely articulated system of coherent connected series, and thought when it moves from one member in the system to another, choosing its path at pleasure but always observing its own laws, resembles in some sense a melody whose course we cannot predict yet which strikes always definite intervals in the scale each with its determinate harmonic relations.

343. When we come to the *judgment* we find that not only its logical form but its content for knowledge which is expressed through that form has in itself no direct Real significance. We give utterance to the categorical propositions 'This tree blossoms,' 'Atmospheric air is a permanent gas,' 'Every triangle has its angles equal to two right angles.' In the first case it is merely thanks to the subject-matter of the proposition that we are able to ascribe to the tree an existence which really is independent of the temporary condition of blossoming, that is to say that the subject and the predicate are actually related and separated as we divide and connect them in the form of the judgment. In the two other cases this separation is not to be found in the thing itself, it is a purely subjective movement of thought arbitrarily selecting one particular constituent in a whole which is really a unity, to be made the object of separate attention. The differences in the copula again in the three propositions are due merely to the imagination, which adapts itself to the peculiarities of each separate subject-matter, and finds an expression for them in language; logic itself testifies by representing all judgments under the symbolic form *S* is *P*, that in the uniform copula 'is' all objective distinctions in the connexion between *S* and *P* are obliterated. They may be related as whole and part, as a thing to its transient states, or as cause to effect; in the form of the judgment they appear solely as subject and predicate, two terms which denote merely the relative positions which the ideas of them assume in the subjective movement of our thought, and tell us nothing as to the objective relation in itself which *if* it becomes an object of thought compels the ideas to assume those positions in our thoughts. Once more, in Hypothetical judgments we do indeed appeal to an objective relation of this sort, but in the form of the judgment we neither express it nor make it intelligible. The conjunction of antecedent and consequent in the form, 'If *B* is true *F* is true,' in itself affirms no more than the proposition that *B* and *F* belong both together and in some way not defined to a single notion *M*. The fact that we notwithstanding divide this coherent unity and place one part of the notion in front of the other, so that by reason of the inseparable connexion between the two, the one becomes antecedent, the other consequent,—all this is once more simply one of those subjective movements of thought which do *not* take place in the content of the notion. And this subjective character of the movement is shown by the fact that we have it in our power to reverse its direction. We say, 'Every equilateral triangle is equiangular,' or 'If a triangle is equilateral it is equiangular,' but we might say equally well 'If it is equiangular it is equilateral.' That which constitutes the objective content here is the undivided thought or the intuition of the equilateral and equiangular triangle; the two constitutive elements, equality of sides and equality of angles, are simultaneously present in it, but thought taking an arbitrary starting-point at one or the other moves up and down between them dividing and uniting in its own fashion. This holds of all judgments which like those of mathematics are occupied with the ideal and not with the actual. They would all admit of simple conversion, if their expressions in language through the medium of propositions allowed of all the conceptions which occur in them being as precisely defined as is the case in the form of the equation.

If on the other hand our hypothetical judgments relate to data of reality, in such cases our intention is certainly that the antecedent and the consequent are to be taken as not interchangeable, but the hypothetical form of the judgment does not in itself express the condition which makes that assumption true. For given the antecedent B there is logically no interval left which separates its validity from that of the consequent F; the two together constitute, in perfect accord with that which the hypothetical form of judgment itself affirms as its result, a single process M which can be expressed in a judgment. And further inasmuch as if we take our conception accurately, leaving nothing out and adding nothing to it, no F^1 can be connected with our B but F only, and no B^1 with F but B only, it follows that we pass in thought with equal right and necessity from either of the two starting-points taken at pleasure, to the other, from B to F just as much as from F to B; we know the consequent from the antecedent and the antecedent from the consequent. That in actual fact there is here some circumstance which makes B and B alone the *Antecedens*, and F and F alone the *Consequens*, we are very well aware, because we are acquainted with the subject-matter under consideration, but it receives no expression through the form of our logical act. For that form depends upon nothing more than the abstract notion that F is in a general sense *conditioned* by B; but this, a mere abstract relation, is as shown already, something less than anything that we obtain in reality between *B* and *F* as things or events. A relation through which *B* and *B* only is to be the antecedent, and to be a real antecedent, can only actually obtain if *B* is cause and *F* effect; but in the hypothetical judgment instead of this real and specific relation of causality we have nothing but the vague and general relation of conditioning in the abstract, which thus has no significance for reality whatever.

Finally Disjunctive judgments do not even purport to express any reality at all; the process of wavering undecided between several mutually exclusive predicates can answer to no process in the real world; it remains a state of our thinking, to which the adequate data for the knowledge of reality are lacking.

344. A brief consideration of the various forms of Syllogism leads us to similar results. We shall be most readily disposed to ascribe a Real significance to those Figures of Subsumption which arrive at their conclusions by bringing the particular under the universal, for this subordination we do certainly regard in the sense already sufficiently explained as a notion which possesses an objective validity in relation to everything that can be presented to the mind as an idea.

Still here also the logical form of the argument does not correspond to anything that takes place. In mathematical syllogisms the universal major premiss, from which we derive our more particular conclusion, has no priority of truth as compared with the conclusion or with the minor premiss; all three are parts of one eternal truth, all possess a simultaneous validity. The priority of greater simplicity or more immediate evidence the major premiss may indeed possess, but both the one predicate and the other would belong to it in relation to our thought only, without giving it any superiority in itself over other propositions of equal certainty. Lastly there is nothing in the form of inference by Subsumption which obliges it to start from a major premiss of this simple character at all; on the contrary the simultaneity of the connexion which obtains between the entire body of mathematical truths allows the simpler among them to be derived as limiting cases from a logical connexion of less simple, no less than the other way, and always in this figure of Subsumption.

This purely subjective significance of the form of the syllogism we sometimes forget in applying it to matters of fact. So long indeed as the universal major expresses a highly concrete and specific truth, when for instance we say 'All animals breathe,' we never question that such a major premiss cannot designate any reality which is prior to the validity of the conclusion 'Fishes breathe,' anywhere but in our thought. Yet when we turn to the most universal principles of the system of things, the impulse comes back upon us to give to the expression of those principles, the most universal laws of nature, which present themselves as major premises in our enquiries into the order of the world, a real priority, which is in fact wholly inconceivable, to the processes in which they are to hold good. This impulse is not without danger to the soundness and consistency of our metaphysical theories; it leads to a superstition which has far reaching consequences, that the reality of the world may be derived from something which is unreal and which is yet essential and possessed of a regulative power, whereas on the contrary we have thoroughly to convince ourselves that all necessary truths, to which we imagine that we can subordinate the existent as if it were something merely secondary and additional, are simply the nature and self-consistency of the existent itself, and are only disengaged from it by a reflective act of thought and credited accordingly with a prior and regulative character to which they have no claim.

Inferences by Induction do not give occasion to this sort of misunderstanding; no one fails to see that the synthesis of particular facts in a general, not merely a universal[1], proposition is not the real ground of the validity of the general proposition but only of our apprehension of that validity. Still more convincingly does the variety of forms, which a Proof may assume, witness to the merely subjective significance of the several inferences of which it is made up. How many different proofs, direct and indirect, progressive and retrogressive, all equally adequate, may be given for one and the same proposition! How many even in the form of direct progressive argument alone! And supposing that in fact one out of the many could possess the prerogative of alone exhibiting the essence and actual structure of the thing, still the mere fact that other forms of proof are possible would always show that it is not the logical Form by itself which occasions or expresses the Real validity of this particular form of proof, but that its superiority over other forms of proof lies in the content which we have taken and conjoined in this form. Lastly in regard to the final operations of thought with the account of which the doctrine of pure Logic concluded, we saw there that Logic does in those operations strive to discover some Forms in which the proper essence of the thing, as distinguished from our mere subjective and haphazard notions about it, may be exhibited. But there too we come to the conclusion that those Forms turn out to be far wider than that which they purport to contain. If the proper essence of the thing does make its way into our thought, it can only be apprehended under these Forms, but the Forms do not create it and do not fully express it; they admit always of fresh applications which issue as we are ourselves conscious in merely subjective notions, and from among which the selection of the more trustworthy in relation to reality cannot be made by the help of Logic but only through knowledge of the subject-matter, if such knowledge is forthcoming.

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[1] [See p. 73, sup.]

345. It is now time to determine more exactly the meaning of certain expressions in the use of which I have hitherto been somewhat less precise. We have spoken of Subjective and Objective, of Formal and Material[1], of Formal and Real significance, as applied to the Forms of Thought. The three pairs of antitheses do not coincide. If we distinguish, as we have done, between the logical act of thinking, and the thought which it creates as its product, the former can claim only a *Subjective* significance; it is purely and simply an inner movement of our own minds, which is made necessary to us by reason of the constitution of our nature and of our place in the world, and through which we make that Thought, for instance the distinction which exists between *a* and *b*, or the universal *C* which is contained in them both, an object for our own consciousness. In the same way every one who desires to enjoy the prospect from a hilltop has to traverse some particular straight or winding path from the point at which he starts up to the summit which discloses the view; this path itself is not part of the view which he wishes to obtain. The Thought itself on the other hand in which the process of thinking issues, the prospect obtained, has *Objective* validity; the various paths followed by various travellers once traversed and left behind, the scene which opens before them is the same to all alike, an object independent of the subjectivity of the individual; it is not merely one more affection of his consciousness which he experiences, but an object presented to his thought which also presents itself as the same self-identical object to the consciousness of others.

The second antithesis[2] throws light on the same state of facts from another side. It would not be sufficient to call the operations of our thought Subjective and nothing more. The term would simply separate them from that which actually goes on in the subject-matter[3] with which they deal, leaving it quite obscure what the relation is in which they stand to it; yet after all some such relation there must be, if the Logical Thought in which they issue, is to possess an Objective validity which does not belong to the thinking act which issues in it. Accordingly we call the logical operations not Subjective *merely* but *Formal* because their characteristics though not the actual determinations of the matter they deal with[1], yet on the other hand are Forms of procedure the very purpose of which is to apprehend the nature of that subject-matter, and which therefore cannot stand altogether out of connexion with that which there has place.

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[1] ['Sachlich,' opp. to 'formal.']

[2] ['Formal' and 'sachlich.']

[3] ['Von dem Verhalten der Sachen.']

Upon this point the illustrations adduced above will remove all uncertainty. The limitation to a *merely* Formal validity showed itself in the fact that there may be several processes of thought equally successful in view of the result arrived at, all, that is, leading to the same final thought-product, or the same material result. No one of them therefore can have an exclusive significance as regards that determinate matter and content with which all are equally concerned; all alike are merely forms of procedure, employed to reach a certain result which once obtained is valid independently of the path which led to it. But clearly it would be impossible to arrive by all these different paths at the summit from which this prospect opens, if they were not all included with their determinate positions and relations the one to the other within that same geographical territory, the remaining part of which is what constitutes the landscape which is commanded from the summit. Herein consists the positive element which this second antithesis affirms of the processes of thought[2]; each is one among the various ways in which the variously ramifying systems of the world of fact makes it possible for us, by reason of its universal interconnexion, to arrive by a process of movement from point to point within that world, at a determinate objective relation, although the particular movement chosen neither is nor yet copies the way in which *this* relation itself arose or now obtains.

The third antithesis[3] is not merely another way of expressing the second; it relates to a specific question. We regard every content of thought as having a material value[4] if it has a fixed Objective significance in the sense above explained—ideas of the non-existent no less than of the existent; by the term *Real*[5] we should have to understand only things and events in so far as they exist and occur in an actual world of their own beyond thought. Now it is out of the question that this kind of Reality should move and have its being in the forms of the Concept, of the Judgment or of the Syllogism, which our thought assumes in its own subjective efforts towards the knowledge of that reality. But even the logical thoughts which are the issue of those operations have not in relation to Reality in this sense the immediate and material validity which belonged to them in relation to every content of thought as such. It will be better to reserve for the Metaphysic[1] the fuller discussion of this important point; a reference to the illustrations already adduced will suffice in the way of a preliminary elucidation.

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[1] ['Der Sachen.']

[2] [They are not merely 'subjective' but also 'formal.']

[3] [In German 'Formal' and 'Real.']

[4] ['Sachlich gegeben.']

[5] [German 'Real.']

We saw that the notion of a condition is inadequate to denote that which we mean by a relation which subsists in actual fact between two real elements; so to subsist, it would have to be more than a relation, it would have to be nothing less than interaction. This being so, it was in that Real connexion between the Real elements that the cause resided which brought their phenomenal appearances for us into that particular formal relation which we now, employing a merely logical term, *call* a conditioning of one by the other.

The same is true of all logical Forms. No real S can be subject and nothing more to a real P, which is its predicate and nothing more; in actual fact P can only attach to S either as a state which it passes through, or as an influence which it exerts, or finally as a permanent quality which belongs to it in the sense (a sense it is true at present somewhat obscure) in which we contrast the metaphysical notion of a Quality with the merely logical notion of a Mark. It is not till one of *these* three relations has been affirmed that we understand what the meaning is *realiter* of the logical conception of S as subject and P as predicate. It is not till then that we have an actual state of things answering to the logical copula, which in itself leaves it quite undetermined what precisely we are affirming to have occurred to the real things in question, when we feel ourselves necessitated thus to connect the ideas of them. When then we employ such expressions as unity, multiplicity, equality, contrariety, relation, condition, so long as we use those terms by themselves, we have said absolutely nothing about the existent. We have still to show how it is brought about that the unity of the One is proved to be an actual reality, not merely a barren logical title; how it is that what are many but identical, although in thought they simply are identical, nevertheless in real existence break up and become many; what is the one kind of reciprocal influence in which the opposition, what is the other kind of reciprocal influence in which the relation, of different existing things, shows itself to be real.

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[1] [See 'Metaphysic,' p. 142 ff.]

CHAPTER V.

The a priori truths.

346. LET us put together once more the conclusions to which we have been brought. Neither in the content of our ideas nor yet in the reality which we regard as its source outside, was there anything to correspond to the logical processes of thought, which choosing their path at will, connected or separated the several constituent elements of which that content was composed. On the other hand, at least in relation to this content, without regard to that reality which may be its cause in the world outside, the Thought-product, in which it was the aim of the Thought-processes to issue, had, we saw, an objective significance. The differences, the resemblances, the contrasts, the subordinations, of which we could only possess ourselves in consciousness by help of the discursive activity of Thought, passing backwards and forwards from point to point, had we saw an actual *validity* as applied to the apprehended content, although the content itself in no way participated in such movements. They subsist, as we saw, independently and objectively in the sense in which any other relation may subsist between the terms related. Real existence, that is to say, they can never claim except at the moments in which they are thought; but on the other hand, such is the common constitution of all minds, that whenever the given terms of the relation *a* and *b* are thought, one and the same judgment *C* affirming this relation between them is immediately and invariably pronounced.

We are here brought back to the Platonic world of Ideas. All contents of possible ideas stand in fixed and unalterable relations, and by whatever processes or movements of thought, as our own pleasure or as chance determines, we may carry our attention from one to another, or in whatever order they may be one after another brought to our perception by occasioning causes even unknown to ourselves,—we shall invariably find the same relations obtaining amongst them which are given us once for all in the objective and endlessly complex structure of the world of Ideas.—So often as this proposition is insisted on it will be regarded as an entirely superfluous affirmation of that which is perfectly self-evident, and just as often I must repeat that the very existence of this self-evidence is the most astonishing thing in the world. Although an indispensable foundation of all thought, and just on that account passed over by us in our presumption as a mere matter of course, it is not even, as I observed before, a necessity of thought in the sense in which that character may be claimed for the particular relations which it includes within it. We cannot indeed fully realise in thought what the state of things would be if this fact were wanting, but still we can imagine a world in which it did *not* obtain; in which countless contents presented themselves for our minds to form ideas of, but each one standing in no relation to the rest, all so entirely disparate in nature that no two of them could be combined as allied species under any common universal, nor any two of them be pronounced to differ from each other more or less or otherwise than any other two. One postulate alone, in such case, Thought would be in a position to make, in obedience to its own law of Identity, namely, that each one of the contents must be identical with itself. This postulate would be the condition of their being presented to thought at all, and it might be fulfilled by such a world as I am supposing. But beyond this we cannot go. Thought may wish, in order to the possibility of its further operations, but it cannot demand as a necessity of thought, that between the different objects there should be found that graduated scale of affinities which alone enable it to accomplish the ends after which it strives,—it is not a necessity of thought that thought itself should be possible. And even supposing that by its own intrinsic power it could postulate those affinities, still it could not make them; it would always have to trust to their being given it by the grace of facts, ordered and arranged on principles which it could never have itself contrived, as series of tones or colours, or as differences in degree among things qualitatively the same, or in any other way.

But strange and important as is the fact that such affinities in the world of experience are actually found, it is not in this fact or in the consequences which follow from it, that the final goal of our enquiries lies. All that it guarantees us is the security with which thought is able to move within the world of ideas as such, to investigate the systematic and invariable connexions obtaining among the elements of that world, and by conjoining them one with another to construct new forms which will be found without fail in another and a predictable place in the world of ideas, so connected, finally, one and all together in various directions and at fixed distances, that the most diverse and the most roundabout tracks of thought may lead to the certain discovery of any one of them. This however by itself is not all that we are concerned to know. What we want to arrive at is the significance which is to be attached to this systematic arrangement of the world of knowledge in relation to that empirical and unsystematic order of events, in which a causal reality independent of thought presents contents of possible ideas to our perception. What we wish to understand is not only the classification of things which is eternal, but also the course of things which is in change.

347. The two are completely distinct. Perception does not present those objects to us in connexion which stand side by side as akin in the system of knowledge, nor is its entire history a periodically recurring procession of orders, genera, and species, following one upon the other in a descending scale as they do in the order of classification. Contemporaneous in different points in space, succeeding one another at different points of time, we find the most heterogeneous elements of that realm of contents phenomenally connected; if laws in this scene of change there are, they are of a different kind altogether from the logical laws which have hitherto been engaging our attention. If we agree henceforth to designate the empirical course of phenomena as it is thus presented to us, Actual Reality [1], then the question is as to the significance which our thought can claim in relation to it, since its affirmations even though retaining their validity, seem nevertheless to be incapable of controlling the order of connexion which the reality presents. For even supposing it to be true that a and b will exhibit, when given in actual perception, the same distinction and the same affinity which belong to them in our thought, still this tells us nothing as to whether they will actually be found in conjunction in perception, or whether that conjunction may not be an impossibility. Admitting the law of identity to hold without exception, still it does not profess to do more than affirm that now and always every $a = a$, and every $b = b$, *whenever* and *wherever* they may be found. But here at once the last clause is no longer part of the law of identity itself; we append it because we know on other grounds that possible objects of thought are susceptible, over and above their eternal validity in the world of ideas, of an alternation of temporal reality and unreality in the world of phenomena. Of this the law in question contains no indication, and cannot therefore in the least determine the order in which in that world whether in the way of simultaneity or succession, the two phenomena necessarily introduce or necessarily exclude one another. Again, the classifications by which we range our conceptions one under the other will be valid equally of our perceptions and of the timeless content of our ideas; but when we bring a perceived object *S* under the general conception *M*, although all the higher universal conceptions *NLK* which are contained in *M* are now valid of *S* too, still this deduction gives us no new objective knowledge, but only a logical analysis of what was already implied in bringing *S* under *M*,—correct if this was correct, incorrect if the contrary, but in neither case enabling us to combine the *S* given in the perception with a *P* which has not been so given.

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[1] ['Reale Wirklichkeit.']

Hypothetical judgments seem better adapted to an extension of knowledge. In so far as they apply to a subject *S* a condition x, and derive from the two together a predicate *P*, which was not already contained either in *S* or in x by themselves, they make at least a formal approach to that which we conceive to take place in reality. In the problematic antecedent they express the connexion of *S* and x as a possibility, and accordingly distinguish the thought content of it from the realisation which may be in store for that content in the actual course of events, and as to which they abstain from affirming anything. On the other hand, that condition once given, they do seem to anticipate the after perception and to define the new result which will necessarily follow in this perception. But now what is it that justifies us in subjoining to, or equating with a determinate $S+x$ a determinate *P*? In thought it can rest only on this, that by means of a logical determination x we transform the notion *S*, which previously did not contain *P*, in such wise that now it does contain it; and now it is of this new subject, not of the one we began with, that we affirm the predicate *P*, which in fact we have already taken into it. But that which is directly presented to us in perception is something different from this. When in actual perception a new phenomenon x enters into relation to a previous phenomenon *S*, what happens as a rule is not that from the conjunction of the two in thought there results the subject $S+x$, from which thereupon the resultant phenomenon *P* would follow as a matter of course as if they were equivalent expressions. On the contrary the question has still to be solved, how it is possible for x so to transform *S*, that there may spring from it the conditions for the realisation of *P* which were before wanting. Thus, wherever we apply hypothetical judgments to questions of the real world, they are always found to rest in the last resort upon certain presuppositions. They always assume the validity of certain propositions affirming the connexion of a particular condition with a particular consequence—a connexion which cannot be deduced from conceptions—to be a universal fact. If it is really universal, then thought can draw it out into its particular instances by a purely analytic procedure, but its real content appears, to begin with, as a synthetical judgment, which binds together as subject and predicate two conceptions, the contents of which mere logical analysis can never prove to be identical.

348. Our hope then of mastering by thought the course of events in the real world, rests on three points. First, to no single constituent *b* of the ideal world can thought ascribe, over and above the eternal validity which within that world belongs to it, a necessity of realisation in the order of events in time; it is only if this reality belongs as a matter of fact to a second such element *a*, with which *b* stands in necessary connexion, that it can then pass over to *b* also. All our knowledge therefore is in this respect hypothetical; it strikes in at a particular point in a reality which it finds as a matter of fact given to it, in order to deduce from this real premiss as themselves *real* the consequences which attached to the thought premiss as *necessary*; but it is never possible, starting from mere conceptions of thought, to prove the actual reality of that which is contained in them. And in fact the attempt has never been ventured upon except in the single instance of the ontological argument for the being of God. The temptation in this case was very intelligible. The conception of God as a necessary consequence *b* following from a reality *a* other than Himself, and given in perception, contradicted our necessary idea of Him, for this very idea demanded that He should be conceived as the ground of all consequences. Hence, it seemed, nothing remained but to seek the reality of God in the idea itself of Him. True all that could really be found was the *claim* to reality which the idea carried with it. Beyond question the idea of God includes the idea of Being, and more than this, the idea of living Being; for all other predicates by which we think of God as God, can only be unified, or even thought, when they are conceived as belonging to a real Being who fills time, and is capable of undergoing a change of states. But in this sense the idea of any being whatever includes the thought of that particular kind of reality which the nature and the mode of combination of its content require. The very notion of an organism is unthinkable without this assumption; the properties of nutrition, growth, propagation of its kind, have no meaning when applied to a subject which does not exist, and just as little when applied to one which exists merely and has no faculty of development. If therefore the objects of our conception are to have reality at all, they must have that kind of reality which answers to their nature, Beings that of existence not of occurrence, Events that of occurrence not of existence, Relations neither the one nor the other, but a reality which consists in being valid of reality. It was a mere illusion to suppose that the case was different with the idea of God, and that it was allowable to look upon that notion of the highest reality which is necessarily included in that idea as equivalent to the reality of the whole content which included it.

A class of arguments nearly allied to this, which pass from the incontestable *value* of an object of thought to the belief in its reality, have an appearance of committing a fallacy of the same kind, but in this case it is an appearance only. It is not altogether just to maintain that we believe in a supreme Good, in a life beyond the earth, in eternal blessedness, merely because we desire them. In reality such beliefs rest upon an extremely broad, though an unanalysed foundation of perception. They start from the fact of this actual world as it is given us in experience, in which we find certain intolerable contradictions threatening us if we refuse to acknowledge that these ways in which the structure of the world extends beyond our perception are real complements of that which we perceive. In form, therefore, this class of inferences is quite legitimate; starting from the reality of *a* as given in experience, they connect with it the reality of *b* which is not so given, but which appears to follow from *a* as a necessity of thought.

349. The second point alluded to is tacitly assumed in every argument, but seldom explicitly acknowledged as a necessary logical assumption. Clearly we could never hope to work upon reality through the medium of thought, if we were not in a position to assume in the empirical order of things the presence of universal law, which alone makes it possible for us to turn the formal laws of our thought to positive use. We saw that the real causes which determine the succession of our perceptions of possible contents of thought are wholly independent of the systematic relations which we find between those contents when regarded as objects of thought simply. Whence then do we derive our assurance that there are reasons of universal validity at all determining this order of succession, and that the unknown cause of the experienced series of our perceptions is not simply playing with the elements of our Ideal world and its systematic classifications, itself void of all principle, bringing before us like a self-acting kaleidoscope now one arrangement of the picture and now another, but observing no law or order in its combinations?

We have no ground whatever for representing the wild disorder which this supposition implies as unimaginable; there is a very great deal in the empirical world which we do not yet understand, that actually does still so appear to us; if throughout the world of reality all regular law and regular relations were altogether absent, all we can say is that the same spectacle would then be presented to us everywhere which meets us now in cases where the laws are concealed from us. The laws of our thought would still hold good, but in the sense of an empty postulate, to which reality would offer no counterpart, just as there are many events even now to which we seek in vain to apply them, events which seem with their like conditions and unlike results to mock at our principle of identity. Nevertheless this assumption of an independence of law in the real world is maintained by no one; in every case where observed phenomena might seem to force it upon us, we regard the state of facts so presented as simply a problem which awaits solution, and we never doubt that a wider experience will furnish links of connexion hitherto unobserved to restore order and regularity to the observed parts in which at present they are not to be found.

Now on what does this confidence rest? The universality of laws in the real world is neither in itself a necessity of thought, nor can it be deduced as a necessary consequence from given facts. We might have the right to say that the laws of space, even supposing space to exist only as an innate intuition in us, still must of necessity hold good of all objects of our experience, for nothing will ever make its way into experience without having been already moulded in that form of space through which alone it becomes an object for us at all. But we cannot attempt to prove in the same way that unless there was a connexion according to law in the real world the experience which we possess would be impossible. That which we actually possess is merely a succession of ideas; that this succession constitutes an unbroken connexion in accordance with universal laws, that is to say that experience in this heightened sense, as distinguished from mere perception, is also actually given us,—to affirm this is to confuse that which we know as a fact with preconceptions of our own which we bring to the facts. For our actual knowledge amounts to no more than this, that a large number of occurrences admit of being regarded *as if* they were conditioned by universal laws; there remains always a far larger number which we have not yet succeeded in thus reducing to order. A reign of law embracing all reality, and admitting of no exception to its rules, is therefore neither an actual nor a possible outcome of experience, but only an assumption with which every enlargement of our experience is accompanied.

We have therefore only two alternatives. Either we may acknowledge this assumption as an assumption and trust it, and thus credit ourselves with this one piece of certain knowledge, by the help of which our thought, crossing the boundary of its own domain, reaches one certain result as to the nature of reality; or we may look upon it equally as a mere assumption, and on that account *distrust* it, accepting thankfully such instances as confirm it, but always bearing in mind the possibility of finding ourselves stumbling at any moment upon ground where it no longer holds good. Whenever human reflexion has reached the point of a scientific view of the external world, it has without exception preferred the first of these two alternatives. Even those who are most careful to resist any undue encroachments of reason, and pride themselves on interrogating nature and nature only as to her own laws, never question the fact that such laws do universally obtain, they only insist that we know nothing about them. Only they do not observe, that in thus affirming the universality of law they are passing beyond the data of reality, and are making in one clause an *a priori* assertion about it which the next declares to be illegitimate.

The alternative theory may be thought to be discoverable in one particular instance, the belief in the freedom of the human will. As to the material rights and wrongs of this hypothesis, I am not here called upon to decide. But with regard to its form, it is only in appearance that it comes under the point of view in question. It does not assert that the same thing is free at one time and conditioned at another without any reason. On the contrary, subjecting as it does one sphere of reality permanently and without exception to determination by fixed laws, and connecting the fact of freedom exclusively with the presence of a particular spiritual nature in the subject which wills, it does in fact assume that the system of the world is throughout a system of law, and merely ascribes to it the peculiar property of admitting at particular points in its course of the entrance of unconditioned elements, which once admitted into the world of reality thenceforth produce results which are conditioned by law. This theory also then, and more clearly still any theory which, denying freedom, brings the inner world as well as the outer under a system of determinate laws, permits itself in so doing to make an *a priori* affirmation concerning the real world, the universal validity of which experience as such can never prove. Whether it is justified in so doing, can never be decided by strict logical argument, for every attempt to prove this affirmation a necessity of *thought*, would leave the question of its validity as applied to the real world undecided. On the other hand, to attempt to exhibit it as agreeing with the nature of reality, would only be to repeat in a new form the old claim which it is desired to establish, the claim to be able to make *a priori*, that is to say *universal* statements on the authority of thought alone, about that real world, of which experience can never give us universal knowledge. We have therefore the right to say that all our conclusions concerning the real world rest upon the immediate confidence or the *faith* which we repose in the universal validity of a certain postulate of thought, which oversteps the limits of the special world of thought. In point of fact this confidence which logic can never justify lies at the foundation of all logic, as it does also of that formula in which we described it as the universal tendency of thought to turn the observed fact of co-existence into coherent connexion. The methods of applied logic one and all have a meaning only on the assumption that that inward coherence and connectedness which this tendency ascribes to the real world does actually belong to it. To suppose it otherwise would be to cut away the logical standing-ground on which induction relies whenever it pronounces one inference drawn from experience to be even more *probable* than another; it would have to be content with rehearsing the premises, the conclusion would be wanting.

350. There remains the third question. The assumption of a connected system of uniform laws embracing all reality does not by itself teach us what the particular laws are, in accordance with which a definite event *b* is conjoined with another event *a*. Further we have already satisfied ourselves that the mere analysis of the contents of the notions of *a* and *b* as such could never enable us to affirm that the realisation of the one must necessarily be followed by that of the other. Two courses remain open to us: either to lay claim to an immediate certainty of the universal and necessary validity of synthetic judgments which nevertheless demand such a connexion, or else to content ourselves with extracting all the particular laws of reality one by one from the evidence of experience by the help of the methods expounded in the last book. At this parting of the ways I wish by one general formula of ready worship to purchase a dispensation from any further glorification of the second of these two alternatives. It becomes in time wearisome to be told over and over again in endless iteration, how reason is to come to nature in a spirit of self-renunciation, how indeed from her own resources alone she cannot possibly decide a single question, and how she at once wanders off into a world of brain-spun phantasies if she does not at every step apply to experience for her *data*. Unhappily we cannot affirm that such warnings are superfluous, or that they are nowhere applicable, for errors enough have been due to the neglect of them. Still any moral sermon becomes intolerable if it goes on for ever, and at last its only effect is that it moves us, as we are moved here, to ask the question whether the claims which the doctrine advocated holds up for our acceptance are not just as one-sided as confessedly those are which it undertakes to disprove. Can then, we ask, the purely empirical investigation of the laws of the actual world really solve its problem entirely from its own resources, calling in perhaps the aid of the law of identity, but otherwise without making assumption of any synthetic judgments *a priori*? That it cannot do this, was the doctrine of Kant; if we arrive at a similar conclusion, we shall be championing a characteristic tenet of German philosophy, which has brought on us assaults from all nations.

351. English scepticism in the person of Hume endeavoured to restrict us on the one hand to the expression of mathematical truths, which appeared to Hume to rest simply upon the principle of identity, and on the other to the narration of the facts of history, which having once occurred are thenceforth matter of actual experience, and can be expressed in synthetic judgments *a posteriori*. No scientific inference was possible, he thought, which should predict the occurrence of a *b* in the future on the strength of a given *a* which was not identical with it.

Before I go on to discuss the last-named contention, it may be useful to point out, that if it is valid, then the previous contentions made with regard to mathematical and to historical truths cannot be. The possibility of synthetic judgments *a posteriori* is a point which does not sufficiently arouse our suspicions, because they are taken for simple expressions of experience, into which no admixture of too forward thought has made its way. But so long as they are judgments at all, no matter whether expressed in language or not, they are still not the facts given simply, but a preparation of the facts, made by reading into them an inner connexion which in immediate observation is not to be found. No narration of an event is possible except by combining together as subject and predicate one portion of the sensuous images which arose in us when we witnessed it, with another, and then going on to think in between the contents of these two conceptions a relation of action exerted on one side and received on the other, or again of mutual alteration of states, none of which relations are in the least degree given in the perceptions as such.

It may be contended that the proposition Cæsar crossed the Rubicon, means no more than that a certain partially changeable, but still coherent group of sensible impressions, which for shortness we call Cæsar, changed its position in space in relation to a second group of sensible impressions, which we call the Rubicon, in such wise as to be perceived by one and the same spectator first to the right of the latter group and then to its left. I answer with no less obstinacy: that this group was the same group on the left as on the right, that is to say that it has changed *its* position,—this does not lie in the simple data of observation, but is a hypothesis which covertly introduces under a connected and continuous alteration of the appearance a permanent substratum with merely changing relations. Whenever in recounting an event we speak of any sort of movement in space, we are giving not our perception, but a hypothesis about it. That one and the same real a[1] passed through one after the other the places m, n, p, is not a fact we have seen; the fact perceived is only that in successive points of time similar appearances a were observable in successive points of space. One who was under no necessity to explain this fact to himself by the hypothesis of a permanent subject, could not venture to affirm the proposition a has moved, as a description of the facts, but merely as a convenient mode of expression, having in relation to fact no significance whatever. If he denies himself this introduction of certain points of view into the interpretation of the content of perception, then he must acknowledge all synthetic judgments *a posteriori*, all judgments in fact of whatever kind, to be inadmissible, and instead of a recounting of past facts there remains in fact merely the possibility of recalling in memory a series of perceptions, a reproduction of the raw material, out of which judgments might be formed, if only such a proceeding were allowable.

352. Turning to the question of the discovery of mathematical truth, we shall not dispute the validity nor yet the importance of the principle of Identity, but we must dispute its fruitfulness; we must insist that if it were the only principle we had to start from, mathematical truth could never be discovered at all. It is no doubt true that in any proposition affirming equality or inequality, $a = b$ or $a \gtrless b$, we have always to assume the validity of the principle of identity, according to which $a = a$, and $b = b$, in other words that every quantity which we desire to bring into any relation with other quantities, is identical with itself, for obviously every such comparison of different quantities loses its meaning if the quantities compared may have an unlimited variety of meanings. Here the principle of identity has a validity which is manifest enough and is the necessary security for truths of whatever kind. But it is precisely from this point of view that least attention has in fact been paid to it; that which has been more especially emphasised is that very different application of the principle, by which the two quantities compared are pronounced equal to *each other*. It is in this application of the maxim of identity that philosophers have found not only the guarantee of truth, but when repeated in frequent succession through a long chain of such equations, a fruitful method for its discovery.

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[1] ['Dasselbe reale a']

I cannot think that either the one of these contentions or the other precisely expresses that which is intended. Equations either as in $\sqrt{4} = 2$ express simply the definite quantitative value which is arrived at by an act of calculation as applied to a given quantity, or else they express the fact as in $\sqrt{ab} = \sqrt{a} . \sqrt{b}$, that certain operations, different in form, applied in a prescribed order of succession or of connexion to any given quantities within defined limits will give identical results. Now in both these cases the value of the entire mathematical process depends not solely upon the discovered equality of the result but rather upon the fact that different paths have led to the same goal, that is to say that it has been found possible to affirm the equality of different things. If I am answered that the quantitative values of the two different terms in the comparison have not been made identical as an after result of that operation but were so always, and that the identity was merely concealed under the different forms in which the two were originally presented, or that the one form of expression merely sets the problem of which the other gives the solution,—such a reply expresses precisely my own view, only that it takes as self-evident that which I cannot regard as being such.

For whence do we derive our confidence in the possibility of one and the same self-identical value being presented under different forms? Certainly not from the law of identity alone; for it contains not the slightest hint of an antithesis between Form and Content or Form and Value; nor supposing that we derived our idea of such an antithesis from some other source, could the law of identity even then tell us anything whatever about it. It could only tell us over again, every Form is identical with itself and every Value with itself. That one and the same Value can be present under different forms, it could never affirm, because it could never fix any limit to the validity of such a proposition except one which would reduce it to a barren tautology. For to the question *what* different forms of expression designate identical values, it could only answer, those in which one and the same identical value is contained.

I need not here enlarge on the fact, that it is in this possibility of affirming the equality of the different, and not in the bare application of the logical law of identity as such, that the motive force of all fruitful reasoning in mathematical science is to be found. We should never get any further, if we could never subsume under the subject of a given major premiss anything but a term absolutely identical with it; we do make progress just because by means of innumerable substitutions, by a process of analysis on the one side and recombination on the other we are able to bring a quantity given us in the form a into the form b, and thus to subsume it on any occasion under such a major term, as then enables us further by known methods of calculation to give it a predicate which was not deducible from it in its original form. Everything turns therefore on our right to affirm identity of the different, and this right does not follow, at all events as an immediate consequence, from the purport of the law of identity.

353. The remainder of my argument here must be taken in connexion with the considerations which I urged when dealing with the subject of pure Logic as to the nature of judgments synthetic in form but identical in content. I there made allusions to Kant, who in endeavouring to prove the presence of synthetic judgments *a priori* in all branches of reasoning included under that category the arithmetical judgment $7+5=12$. My object at that point was to insist on the identity of content which must necessarily obtain in any true proposition between the subject and the predicate taken in their entirety. I was dissatisfied that this point should not have been more expressly insisted on by Kant, but I then reserved the right to revert again to the truth which his doctrine contains (§ 58). Kant held that we could not possibly recognise in the predicate 12 the solution of the problem expressed in the subject $7+5$, without an act of Perception[1]. Perception alone, that is, he insisted, can establish for us that the identity required between the two sides in order to the correctness of the equation is actually the fact. Considered for purposes of illustration, indeed, I think that Kant's example was not happily chosen, because it does not

[1] ['Anschauung.']

bring the formal difference which exists between the subject and the predicate, and upon which stress ought to be laid, into sufficiently clear prominence. It is true, indeed, that 12 is not merely another name for $7 + 5$, but expresses something quite distinct, viz. that the same quantity which is produced by the addition of 7 and 5 also occupies a place as a definite term in the numerical series between 11 and 13. But then the simplest idea which we can form of that series itself is to conceive it as arising out of repeated additions of the unit, that is to say out of the very same operation through which 7 and 5 themselves were put together. So that we conceive the left side and the right side of the equation equally as a sum of units and we merely analyse, on the left side, into two steps, as the idea of a sum allows us to do, that which on the right we take as a whole.

On the other hand such a formula as $7 + 5 = 4^2 - 2^2$, though not in fact expressing any more completely than the other that which is essential in Kant's thought, yet would have better illustrated the point that there are various ways by which we may arrive at one and the same quantitative value. For that which all turns upon is in fact nothing more than the assertion which is contained in the sign of addition,—viz. that quantities *can* be summed so as to compose another and a homogeneous quantity; a proposition the importance of which we may once more be tempted to ignore, because it seems to us self-evident and a mere identical proposition defining the nature of numerical quantity as such. And so it undoubtedly is, but how do we arrive at this piece of self-evident knowledge? Not every ideal content will submit to the same operations; we cannot add red and green together and produce blue; the notes *c* and *d* do not admit of being summed so as to produce a third note *x*, such as to stand higher than *d* in the musical scale by the interval *c*, just as 12 stands higher than 5 in the series of numbers by 7.

But here the question may be asked in surprise, what does this last remark lead to? Of course, it may be said, mathematical operations can only be applied to quantities, whose nature it is to admit of them, and not, or at all events not immediately, to impressions which are qualitatively different. But this is really to be blind to what lies under our very eyes. This very fact, that there *is* such a thing as quantity to be found in the world of ideas, while yet our thought itself is not bound, on pain of not taking place at all, to be the thought of just these comparable quantities—this very fact is a fact of *immediate perception*, which if it were lacking to us, could be as little supplied through logical operations working on a different set of ideas, as could the conception of qualitative resemblance if the world of ideas presented no comparable impressions of sense such as colours or sounds. The proposition therefore that quantities can be summed is undoubtedly an identical proposition; but that the subject and predicate of that proposition appear as valid in the world of ideas, and that it has quite a different value from the equally identical proposition, all wooden iron is wooden iron,—this does not follow from the principle of Identity. It is not then the bare logical principle of Identity, but the perception of quantity, the peculiar nature of which makes it possible to frame a countless number of propositions in content identical yet in form synthetic, which at once guarantees the truth of arithmetical reasoning and is the source of its fruitfulness.

That which might here be added in the interest of mathematics, I must pass over; with regard to the logical question I confess myself in entire agreement with Kant in a further point, namely in maintaining the pure or *a priori* perception of numerical quantity in the sense of the word *a priori* explained above. It is true that neither the idea of quantity as such, nor the more defined conception of its capability of being summed, nor finally any one arithmetical proposition, ever enters into our consciousness without being occasioned, and the occasion can always be traced in the last resort to an external stimulus. We think them only when we are led in one way or another to frame the idea of numerable objects. But, when the occasion arises, we do not learn that $7 + 5 = 12$ from the content of this perception, in such a way that our knowledge of the truth in question would gain in certainty with every fresh confirmation by subsequent experience; but as a matter of fact the single presentation of the idea $7 + 5$, no matter whether mediated through external perception or not, is sufficient to teach us its identity with the term 12 once for all and as a fact of universal validity. And supposing that we found when we came actually to count external objects in a variety of further instances that our arithmetical proposition was in some cases confirmed and in others not, we should certainly all of us, even the most decided adherents of empirical theories, agree to correct not our arithmetical proposition by our countings but them by it.

354. The case is perhaps still clearer if we turn to geometry. As to Kant's particular instance of a synthetic geometrical proposition, a straight line is the shortest line between two points, I have alleged similar scruples as in the case of his arithmetical example just discussed. Here again the example is not happily chosen, because we have no other direct standard of measurement for the conception of distance which is contained in the predicate 'shortness' but the straight line itself, and thus the proposition suggests before everything else the complete identity of its subject with its predicate. And such identity does undoubtedly, in respect of their content, exist; the proposition would not otherwise be true at all; but once more, how do we establish that identity? By connecting the two points through a something which we say is 'between' them. Now it is clear that this expression does not mean merely that the two points are logically designated as not identical or as merely in some way or other different, for that is equally the case with green and acid, out of which a proposition of this kind could never be formed. Nor again is it merely that they can be compared, for so—again with no such consequence following—can green and red. What it means is that they are connected in a manner completely *sui generis*, which is thinkable and has a meaning to us through an original faculty of spatial perception and so only, and which in the absence of such perception could never be made intelligible to us through any logical operations working on a content of a different kind, and of which, finally, even now when it is familiar to us all, no form of words, unless tacitly implying such spatial perception, can by any possibility give us a clear idea.

Other instances of Kant's are more expressive. Take, he says, the proposition that two straight lines cannot enclose a space, or, therefore, make up a figure, and try to derive it from the conception of straight lines and the number two; or again the proposition that out of three straight lines a figure *can* be formed, and try to deduce this in the same way from the conceptions it contains. Your labour is all in vain, you find yourself driven to have recourse to Perception, as Geometry in fact always does. These words remain true, even though a slight inexactness in the form of expression may offer a handle to controversy. The complete subject, in the second example, to which the predicate of 'forming a triangle' belongs, is not simply 'three straight lines'; the lines must be in the same plane, they must not be parallel, they must admit of being produced at pleasure. Again in the first of the two examples, we have no right to require the impossibility of the closed figure to be deduced from the isolated conceptions of the number two on the one hand and of the straight line on the other; we must begin by representing 'two' as the number of the lines, and the lines themselves as included in the same world of space. If we add these fresh points, the predicate will be seen in both cases, though not in both with equal obviousness, to follow identically from the subject when taken in its full meaning, and so the identity of their contents, which is essential to the truth of the proposition, will be established.

But this mere matter of statement does not alter the question at issue. All these conditions, that the lines must belong to the same plane, must not be parallel, must be capable of being produced, have absolutely no meaning whatever, unless we assume the spatial perception to begin with. It is this and nothing else which is our evidence that anything answering to those expressions is to be met with in the world of ideas, and which alone assigning a thinkable meaning to the complete subjects of the propositions in question, gives a reason in so doing for the predicates identical with their subject, which in each case they contain. These propositions then are undoubtedly identical propositions, although under a synthetic form; but that their full content and the manifold relations contained in it exist, is not due to the principle of Identity. That is to say, it is not by means of the principle of Identity that we can pass from one form of expressing a geometrical fact to another equivalent to it; rather it is the peculiar nature of space which makes it possible for identical facts to be variously expressed. It is upon this fact, and more especially upon the unlimited power we possess of bringing any given figure in space by the help of artificial combinations under fresh mathematical points of view or fresh general ideas, and thus constructing predicates for it, which were not contained in our original conceptions of the figure,—it is upon this fact, and not upon the mere application of the bare principle of Identity as such, that the fruitfulness of geometrical procedure depends.

355. At this point I may expect the criticism that my argument has brought me to a different conclusion from that which it was aiming at. I began I shall be told by maintaining the necessity, in order to any extension of knowledge, or to the discovery of the laws of nature, of synthetic principles *a priori*. And now I am invoking the aid of Perceptions to supply both subject, predicate, and copula of the judgments in which we express those principles, a proceeding which seems after all to amount to no more than the not very helpful proposition, that we cannot think without having some idea of that which is to be the content of our thought; the fact still remains that the object in question is given not *by* thought but *to* thought, in a manner not essentially different from that in which every other object of consciousness is given, namely through experience.

In regard to this last point I repeat once more in one word what

I have said already, that all knowledge whatever, whether innate or not, which as a matter of fact whether constantly or upon occasion makes its appearance in the consciousness of any one, is for him in the broader sense of the word, an object of experience. And further we have admitted from the first that no one of the principles which we regard as innate, can be operative in us even in the sense of a major premiss unconsciously followed in our judgments, until an incitation so to follow it comes to us in experience, while it can only become in the full sense an object for our consciousness through a definite act of reflexion upon those applications of it which have already been made unconsciously. In this sense then I have no objection to offer if any one insists on calling the apprehension of *a priori* principles itself an inner experience; I only regard it as a perfectly barren contention. Nor can the controversy between the *a priori* and the empirical view turn on the further point that the latter ascribes to outward that which we would rather attribute to inner experience. For this antithesis simply does not exist; whatever notions we may form about a supposed external world, our experiences can only be of the representations of it in our own minds, of the order and connexion of our own ideas. Upon this point I may be allowed to be brief. In Germany at all events the fallacy which has been imported from abroad does not yet find favour, that by measuring the solid and superficial angles of material forms we can confirm the propositions of geometry, or discover any others than those which we can develope with our eyes shut from relations assumed to exist between mere points of space. We are still aware that such measurements, supposing that we make them, relate directly not to the nature of the bodies which fill the space in question but to properties of the space which they fill; finally that they can only be made at all by the aid of contrivances and methods which are all founded to begin with upon the essential order and regularity of our spatial perception, and that we can never therefore employ the process of measurement to test this our geometrical knowledge by the standard of a knowledge which has a different and independent source, but that so far as we do employ it we are merely bringing a particular case of spatial Perception under the laws of geometrical Perception in general.

Thus the difference between us and our opponents comes back merely to this, that to us the simple principles of geometry, that every straight line may be produced to infinity, that the opposite angles of two straight lines intersecting one another are equal, that the sums of any two adjacent angles are equal,—such principles are to us truths which once presented to thought are valid always; whereas in the view of empirical philosophy each particular apprehension of them must in consistency be regarded as a psychical fact and nothing more, as to which there is no certainty whether it will recur in a similar case or not, and of which therefore universal validity can never be established as true, and can only be established as probable on the strength of the agreement of a very large number of instances in which as a matter of fact it has so recurred.

356. I must once more summarise my position in relation to this point of view. In the first place the contention that every truth of whatever kind requires this test of experience in order to be received as universally valid, would contradict itself. For on the one hand it must itself come under its own rule and by consequence cease to be universal; on the other hand, as we have already seen, without the assumption of the unconditional validity of some absolutely certain principles not drawn from experience the very deliverances of experience itself could be no one more probable than another [1].

On the possibility of an immediate knowledge therefore of *some* universal truth all certain belief depends, that of our opponents no less than our own; the difference between us can only be as to what the truths are which we hold to be accessible to this form of knowledge. But it is self-evident that in the case of truths which are to be recognised immediately as universally valid, their sole credentials must be the clearness and strength with which they force themselves upon consciousness and at once claim recognition without constraining it by any process of proof. Now any one has perfect liberty to allow this claim or to resist it; it is open to every one whether in all honesty to distrust the self-evidence with which this or that object of knowledge presents itself to his consciousness, or to insist (at all events for sophistical purposes) that no self-evidence in the world affords a proof of the truth of the thing evidenced; only in the latter case he must allow that a like vein of sophistry may contest the validity of any process of proof whatever and of his own contention along with the rest.

This sort of idle disputation for disputation's sake we may leave to itself; the former more honest variety of scepticism on the other hand is not without its justification, for undoubtedly that state of repose and peaceful equilibrium of the mind, in which the self-evidence of knowledge, regarded as a psychical fact, consists in the last resort, may also be produced by conjunctions of ideas of by no means universal validity. These false forms of self-evidence we have admitted to exist, and the logical processes have been given through which we seek to free ourselves from such illusions. These processes all resolve themselves into this—by shaping our investigations in various ways, adopting various starting-points and various methods, we arrive at separating from a subject S to which it is our object to ascribe a predicate P, all associated ideas x, not really contained in S but secretly affecting our conception of it, which might create in us the impression that our P which in fact belongs only to $S + x$ is an invariable attribute of S as such. Our method does not always assume the form of a direct proof; the proposition that a straight line may be produced to infinity is too simple to admit of any argument except one which brings us back by a complete tautology to immediate Perception; in other cases again proof will take the apagogic form of a *reductio ad absurdum*, a form of argument which does not deduce the truth of the given proposition from some other acknowledged principle but merely establishes the impossibility of denying it. When this has once been accomplished we regard the proposition in question as a truth of universal validity, needing no empirical confirmation from particular instances in which it is found to hold, but on the contrary standing over against all particular instances as certain *a priori*. We do not deny the possibility that this trust in reason may now and again deceive us; but we should not surrender the presumption in favour of a principle thus arrived at being true merely because it is *possible* to distrust it; we shall hold fast to it until either the results to which it leads involve us in contradictions, or until some other truth becomes plain to us, from which we are able to understand how a proposition now seen to be false came to present the appearance of a self-evident truth.

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[1] [p. 464.]

357. There are various points here which still need elucidation. The terms in which in the Kantian school pure Intuition[1] has been spoken of in contradiction to Thought, have led to its becoming associated with the idea of a peculiar and somewhat mysterious form of *procedure* through which the apprehending mind accomplishes something which is impossible to its discursive thought. The obscurity which attaches in consequence to this idea is due to this, that in fact it is just in the case of Intuition that no sort of procedure consisting of the connecting of various single acts is describable, whereas there is one in the case of Thought. The attitude of Intuition towards its content is that of passive receptivity, and its work is done so completely at a single stroke, that no steps or stages in it can be distinguished or could be described. This must not be misunderstood.

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[1] ['Anschauung.' 'Perception' is usually a better rendering of this word than 'Intuition'; but the latter is preferred in this passage for obvious reasons.]

When geometrical intuition teaches us that two straight lines intersecting each other can only have one point common to both, there does undoubtedly take place, regarding the act as a psychical event, a certain succession of ideas, which we might describe if in any particular case it were exactly known to us. We might explain how we first think each of the two straight lines in itself, then place them in the same plane, make them from a parallel position converge, follow each to the point of section and then beyond it,—all this we can describe, but this is not the geometrical intuition itself: so far we have only brought into consciousness all the different points which go to make up the relation in question, and now intuition pronounces on these points of relation, as by a single instantaneous revelation,—the two straight lines can only have one point in common. How this final step is accomplished, the immediate apprehension of the necessary truth which is implied when once all the members of the relation are completely given, is a point upon which certainly at present, and in my judgment no less certainly for ever, any further psychological analysis is impossible. It is only in this sense of absolutely immediate apprehension that I have here employed the term intuition, and it leads me to a further observation as to the meaning of the expression *a priori* as applied by us to intuition.

I have explained before why it is that knowledge must necessarily consist not in the mere passive reception of impressions but in a reaction, the form of which reaction will depend on the nature of the mind which is stirred to it. I did not conceal my agreement with Kant in accounting Spatial Intuition as a form of such reaction, and therefore as *a priori* or innate in the sense in which that term may legitimately be used. For the question before us however this point is of no importance. It is not because the idea of space is innate in us, that we are in a position to frame universal propositions in geometry, which once thought are valid always; if it were at all intelligible without any such hypothesis how the idea of a particular combination of spatial points of relation could arise in us purely through external impressions, still, in presence of such an idea, the immediate apprehension of the universal truth contained in those relations, which is the service of intuition, would be not more inexplicable (though it would be equally inexplicable) and not less possible than if those same points of relation could only be brought into our consciousness by the help of an innate mode of reaction and spontaneity in the mind itself. I therefore reserve the question of the *a priori*, in the sense of the innate character of spatial Intuition, with any further question which may arise out of it, for the Metaphysic, and apply the term *a priori* to spatial intuitions in a restricted sense only, viz. to indicate that they are not derived by a process of induction or summation from particular instances which exhibit them, but are thought to begin with as truths of universal validity, and are thus prior to the particular instances in the sense of being rules by which they are determined.

358. This brings us to the last point which we have here to consider. Philosophers have spoken of pure Intuitions as an innate *possession* of the mind, in terms which could not but lead as a natural deduction to the idea that all truth which rests upon any such intuition is also an intellectual treasure always at hand, which we take with us to experience, and through which we judge it. And in fact Locke made use of this deduction as an argument against the doctrine of innate Ideas. It needs however only a brief consideration to see that such a deduction is illegitimate. Every one who speaks of innate knowledge includes in it most certainly mathematical truth, but mathematical truths had all to be discovered before they were known, and the universally innate possession of a spatial intuition was not the same thing as the possession of a knowledge of geometry. But the most elementary of these truths were discovered as soon as ever the mind was drawn to turn its attention away from the infinite variety of figures presented by bodies in space which surround us in the world of perception, to the simplest relations which are contained in all of them alike. Then at once the truth of each several principle one by one sprang to light self-evident and self-proved, just as Plato so admirably represents it in the Meno, only that it was superfluous to refer us to a previous state of existence from the memory of which this sudden emergence of knowledge was supposed to come, inasmuch as there also the *conviction* of the certainty and necessity of the truths which there were given to intuition in a universal form could only have arisen in the mind through the same immediate act of apprehension by which in our life here we recognise it in particular instances.

It is still easier to understand how it is that the more complicated mathematical relations should have had to wait for their discovery, and that an immense tract of ground should always remain before us, in which new discoveries are to be made. The consequences which follow from simple mathematical principles become science only by being deduced from them by reflexion, and this operation involves a labour of a most extended and constantly progressive character, the application of processes of exact definition, of analysis into varied elements, of synthesis into well-defined forms, to abstractions made by the mind itself, and this in order to establish even the subjects of the propositions required, the predicates being obtained, it may be, by processes no less elaborate.

Paradoxical therefore as it may seem we must disabuse ourselves of the false idea that the world of the self-evident lies of itself plain in its self-evidence before us, and that all we have to do is by the help of this comfortable possession of a self-evident truth to go on to subdue the intractable world of our perceptions. The fact is that even universal truths, for the apprehension of which the mind requires nothing outside itself, have yet to be found by searching, have to be abstracted and separated off from among the measureless host of ideas which form the world of consciousness. Nor can we even expect that the very simplest of innate truths, the highest principles of all, will be revealed to consciousness first of all by this process of self-reflexion. On the contrary, their first appearance is invariably occasioned by some particular instance which exemplifies them, or some particular case presented by perception or by imagination that the mind may pass judgment on it. But it may happen that our perceptions may be of such kind as never to present to us the case required in its purity, and in the same way to debar the imagination from conceiving the idea of it, and this though if once it were presented to consciousness, the mind would at once feel the conviction awakened of a truth of the most universal and fundamental kind, and would judge accordingly. Thus then it may be an extremely difficult task for knowledge, to remove all the obstacles which the actual connexion of our ideas, imposed upon us by experience, plants in our path, and to fight its way through to the knowledge of the self-evident.

359. In mathematics, where the matter of investigation can most easily be separated from the real objects to which it is attached in experience, it has speaking generally been possible to advance from the simplest truths to their derivative consequences, although, in spite of this, the fresh knowledge has afforded new and more comprehensive expression even for the principles which were known before. It has been otherwise in the science of mechanics, which applying itself directly to actual occurrences, seeks to prescribe laws to the interactions which obtain between real things. I use this much criticised expression[1] of

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[1] ['Vorzeichnen' (to prescribe).]

Kant, in order to reduce the objections which have been made to it within their proper limits. No one could have intended it to mean that human reason can invent laws at its own good pleasure which nature is bound to follow. But supposing the idea of a relation between different elements to be presented to us in so simple a way as to exhibit an instance of the perfect purity required, in which true laws of nature are seen producing their simplest result, with no multiplicity of extraneous conditions to obscure it, why should it not be possible in such a case for reason, itself a member in the system of the world in which these operations take place, to have an immediate apprehension of the result in which the relation supposed must necessarily issue? This is not to thrust its own subjective laws upon nature, but to detect the real laws of nature herself, which become to it binding rules which it brings with it to the confused tangle of separate events wherewith to estimate and interpret them.

In this sense pure mechanics is an *a priori* science; it is quite true that many of its principles may have been first suggested and the enquiry after them occasioned by experience, but it was not by the testimony of repeated perceptions that they were discovered and reduced to the exact form of a law, it was by an operation of thought, apprehending with the clearness of immediate vision the self-evident law in an instance where it is presented in its purity, and in complicated cases finding means to reduce them to a similar simplicity. This is commonly expressed by saying that within its own province mechanics is an absolutely demonstrative science, which from presuppositions of its own creation evolves necessary conclusions with irresistible logic; but that, to compensate for this it has in relation to experience only a hypothetical validity, that is to say it is valid only on the assumption that real things exist which admit of being subsumed with perfect exactness under the conceptions from which its conclusions are drawn.

But such language allows too much to an unjustifiable scepticism as to the tenableness of the hypothesis, and does not really answer to the facts. For the science of mechanics did not spring up in some meditative consciousness, playing with possibilities before experience existed, it arose under the persistent pressure of experience which called for explanation. The abstract universal conditions, from which in mechanics we derive definite consequences, are not Problematic schemes of something which might perhaps be found in reality, but reductions of that which is Assertorically given in experience to a form in which its validity is universal.

But that reduction to a universal form was necessary by reason of the one actual postulate with which the science of mechanics stands and falls, that a uniformity of law does hold good in the world of events. If this assumption is justified, and if there are many elements $A\ B\ C\ldots$ operating together in the order of nature, each under various forms $\alpha\,\alpha^1\,\alpha^2\ldots\ \beta\,\beta^1\,\beta^2\ldots$, finally all of them in varying relations $M\ N\ldots$ each of which again may assume different values $\mu\,\mu^1\,..\ \nu\,\nu^1\,..$, —then any single event must be the joint effect of many single laws, each law concerning two elements $A\ B$ only and their relation M, and determining the particular operation E which results from these data, and which in turn will change to $e\,e^1\ldots$, as $A\ B$ and M pass through their several changes in form or in value.

It may be that experience never affords a perfectly pure instance of one such single law; still it would be folly to find fault with mechanics for speaking of motion in the first instance without taking account of resistance which nevertheless invariably attends it; or of a homogeneous mass which is nowhere to be found, or finally of a perfectly rigid body, whereas perception presents us only with bodies which are elastic, yielding, of various degrees of hardness. It will be time enough to take account of the influence of these secondary conditions when we have learnt the universal laws upon whose consequences they exert their modifying operation; but even supposing that the theory of the resisting mediums, of the specific qualities of matter and its molecular properties, were never to reach the simple clearness of the other departments of mechanical science, it is certain that a philosophy of nature which was not even acquainted with the laws of the simple and pure cases from which every individual mixed case varies by a determinate amount, would be attended by still less success. For it is by no means for the mere convenience of shortening our procedure at the cost of its exactness, that we ignore the special peculiarities of the concrete instance, and begin by looking for the law of a universal and abstract instance; our assumption of the presence of law in actual events involves as a necessary and objective consequence that the joint operation of many elements is made up of the several operations, which each pair of elements, combining in a specific relation, generate on their own account, and which they alter in accordance with a fixed law as this relation alters.

360. The empirical content and course of our perceptions has rendered it by no means an easy task for mechanical enquiry even to form the ideas of the simple and pure cases, upon which an immediate intuition of the truth could at once pronounce a judgment self-evident and universal; on the contrary, it is here more than anywhere else that experience has exerted the injurious influence already alluded to, drawing the mind away from the apprehension of the universal and the unconditioned, by constantly introducing to it the particular and that whose validity is conditional.

The entire period of antiquity passed away without the conception of motion, the central point in mechanics, having been reduced to a form simple enough to be immediately apprehended by the mind in its abstract character. Three great examples of motion were presented by experience to the imagination, the perpetual motion of the heavenly bodies, the rapidly ceasing motion of terrestrial bodies caused by external impulse, finally the energy of living beings, originating within but after a while wearying. The mind of antiquity never succeeded in separating the simple process in which all motion consists, continuous change of place, from the conflicting peculiarities of these different classes of instances in which it occurs. The phenomenon was never disentangled from certain assumed causes of it; the course of the stars was represented as a divine motion exalted above the general laws of nature, or else the motions of terrestrial bodies were attributed to an extraneous necessity and thus degraded to a position below the due and natural order of things. Add to this that the analogy of the wearying of human activity led men to regard cessation of all motion as such as the natural and self-evident law, its eternal continuance as a divine exception.

It was reserved for a much later epoch to conceive the essential features of all motion of whatever kind, as consisting simply in a relation between the three elements of velocity, duration of time, and space traversed, and by the modest formula $s = vt$ to lay the foundation of a scientific theory of motion. That formula once given, the law of the persistence of motion followed of itself: for although the discovery of the law was due to a generalisation from particular results obtained by experiment, showing that motion always lasted for a longer time in proportion as all external hindrances were removed, still no one doubts that directly it was discovered it expressed a tardily apprehended necessity of thought. That there is such a thing as motion had to be learnt from experience, but if it exists or is to exist at all, the idea of its persistence becomes a necessary postulate in order to make it even a possible object of intuition[1].

Similar difficulties had to be encountered in forming the conception of mass. The bodies with which we most ordinarily deal, whether solid or fluid, were observed to follow the downward tendency of weight, whereas vapours and fire tended upwards; thus the idea arose of two opposite impulses, both belonging essentially to the nature of bodies, but leading away from one another in two opposite directions, directions which might indeed have been correctly distinguished by a qualitative distinction of the ultimate points towards which they tend, but which were in fact confounded with the unintelligible antithesis of an above and below in an absolute space. It took a long time before the combination of more extended observations was able to compensate for the one-sided character of the facts as experience at first presents them, and to show that neither the direction nor the intensity of weight-pressure was everywhere uniform. Not till then did the natural idea make its way that the beginning of any new motion whatever must necessarily require something to determine its direction *a fronte* or *a tergo*, in the way of attraction or repulsion along a straight line, that is to say, that it takes its origin always from an interaction of different elements in space, and that the amount of such interaction depends on the quantities of a homogeneous real existence which are united in each one of the elements in question. The idea of mass, again, which was thus arrived at, in which regard is had only to the amount of the resistance of inertia on the one hand which real existence in space offers to any motion which is demanded from it, and to the magnitude of the power with which it enforces every motion of other elements which originates from itself,—this mechanical conception may very well stir new questions to which philosophy would have to find an answer; still when once a regular order of natural events is given or is assumed, in which each single event is taken as the condition determining the definite degree of another event following upon it, it is easy to see that such a conception involves as a self-evident postulate the commensurability of all real elements in regard to the magnitude of the effects they may be expected to produce, a principle which is expressed in the conception of Mass. But how great the power in which one-sided and partial observation exerts over our conceptions, is attested by the difficulty which the common imagination finds even in the present day in believing in the possibility of the Antipodes, and again by the errors of certain schools of natural philosophy, to which not indeed the eternal downward motion of the philosophy of antiquity, but still the concentric pressure of gravity, formed so essential a part of the general notion of material substance that the idea of mass without weight always seemed to them a contradiction.

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[1] [See sect. 247.]

Here I must break off; but any one undertaking to write the history of the development of mechanical conceptions would find it a suggestive task, instead of being content perpetually to repeat how we have come simply through the connexion of particular experiences to our knowledge of natural law, to go on to trace and explain how at first the partial and one-sided character of those experiences forced upon men's minds a number of false ideas, and hindered them from arriving at an earlier apprehension of self-evident truths.

361. There are conflicting opinions as to the logical character of the simplest mechanical principles. Just because they concern in the first instance not actual bodies but a certain postulated subject-matter whose nature is wholly determined by our definitions of it, we either consider ourselves bound to look upon them as analytical judgments the truth of which is guaranteed by the law of Identity, or else we regard them, even as taken in their purest and most abstract form, as still synthetic and therefore mere probable hypotheses, whose truth can only be established by their agreement with experience and the complete internal harmony of the conclusions to which they lead.

My own judgment in this controversy can be no other than that already given in reference to the kindred problems in Arithmetic and Geometry, but I must content myself with briefly indicating my point of view without developing it in all the detail which might be desirable. In general I might express my position thus: the two given data A and B, as to whose connexion a mechanical judgment is to be affirmed, are not given to us merely one by one; our ideas of them are only intelligible and are only understood in and through a single Intuition, which embraces both together and which determines also in one and the same act the relation between them.

Let us, to begin with, turn once more to an example from arithmetic. The proposition $3a - 3a = 0$, we shall be disposed to refer immediately to the principle of Identity; nevertheless all that that principle tells us, taken simply by itself, is that $3a = 3a$, $-3a = -3a$, and finally $3a - 3a = 3a - 3a$; that this last expression $= 0$, we can only maintain on the strength of a direct intuition of the fact that there are two operations lying within our power, the addition of a to a and then the subtraction of a from $2a$, which exactly cancel each other, and in the repetition of which an equal number of times the subtraction will annihilate whatever quantity the addition may generate. For in fact in the expression $+a - a$ the sign $-$ represents not merely an opposite to $+$, it indicates at the same time the mode in which this opposition is able to operate and is to operate, namely by subtraction. If we knew nothing of the possibility of such an operation, or if it could not be carried out, then we could as little evolve the result o from $a - a$, as we could arrive at a result from the mere combination in thought of the contradictory notions of possibility and impossibility; in their character of opposites these two notions can equally well be represented by a and $-a$, but this cannot be interpreted by a subtraction.

We see therefore that the proposition $a - a = 0$ may be regarded with equal truth as at once identical and synthetic. It is an identical proposition, because it would be actually false if the two sides of the equation did not represent precisely the same content; but that the identity is there, no mere logical analysis of our a, $-a$, and — signs, can possibly inform us; we learn this solely through the immediate intuition of the meaning which the sign — is in this particular case capable of bearing, because it is related to the increasing or the diminution of quantities. Hence the proposition is a synthetic judgment of identity between two contents different in form, between a problem and its solution.

A similar instance is presented in the field of mechanics by the determination of the resultant of two motions the lines of which include an angle. I confine myself here to the postulate from which the ordinary attempts at demonstration start, namely that where two such forces are equal the resultant bisects the angle between them. This proposition is commonly regarded as self-evident, and we suppose ourselves to possess in this simplest possible instance an immediate certainty of a conclusion to which any more complicated problems would have to be reduced. And undoubtedly the most cautious mind will agree to recognise in it not merely a probable hypothesis but a truth which only cannot be proved, because it is too simple to admit of being proved from anything simpler. But the observation which is commonly added by way of elucidation, that there is no reason why the resultant should approach more nearly to the line of the one force than of the other, may serve to illustrate the logical character of the proposition in question. For it cannot in itself be a positive ground for the necessity of the assumed direction of the resultant, that grounds are absent for two other classes of directions, unless we start with the position that some direction must necessarily be taken, and that it cannot coincide with either of the two forces. But now it is precisely this that we know from Intuition; a merely logical analysis would only teach us that under the condition a the element M moves in the direction α, under the condition b in the direction β. Supposing both conditions operating together, then M can neither move in the direction α nor in the direction β, because either one or the other would suppose one of the two conditions entirely inoperative. What then would happen? The two conditions being supposed of equal strength, it follows that either both the one and the other must be inoperative and M remain at rest, or else both must act and be counteracted in equal measure,—supposing always that there are ways and means by which that result can be brought about. But this last is the important question; that there are such ways and means, and what they consist in, this is what no method which thought can provide is able to inform us. But when we turn on the one hand to the Intuition or Perception of space which gives us the connexion between the different directions which are possible in it, and on the other to the Intuition of motion, there it lies all clear before us; there we find that M can satisfy completely both the two conditions at once, by so moving as at the expiration of the unit of time t to arrive at the same point (being the end of the diagonal of the parallelogram) at which it would have arrived in two such units of time taken in succession, had it pursued first the direction α or β simply during the one, and then the direction β or α simply during the other; that the path finally by which it reaches that point is the diagonal itself, follows from the fact that for any small fraction dt of the time precisely the same principle holds; the diagonal is the geometric locus of all the points at which M must necessarily arrive at the ends of the times dt, $2\,dt$, $3\,dt$ and the rest. Here again, then, and this time in a proposition of Mechanics, we have a synthetic judgment, which establishes the identity between a given problem and its solution through the instrumentality of immediate Intuition.

362. This for the present must suffice; I glance at the more advanced part of mechanics for a different purpose. Whereas its first beginnings by their very simplicity render formal methods of demonstration impossible, the problems later on become so complicated, that the solutions, although strictly following from those fundamental principles, do nevertheless, owing to the large variety of the points of relation which have to be kept in sight, necessitate very lengthy and circuitous processes of abstraction and calculation. Now indisputable as are the conclusions which are thus arrived at, yet nowhere has the desire been more keenly felt than in this exact science, to dispense with the scaffolding of the Calculus and reduce the results obtained to simple conceptions which only need the help of computation so far as is involved in their application to the conditions, determinate in respect of quantity, which particular cases present. I would only remind my readers of Gauss' principle of least constraint, which expresses in the most universal form the law of all motion as follows : a system of material points, however connected with each other and whatever may be the external limitations by which they are controlled, moves at every instant in the greatest possible accordance with the free movement of the points, or under the least possible constraint; taking as measure of the constraint which the whole system endures in every minute portion of time the sum of the products obtained by multiplying the square of the deviation of each point from its free movement into the mass of that point.

The second clause in this law supplies the general conception which is expressed in the first with the mathematical form by help of which, for every individual case, the purport of what the conception requires is precisely defined and made applicable to the given quantitative relations of that case; but in the first clause we are convinced that we possess not merely a general rule which is found as a matter of fact to hold good, but the veritable *ratio legis*, from which all the special laws of the various kinds of motion are derived. Applying it to the simplest case of all, the case of the resultant of two lines of motion, we have seen (§ 232 seq.) that various chains of reasoning will lead us with equal certainty to our conclusion. These forms of proof however serve only with greater or less cogency to *constrain* belief; on the other hand the reflexion that the motion in the line of the diagonal is that by which both motive impulses are completely satisfied, and in which no part of either is lost, presents itself to us, when once we comprehend it and find experience to confirm it, as a ground of judgment of an entirely different order, and of quite peculiar significance, which arouses in us at once the conviction that in it we possess not merely one of the rules by the light of which it is *admissible* to regard the order of events observed, but the supreme principle by which they are actually governed.

I added advisedly that we are obliged to presuppose the preliminary corroboration of our principle in experience; and in fact however convincing the proposition might be in itself, that the conflict of all motions is always so ordered that in the final result no element in the effects aimed at by the constituent is lost, still without such corroboration it would be of very doubtful validity. It would represent a principle after which we ourselves perhaps should order the world, if the task could be set us, and provided always that it was possible and that we had found the means really to carry out in every individual case the universal postulate which the principle contains. But that the actual world of reality or even that the world of thought does possess the particular content, form and constitution, and the particular combination of elements which renders it possible to unite under this single supreme principle all the particular events which take place in it, or even the several laws which abstract reasoning has presented to us as necessities of thought—this we learn only at the end of our journey.

We know how often in the history of mechanics attempts have been made to connect the entire course of the physical universe with some such supreme philosophical law; we have heard of the constant sum of motion in the universe, of the indestructibility of force, of a principle of least action, and of a law of Parsimony. All these attempts did not merely express the aspiration after a fundamental and self-evident idea from which the individual laws, mathematically determinable, which govern events, might be derived; they tell us something also of the direction in which the desired end is to be looked for. But it has never been found possible to determine distinctly and precisely, without superfluity or omission, of what subjects of relations so universal a conception could be enunciated as no less universally valid.

How far up to the present time any advances have been made in this direction, I have not now to enquire; all that I desired to emphasise was the fact of the eager ambition displayed by the mind to perfect the circle of its knowledge by the aid of principles of the most comprehensive order—principles once again which affirm in the form of *synthetic* judgments, which are nevertheless self-evident and universally valid, a connexion between two terms of a relation whose connexion no process of logical demonstration can show to be of an analytical or identical nature.

363. The final goal of knowledge is usually represented in different terms from this. What is aimed at is the reduction of all connexions which appear synthetic in character to an analytic form—more properly expressed to the form of identity—and we are even believed to be actually on the way to the consummation of that end.

At the commencement of our knowledge, we are told, a conception S is made up at first of the small number of marks $P\ Q\ R$, which we have already found to be connected together; then supposing that fresh experience presents in a particular instance a further characteristic Z conjoined with S, the proposition S is Z which gives expression to the observation made, is considered to be a synthetic judgment. If however this new fact of experience becomes established as obtaining in all cases of S without exception, Z is adopted forthwith into S, and the proposition S is Z has now with the enlarged signification of S become analytical. This in fact, it is said finally, is the goal towards which all knowledge is striving to reduce those connexions of subject and predicate which at first appear so completely synthetic to this analytic form, that is to say, to resolve coexistence into coherence. And this is a perfectly correct description of the origin and growth of knowledge—for it must, alas! be confessed that beyond this point it seldom advances—yet it has to be remarked that this ideal described in the last words of the sentence is one which is attained only to a very modest extent, and that in the sense of the initial appearance of mere coexistence giving place to an intelligent apprehension of a self-evident law of connexion it is never attained at all.

If we had formed the conception of body to begin with out of the qualities of extension, impenetrability, and inertia alone, characteristics from which the necessity of mutual attraction does not follow, the proposition 'Body is heavy' would undoubtedly have been a synthetic proposition; but the same proposition does not become analytic, even if we take into the conception of body the universally observed fact of gravitation; this last property is just as little to be derived from the others as it was before, and therefore just as much synthetically connected with them as it was in the first judgment which expressed that association as a constant fact. Undoubtedly we are able, taking this synthetic conjunction of all the different marks of S as our datum, to submit them to the analytic method, and bring them one after another before the mind as separate objects of thought; but this recognition of the mere fact of constant coexistence where the coherence is not understood is in fact the renunciation of knowledge; the mind could only rest satisfied if the conjunction of any two such properties of S were a sure guarantee of the necessary presence of any third. And such demonstrations we are able to some extent to establish, and whenever we succeed in doing so it means that an advance in knowledge has been accomplished; but it is clear that no such result is possible, unless in the last resort we assume at some point or other a premiss of the form $A + B = C$, that is to say, a premiss which does not merely affirm identity of what is the same[1] by the bare principle of Identity[2], but affirms identity of the different where no reduction to the principle of Identity is possible. Thus the supposed transformation of all synthetic knowledge into analytical resolves itself after all into the enquiry, what are the simplest forms of synthetic truth?

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[1] ['Gleiches einander gleich setzt.']

[2] ['Princip der Identität.']

364. This contention, though it may perhaps be said to amount to nothing more than a needless change of phraseology, will nevertheless be in the end admitted. But it will be urged in addition that this very necessity of allowing certain synthetic combinations to start with, proves the inability of human knowledge really to come to a final resting-place, and to obtain insight into the inner connexion of the coexistent; everywhere there remains a residue of mere facts, of which the connexion of one with another is unintelligible, and vouched for only by experience. I cannot agree with this opinion, according to which we attain to knowledge only where we can affirm exact identity of what is exactly identical. For after all whence comes the confidence with which we hold the proposition $A=A$ to be an *intelligible* truth, except from the immediate self-evidence with which it forces itself upon us, and which leaves us no room to wish for any mediate demonstration of its certainty besides? But how it happens, by what means it is brought about, or from what inner coherence in the nature of things it follows that A is identically like itself, we do not know, nor will anyone believe that there is any meaning in asking such a question at all.

If then a perfectly simple synthetic proposition of the form $A+B=C$ presents itself to us with a like degree of self-evidence to recommend it, why should a question be raised in this instance which was meaningless in the previous one? Why should the latter act of equation only be allowed to be valid by the help of some intermediary process, to show us *how* C can $=A+B$, when in the former our intelligence was satisfied to know the fact *that* $A=A$? I will not again insist on the point that in the processes of our thought no such mediation could come from the mere law of Identity, that it would always have to begin with a proposition $A^1+B^1=C^1$, analogous in character to the one to be proved—for this reflexion would certainly not meet the complaints of the incompleteness of a knowledge which is said to be incapable of attaining to any supreme self-evident principle. But how are we to understand the requirement that we should accept some such synthetic connexion as given, as valid in itself, and only not accessible to our intelligence? Are we prepared to assume that as a matter of fact M and N are always conjoined in reality without affecting one another in any way? But if this is impossible, and if it is at the same time impossible that out of one and the same self-identical A two different results M and N should arise, what else is left to us but to suppose that there do exist in the real world certain natural and original connexions between things different, original syntheses the members of which are not joined together by any intermediate links, so that the tie between them could appear as even the most distant consequence of the law of Identity, yet are none the less immediately and really connected? If then in the world of Being this must necessarily be the case, how can it be demanded of knowledge that it should exhibit the certainty and the intelligibility of a given relation through a process of mediation which does not exist in that relation itself?

Thus then there may certainly be synthetic truths of an ultimate and absolutely simple nature, which as conceived in their purest and most simple form possess a validity guaranteed not merely by fact but by their own self-evidence, a self-evidence however which if we insist on grounding all logical truth on the principle of Identity, must no longer be called logical but aesthetic, and which accordingly will find the touchstone of its validity no longer in the unthinkableness but in the plain absurdity of its contradictory. To this class of truths belong the simplest principles of mechanics; that we regard them together with all truths of like kind with them not as the earliest constituents of our knowledge which have been there from the beginning, but as its final results, to be won only with difficulty and labour, has been explained above in terms sufficiently clear to make the repetition of it here superfluous.

365. Special lines of enquiry lead in the first instance to single truths of this nature, each one its own evidence and standing in no need of support from others. At the same time nothing prevents us from bringing them as members of one and the same world into connexion with each other and searching for a single supreme principle in which they may find their unity, just as each one of them had already supplied a centre of unity for a body of connected facts. It is possible that many such truths may lose in consequence their independent value, and that even logical analysis may reduce them to particular cases of a more general law, which we have found conceptions of a sufficiently comprehensive and exalted order of abstraction to express. It is just as possible and more likely that the self-evidence with which the coherence of the many single elements of truth enables them to be ranged under a single fundamental idea, may rest upon that very same kind of aesthetic propriety on the strength of which the single laws themselves were formulated, affirming connexions which logic could not prove.

principle—a development itself synthetic and yet at each step necessary—was perhaps the problem of the Platonic Dialectic, though as yet but dimly presaged; it may be truly regarded as the end towards which the Hegelian revival of the Platonic scheme was directed. From these ambitions by which Germany was once inspired, our own age has passed with much sobriety to the order of the day, to that unremitting labour of empirical enquiry, the incompleteness of which paralysed the audacious flight of the Hegelian idealism. Nor was this the only defect of that idealism: unquestionably it was also wrong in regarding that which can only be the ultimate goal of a knowledge approaching towards completion, as already attained or attainable. But in view of the universal idolatry of experience which prevails at present, and which is all the cheaper and all the safer now that the importance and indispensableness of its object are visible to all mankind, I will at least close with the avowal that I hold that much reviled ideal of speculative intuition to be the supreme and not wholly unattainable goal of science, and with the expression of my hope that German philosophy will always arouse itself afresh, with more of moderation and reserve, yet with no less enthusiasm, to the endeavour, not merely to *calculate* the course of the world, but to *understand it.*

APPENDIX[1].

237. In the case we have just considered, a very plausible supposition, viz., the resolution of a motion, led to a correct result, though the conditions of that result really lay in quite another field; there are other cases in which a correct though not quite complete supposition leads to results which are apparently wrong but which can be made right by interpretation. Let a heavy rod whose length is $2a$ and weight W lean against a perfectly smooth vertical wall and make with the perfectly smooth horizontal plane on which it stands the angle ϕ. It will necessarily slip down unless the foot which tends to move away from the wall encounters some lateral check. The amount of this resistance, or, which is the same thing, of the thrust S exerted against it by the sliding rod, is expressed by the equation $S = \frac{W}{2} \cot \phi$. If the rod stand upright, $\phi = 90°$, $\cot \phi = 0$, and therefore $S = 0$; the rod balances itself freely upon its foot, exerts no horizontal pressure at all, and needs no lateral check, and can dispense with the vertical wall. As ϕ diminishes, i.e. as the rod slopes, $\cot \phi$ increases, and with it the thrust; but when ϕ becomes 0, and the rod lies horizontally upon the ground, the thrust according to the formula is infinitely great, while a glance at the facts show us that it must be nothing. This apparent contradiction is easily removed. When we propounded the problem we thought of course of a continuous horizontal plane capable of resistance stretching from the foot of the rod to the vertical wall; but this part of the supposition did not enter at all into the small calculation by which we arrived at the formula $S = \frac{W}{2} \cot \phi$; here we thought only of the single point at the foot which had to carry the weight of the rod; between this point and the wall lay nothing that this calculation took count of. In other words the *general*

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[1] [See Editor's Preface.]

question act in this case. Now, if we do not go beyond what is involved in this calculation, at the moment when ϕ becomes equal to o there is a gap between the foot of the rod and the vertical wall, equal to the length of the rod, and through which it would fall when there is no perpendicular force to support its weight. It can now no longer be said to exert a thrust S; but S signified not only this thrust but also the horizontal force, which in the first place counteracts this thrust, but which also forms the only obstacle that prevents the rod from slipping down into the horizontal position in which its weight no longer meets any resistance. Now that S becomes infinite when ϕ becomes equal to o means that a force acting horizontally towards the vertical wall would have to be infinitely great in order to prevent the rod from falling through the gap; in other words, as infinite forces are never found, there is no horizontal force that could produce that result. We must not be misled by the fact that in practice this result is often attained by squeezing bodies together in a horizontal direction; for this result is then due to the roughness of the surfaces with which the squeezing and the squeezed bodies come into contact, and to the compressibility of the latter which by slight alterations in its shape furnishes points of support which before were wanting.